Supplementary material for Uyar, "Thermoelastic Analysis and Multi-Objective Optimal Design of Functionally Graded Flywheel for Energy Storage Systems", *Engineering Optimization*, 2019.

Appendix. Details of selected optimal designs

The design variables, calculated objective functions, maximum design speeds and the radius of gyration K_G of selected optimal designs in Figs. 9-11 are presented in Tables A1-A6.

The radius of gyration is calculated by the following equation:

$$K_G = \left[\int_{r_{in}}^{r_{out}} \rho(r) h(r) r^3 dr \Big/ \int_{r_{in}}^{r_{out}} \rho(r) h(r) r dr \right]^{1/2} \quad \text{(A1)}$$

Table S1. Selected optimal designs for h_{max}=10mm and ω_{design}=100,000rpm (Fig. 9a)

Optimal Design:	1	2	3	4	5	6	7	8	9
DV: h_1 [mm]	10	10	10	10	10	10	6.446	8.6148	7.1281
DV: h_2 [mm]	9.2106	8.5262	8.1307	9.1435	8.5111	8.1391	4.3096	5.4932	4.9370
DV: h_3 [mm]	7.9235	7.1815	7.3373	7.6969	7.0618	7.2877	2.5909	2.8351	2.8779
DV: h_4 [mm]	7.0176	6.7500	6.9205	6.6635	6.5568	6.9970	2.5668	2.7403	2.6403
DV: h_5 [mm]	6.3481	6.4641	6.3664	5.7553	5.7598	6.1961	2.5094	2.5570	2.5323
DV: h_6 [mm]	5.8074	5.7846	5.5602	4.7983	4.7978	5.4108	2.5025	2.5302	2.5007
DV: h_7 [mm]	4.7154	5.4357	4.8880	3.8713	3.7149	4.4451	2.5011	2.5076	2.5060
DV: h_8 [mm]	6.3776	8.2531	5.4364	3.0940	2.8124	3.5277	2.5016	2.5069	2.5002
DV: h_9 [mm]	9.7606	9.7551	9.4955	2.5460	2.5025	2.8289	2.5005	2.5135	2.5041
DV: h_{10} [mm]	9.8613	9.9178	9.8544	5.1327	5.7624	5.8296	2.5011	2.5027	2.5025
DV: h_{11} [mm]	9.9288	9.9208	9.9062	9.4786	9.2399	9.9817	2.5025	3.0848	2.5032
DV: h_{12} [mm]	9.9332	9.8611	9.7448	9.4541	7.7942	9.1543	2.5030	3.8466	2.5009
DV: MAT_{in}	3	3	3	3	3	3	1	1	1
DV: MAT_{out}	2	2	2	1	1	1	1	1	1
DV: m	9.3997	5.1162	10.852	2.0718	8.1949	1.538	-	-	-
DV: ω_{max} [rpm]	49931	17846	11263	66758	23278	14610	48003	18909	11929
OBJ: F_1 [kJ]	302.47	2112.1	5407	214.46	1506.7	3800.1	48.958	396.48	1050.4
OBJ: F_2 [kg]	2.5733	20.094	48.491	1.3206	10.253	25.56	0.494	3.6976	9.5034
K_G [mm]	90.453	239.87	391.21	79.787	218.10	348.73	85.487	227.01	366.10

Table S2. Selected optimal designs for h_{max}=10mm and ω_{design}=10,000rpm (Fig. 9b)

