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Supplementary material for "Bayesian Conway-Maxwell-Poisson regression models for over and under dispersed counts"

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S1. Conjugate prior for CMP_{μ} distributions

 CMP_{μ} distributions form two-parameter exponential families (Appendix 1.2, Huang 2017). We can therefore obtain the conjugate prior and posterior for the CMP_{μ} distribution by substituting $\lambda = \lambda(\mu, \nu)$ into equation (8) of Kadane *et. al.* (2006). Then, we can write the density function of the conjugate prior for the CMP_{μ} distribution in the form of

$$p(\mu,\nu) = \lambda(\mu,\nu)^{a-1} \exp(-b\nu) Z^{-c}(\lambda(\mu,\nu),\nu)\kappa(a,b,c), \qquad \mu > 0, \nu \ge 0,$$

where $\kappa(a, b, c)$ is a normalising constant given by

$$\kappa^{-1}(a,b,c) = \int_0^\infty \int_0^\infty \lambda(\mu,\nu)^{a-1} \exp(-b\nu) Z^{-c}(\lambda(\mu,\nu),\nu) d\mu \, d\nu,$$

and *a*, *b*, and *c* are hyperparameters restricted by (10) from Kadane *et. al.* (2006). The corresponding posterior is of the same form,

$$p(\mu,\nu|\mathbf{y}) = \lambda(\mu,\nu)^{a'-1} \exp\{-\nu b'\} Z^{-c'}(\lambda(\mu,\nu),\nu)\kappa(a',b',c')$$

with $a' = a + \sum_{i=1}^{n} y_i$, $b' = b + \sum_{i=1}^{n} \log(y_i!)$ and c' = c + n.

S2. Description of predictors for takeover bids dataset

The following description of explanatory variables are taken from Sáez-Castillo and Conde-Sánchez (2013):

Defensive actions taken by management of target firm: indicator variable for legal defense by lawsuit (leglrest), proposed changes in asset structure (rearest), proposed change in ownership structure (finrest) and management invitation for friendly third-party bid (whtknght).

- Firm-specific characteristics: bid price divided by price 14 working days before bid (bidprem), percentage of stock held by institutions (insthold), total book value of assets in billions of dollars (size) and book value squared (size²).
- Intervention by federal regulators: an indicator variable for Department of Justice intervention (regulatn).

S3. Class attendance and takeover bids data analysis examples using improper flat priors

The results of the class attendance and takeover bids data analysis examples using the improper flat priors, $p(\beta)$, $p(\nu) \propto 1$, are summarized in Figures 3 and 4 respectively. These results are very similar to the results based on the Normal and log-Normal pair of priors from the main text, suggesting that the framework is robust to the choice of prior, at least for problems with similar size and complexity to these two examples.

parameter	trace	lag 1	acf	density	summary
intercept				2.36 3.08	posterior mean: 2.72 95% credible interval: (2.36,3.08)
gender(M)				-0.44 0.01	posterior mean: -0.216 95% credible interval: (-0.44,0.01)
program(A)				-0.76 -0.11	posterior mean: -0.433 95% credible interval: (-0.76,-0.11)
program(V)				-1.61 -0.91	posterior mean: -1.261 95% credible interval: (-1.61,-0.91)
math				-0.01	posterior mean: -0.006 95% credible interval: (-0.01,0)
dispersion				0.07	posterior mean: 0.037 95% credible interval: (0.01,0.07)

Fig. 3 Summary of MH MCMC-based inference for parameters in the CMP_{μ} log-linear Bayesian regression model (3) fitted to the class attendance data using improper flat priors $p(\beta), p(\nu) \propto 1$.

parameter	trace	lag 1	acf	density	summary
intercept	Verydllyddyndyddinhau			0.16 1.89	posterior mean: 0.972 95% credible interval: (0.16,1.89)
leglrest	and and the state of the state			0.03 0.53	posterior mean: 0.265 95% credible interval: (0.03,0.53)
rearest	nyeren persita and			-0.5 0.11	posterior mean: -0.178 95% credible interval: (-0.5,0.11)
finrest	And many property and the second			-0.32 0.39	posterior mean: 0.042 95% credible interval: (-0.32,0.39)
whtknght	An product and an an an and a second			0.25 0.74	posterior mean: 0.49 95% credible interval: (0.25,0.74)
bidprem	19 wild yay water and the state			-1.35 -0.11	posterior mean: -0.688 95% credible interval: (-1.35,-0.11)
insthold	parter thank the second of the second			-1.03 0.33	posterior mean: -0.355 95% credible interval: (-1.03,0.33)
size1	Hereider Hillis wirder Hinderigen			0.09 0.27	posterior mean: 0.179 95% credible interval: (0.09,0.27)
size1 ²	AN ANY MARY MANAGER			-0.01 0	posterior mean: -0.008 95% credible interval: (-0.01,0)
regulatn	and meridian provide a second second		-l	-0.28 0.24	posterior mean: -0.027 95% credible interval: (-0.28,0.24)
dispersion	Hand and the state of the second			1.23 2.18	posterior mean: 1.706 95% credible interval: (1.23,2.18)

Fig. 4 Summary of MH MCMC-based inference for parameters in the CMP_{μ} log-linear Bayesian regression model (3) fitted to the takeover bids data using improper flat priors $p(\beta), p(\nu) \propto 1$.