Supplementary material for "Bayesian Conway-Maxwell-Poisson regression models for over and under dispersed counts"

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S1. Conjugate prior for CMP$_\mu$ distributions

CMP$_\mu$ distributions form two-parameter exponential families (Appendix 1.2, Huang 2017). We can therefore obtain the conjugate prior and posterior for the CMP$_\mu$ distribution by substituting $\lambda = \lambda(\mu, \nu)$ into equation (8) of Kadane *et. al.* (2006). Then, we can write the the density function of the conjugate prior for the CMP$_\mu$ distribution in the form of

$$p(\mu, \nu) = \lambda(\mu, \nu)^{a-1} \exp(-b\nu)\, Z^{-c}(\lambda(\mu, \nu), \nu)\kappa(a, b, c)\,, \qquad \mu > 0\,, \nu \geq 0\,,$$

where $\kappa(a, b, c)$ is a normalising constant given by

$$\kappa^{-1}(a, b, c) = \int_0^\infty \int_0^\infty \lambda(\mu, \nu)^{a-1} \exp(-b\nu) Z^{-c}(\lambda(\mu, \nu), \nu) d\mu\, d\nu,$$

and a, b, and c are hyperparameters restricted by (10) from Kadane *et. al.* (2006). The corresponding posterior is of the same form,

$$p(\mu, \nu |\, \boldsymbol{y}) = \lambda(\mu, \nu)^{a'-1} \exp\left\{-\nu b'\right\} Z^{-c'}(\lambda(\mu, \nu), \nu)\kappa(a', b', c')$$

with $a' = a + \sum_{i=1}^n y_i$, $b' = b + \sum_{i=1}^n \log(y_i!)$ and $c' = c + n$.

S2. Description of predictors for takeover bids dataset

The following description of explanatory variables are taken from Sáez-Castillo and Conde-Sánchez (2013):

- Defensive actions taken by management of target firm: indicator variable for legal defense by lawsuit (`leglrest`), proposed changes in asset structure (`rearest`), proposed change in ownership structure (`finrest`) and management invitation for friendly third-party bid (`whtknght`).

– Firm-specific characteristics: bid price divided by price 14 working days before bid (`bidprem`), percentage of stock held by institutions (`insthold`), total book value of assets in billions of dollars (`size`) and book value squared (`size`^{2}).
– Intervention by federal regulators: an indicator variable for Department of Justice intervention (`regulatn`).

S3. Class attendance and takeover bids data analysis examples using improper flat priors

The results of the class attendance and takeover bids data analysis examples using the improper flat priors, $p(\boldsymbol{\beta}), p(\nu) \propto 1$, are summarized in Figures 3 and 4 respectively. These results are very similar to the results based on the Normal and log-Normal pair of priors from the main text, suggesting that the framework is robust to the choice of prior, at least for problems with similar size and complexity to these two examples.

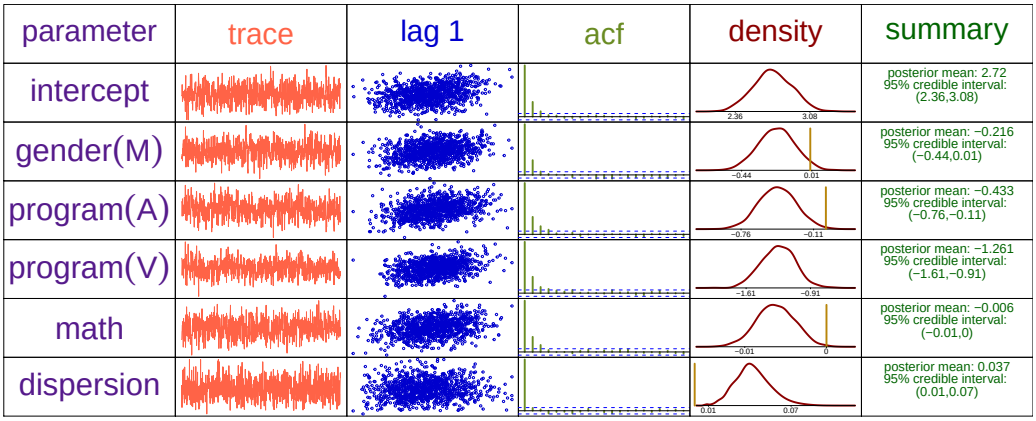

Fig. 3 Summary of MH MCMC-based inference for parameters in the CMP$_\mu$ log-linear Bayesian regression model (3) fitted to the class attendance data using improper flat priors $p(\boldsymbol{\beta}), p(\nu) \propto 1$.

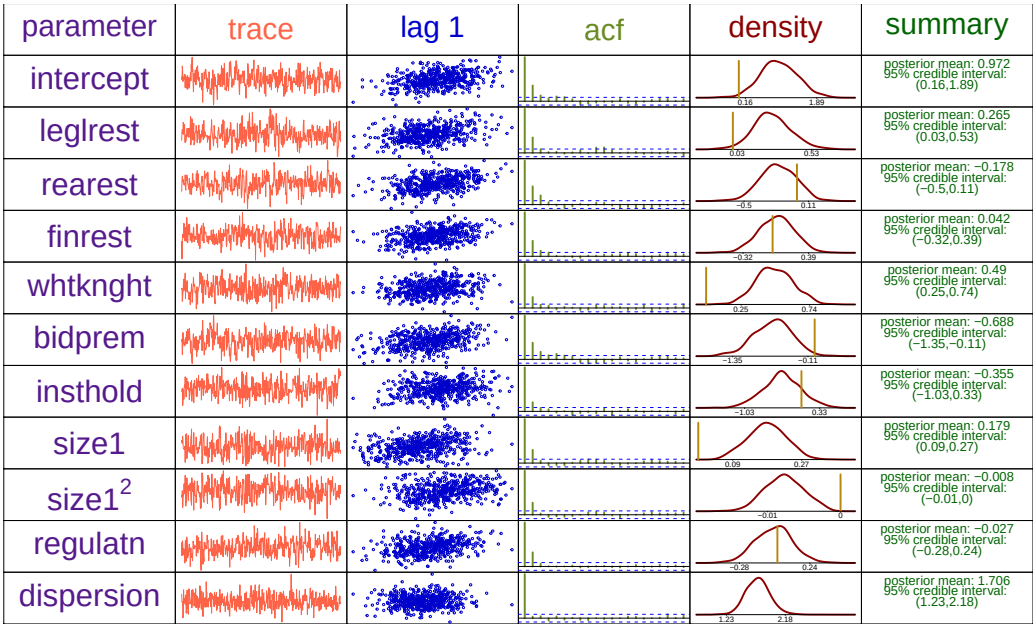

Fig. 4 Summary of MH MCMC-based inference for parameters in the CMP_μ log-linear Bayesian regression model (3) fitted to the takeover bids data using improper flat priors $p(\boldsymbol{\beta}), p(\nu) \propto 1$.