Optimal Design:	1	2	3	4	5	6	7	8	9
DV: h_1 [mm]	2.5858	4.3899	8.2128	2.5781	2.5437	5.9290	2.5300	2.5417	5.8951
DV: h_2 [mm]	2.7161	3.2029	6.7026	2.5524	2.5279	4.4531	2.5192	2.5165	4.0967
DV: h_3 [mm]	2.5554	2.6055	6.0446	2.5073	2.5201	3.2354	2.5037	2.5096	2.5092
DV: h_4 [mm]	3.3215	2.5192	5.8177	2.5076	2.5114	3.0695	2.5055	2.5053	2.5066
DV: h_5 [mm]	2.8660	4.8763	5.9729	2.5105	2.5041	2.9196	2.5027	2.5041	2.5034
DV: h_6 [mm]	8.5230	9.0348	5.9409	2.5133	2.5053	2.6607	2.5002	2.5009	2.5023
DV: h_7 [mm]	9.9080	9.8794	9.3731	2.5096	2.5197	2.5316	2.5002	2.5007	2.5016
DV: h_8 [mm]	9.9776	9.9760	9.8556	4.2920	4.3254	2.6479	2.5014	2.5011	2.5110
DV: h_9 [mm]	9.9858	9.9860	9.9789	9.4374	9.5663	5.1155	2.5018	2.5014	2.5149
DV: h_{10} [mm]	9.9966	9.9826	9.9659	9.9554	9.9268	9.7947	2.5023	2.5048	2.6080
DV: h_{11} [mm]	9.9899	9.9945	9.9439	9.9874	9.9794	9.7883	2.5094	2.5062	3.0329
DV: h_{12} [mm]	10	10	9.9737	10	9.9895	10	2.5130	2.5	3.3673
DV: MAT_{in}	2	2	3	1	1	3	1	1	1
DV: MAT_{out}	2	2	2	2	2	2	1	1	1
DV: m	0.1861	3.3195	1.3307	2.199	2.9251	12.423	-	-	-
DV: ω_{max} [rpm]	10000	10000	9861.2	10000	10000	9964.1	10000	10000	10000
OBJ: F_1 [kJ]	14.029	726.65	4911.7	8.9104	464.47	3039.5	1.9781	100.11	730.98
OBJ: F_2 [kg]	3.0468	23.441	60.169	1.5881	11.81	31.582	0.4875	3.5519	9.6745
K_G [mm]	91.641	237.77	385.85	101.16	267.82	418.95	86.026	226.72	371.21

Table S3. Selected optimal designs for h_{max}=55mm and ω_{design}=100,000rpm (Fig. 10a)

Optimal Design:	1	2	3	4	5	6	7	8	9
DV: h_1 [mm]	55	54.986	55	54.797	55	54.990	8.9217	17.016	21.321
DV: h_2 [mm]	51.119	46.837	46.207	50.921	46.604	45.042	5.2750	10.873	12.028
DV: h_3 [mm]	44.079	39.593	40.093	43.885	37.944	38.853	2.9022	4.1952	4.0093
DV: h_4 [mm]	39.199	37.196	38.524	37.691	33.928	36.996	2.8076	3.2306	3.2627
DV: h_5 [mm]	35.035	34.137	35.704	30.999	28.156	32.005	2.5385	2.8317	2.5801
DV: h_6 [mm]	31.733	30.44	32.388	22.582	21.308	25.343	2.5737	2.5673	2.6522
DV: h_7 [mm]	27.584	26.054	28.559	15.193	14.691	17.915	2.5144	2.5096	2.5144
DV: h_8 [mm]	38.880	32.945	34.033	9.3319	9.5402	11.607	2.5144	2.5160	2.5272
DV: h_9 [mm]	54.657	54.921	53.630	5.8006	6.5344	7.1593	2.5128	2.5144	2.5016
DV: h_{10} [mm]	52.943	51.193	53.359	5.0972	4.3137	5.9207	2.5048	2.5160	2.5176
DV: h_{11} [mm]	54.737	54.101	54.542	8.8112	13.525	14.089	2.5176	2.5369	2.5160
DV: h_{12} [mm]	54.846	52.792	53.571	12.863	23.725	23.138	2.5112	2.5881	2.5208
DV: MAT_{in}	3	3	3	3	3	3	1	1	1
DV: MAT_{out}	2	2	2	1	1	1	1	1	1
DV: m	5.0109	9.1263	5.8948	0.6542	2.308	0.9793	-	-	-
DV: ω_{max} [rpm]	49333	18380	11069	87145	30611	18553	48669	20474	12116
OBJ: F_1 [kJ]	1676.4	11354	30102	776.51	5307.1	13837	50.809	448.87	1093.4
OBJ: F_2 [kg]	14.556	101.38	277.76	4.2093	31.626	83.857	0.5053	3.7935	9.929
K_G [mm]	90.370	240.67	389.74	65.381	177.43	290.66	84.862	220.69	359.76

Table S4. Selected optimal designs for h_{max}=55mm and ω_{design}=10,000rpm (Fig. 10b)

Optimal Design:	1	2	3	4	5	6	7	8	9
DV: h_1 [mm]	2.5064	27.067	47.313	2.5	4.9578	29.904	2.5064	2.6250	6.2316
DV: h_2 [mm]	4.2352	18.466	36.121	2.5064	3.7145	19.955	2.5048	2.5545	4.2913
DV: h_3 [mm]	2.8605	12.020	33.253	2.5048	2.9775	11.391	2.5032	2.5128	2.6698
DV: h_4 [mm]	22.146	17.258	30.206	2.5080	2.8317	10.546	2.5048	2.5080	2.5561
DV: h_5 [mm]	38.141	12.250	35.634	2.5096	2.5160	9.9311	2.5016	2.5080	2.5304
DV: h_6 [mm]	51.323	43.546	27.786	2.5048	2.5481	8.9714	2.5016	2.5048	2.5144
DV: h_7 [mm]	54.968	54.946	52.686	2.5240	2.6202	7.9540	2.5016	2.5016	2.5080
DV: h_8 [mm]	54.800	54.550	51.515	3.2691	2.5497	6.8885	2.5048	2.5048	2.5320
DV: h_9 [mm]	54.946	54.885	54.261	21.735	17.128	10.240	2.5032	2.5048	2.5288
DV: h_{10} [mm]	54.955	54.973	54.872	54.417	51.208	34.576	2.5016	2.5401	2.9214
DV: h_{11} [mm]	54.933	54.886	54.801	53.484	54.349	54.883	2.5096	2.6875	6.5168
DV: h_{12} [mm]	55	54.756	54.375	54.591	52.454	51.560	2.5240	2.8301	9.8766
DV: MAT_{in}	2	2	3	1	1	3	1	1	1
DV: MAT_{out}	2	2	2	2	2	3	1	1	1
DV: m	-	-	2.6825	1.8677	2.3979	-	-	-	-
DV: ω_{max} [rpm]	10000	10000	9944.6	10000	10000	9923.7	10000	10000	10000
OBJ: F_1 [kJ]	78.647	3953.3	26464	33.939	1715.8	10830	1.9791	101.72	920.74
OBJ: F_2 [kg]	17.625	124.16	322.44	5.184	37.07	107.36	0.4876	3.5846	11.062
K_G [mm]	90.211	240.97	386.89	109.27	290.54	429.61	86.033	227.49	389.62

Table S5. Selected optimal designs for h_{max}=100mm and ω_{design}=100,000rpm (Fig. 11a)

Optimal Design:	1	2	3	4	5	6	7	8	9
DV: h_1 [mm]	99.988	100	100	99.997	100	99.967	29.887	23.231	54.614
DV: h_2 [mm]	92.656	86.783	83.578	91.576	86.506	83.605	15.851	14.399	28.307
DV: h_3 [mm]	80.632	74.455	72.607	75.684	75.657	70.465	3.5057	5.0887	3.8896
DV: h_4 [mm]	72.003	70.372	68.992	62.282	69.804	65.323	3.2855	3.3689	2.9047
DV: h_5 [mm]	65.034	65.424	66.049	48.407	59.146	54.138	2.6607	2.7410	2.5952
DV: h_6 [mm]	56.869	59.306	57.854	33.749	44.476	40.031	2.5298	2.5327	2.5387
DV: h_7 [mm]	57.351	58.173	48.050	20.919	29.277	25.596	2.5030	2.5179	2.5327
DV: h_8 [mm]	90.874	91.787	53.727	11.861	16.310	14.539	2.5327	2.5030	2.5506
DV: h_9 [mm]	98.911	98.414	92.778	8.0524	10.495	8.2309	2.5089	2.5149	2.5446
DV: h_{10} [mm]	98.000	99.768	98.128	4.6067	6.7967	5.3268	2.5119	2.5149	2.5149
DV: h_{11} [mm]	99.289	96.239	95.932	5.9754	10.763	11.674	2.7083	2.5357	4.2734
DV: h_{12} [mm]	99.988	92.838	92.502	13.643	18.976	20.359	2.9553	2.5803	6.5408
DV: MAT_{in}	3	3	3	3	3	3	1	1	1
DV: MAT_{out}	2	2	2	1	1	1	1	1	1
DV: m	81.380	7.5131	12.374	1.0321	0.7176	1.0672	-	-	-
DV: ω_{max} [rpm]	48619	17645	11276	94144	32599	20434	52646	20252	11762
OBJ: F_1 [kJ]	3083.2	21380	53892	1277.5	8577.3	21169	60.464	438.54	1162.1
OBJ: F_2 [kg]	27.729	206.84	481.03	6.7247	51.042	125.73	0.5491	3.8440	10.791
K_G [mm]	89.968	238.64	389.59	61.273	166.02	265.84	82.211	219.07	366.51

Table S6. Selected optimal designs for h_{max}=100mm and ω_{design}=10,000rpm (Fig. 11b)

Optimal Design:	1	2	3	4	5	6	7	8	9
DV: h_1 [mm]	2.5149	57.583	82.733	2.5030	5.2881	70.628	2.5208	4.5115	7.5079
DV: h_2 [mm]	2.5208	51.528	67.189	2.5149	4.3954	42.039	2.5060	3.4105	5.0679
DV: h_3 [mm]	2.6726	51.121	52.763	2.5119	3.7944	17.062	2.5089	2.5119	2.7321
DV: h_4 [mm]	6.6747	81.807	50.948	2.5119	3.2379	14.935	2.5060	2.5149	2.5417
DV: h_5 [mm]	39.031	83.203	57.134	2.5179	2.8630	13.495	2.5030	2.5119	2.5030
DV: h_6 [mm]	91.442	97.504	68.349	2.5327	2.7618	12.664	2.5030	2.5030	2.5089
DV: h_7 [mm]	98.137	98.134	67.174	2.6875	2.6369	11.769	2.5030	2.5060	2.5179
DV: h_8 [mm]	99.890	99.316	98.569	2.5060	2.6041	9.6205	2.5030	2.5089	2.5179
DV: h_9 [mm]	99.441	99.967	92.353	33.425	28.661	8.0494	2.5149	2.5060	2.5268
DV: h_{10} [mm]	99.935	99.631	99.286	96.810	90.948	40.798	2.5179	2.5030	2.5714
DV: h_{11} [mm]	99.836	99.679	99.807	98.813	99.304	93.359	2.6607	4.2734	2.7053
DV: h_{12} [mm]	100	100	99.997	100	97.727	95.534	2.8511	6.3414	2.7976
DV: MAT_{in}	2	2	3	1	1	3	1	1	1
DV: MAT_{out}	2	2	2	3	2	2	1	1	1
DV: m	-	-	2.6645	5.6087	6.206	35.405	-	-	-
DV: ω_{max} [rpm]	10000	10000	9905.8	10000	9990.4	9925.1	10000	10000	10000
OBJ: F_1 [kJ]	140.51	7346.9	46347	50.734	2739.8	15105	2.0023	112.4	714.91
OBJ: F_2 [kg]	30.096	252.10	562.34	7.5077	57.608	147.95	0.4909	3.7909	9.6105
K_G [mm]	92.276	230.54	388.63	111.02	294.51	432.14	86.248	232.54	368.33