# Online Supplement for <br> The Rack Orientation and Curvature Problem for Retailers Bradley Guthrie and Pratik J. Parikh 

## Appendix A. Models for Space and Aspect Ratio

We estimate the floor space $a$ of a given layout by the rectangular area that bounds the perimeter of the layout. Let $L e$ and $W i$ be the length and width of this rectangular bounded area. To find $L e$ and $W i$, we must determine the coordinates of the edges for this rectangle along the width ( $x$-direction) and the length ( $y$ direction). Let these be represented as ( $x^{\min }, x^{\max }$ ), and ( $y^{\min }, y^{\max }$ ), respectively, such that $W i=x^{\max }$ $-x^{\min }$ and $L e=y^{\max }-y^{\min }$. We first, however, must derive $x_{n f}^{\min }, x_{n f}^{\max }, y_{n f}^{\min }$, and $y_{n f}^{\max }$, which are the edge coordinates for each face $f$ on rack $n$. Further, let $x_{n f}^{\text {mid }}$ and $y_{n f}^{\text {mid }}$ be the coordinates of the center of curvature for face $f$ on rack $n ; r_{f}$ notates the radius for face $f$. Depending on the combination of $\theta$ and $\alpha$ for a specific layout, these extreme points may fall on faces F2, F3, or F4 (i.e., outer face or either endcap). We propose an exact procedure to determine $L e$ and $W i$, as illustrated in Figure A1, which help in estimating the floor space $(a=L e \times W i)$ and the aspect ratio $\left(r=\frac{L e}{W i}\right)$. Figure A2 subsequently illustrates three example layouts with their respective dimensions (Le,Wi) and locations of values $x^{\max }, x^{\min }, y^{\max }$, and $y^{\min }$.

$$
\begin{align*}
& \text { (1) } \\
& \text { (2) } n=1 \text { or } n=N \\
& \text { (3) }  \tag{2}\\
& \text { For } f \in\{\mathrm{~F} 2, \mathrm{~F} 3, \mathrm{~F} 4\}  \tag{3}\\
& \text { (4) }  \tag{5}\\
& \text { Compute extreme points }  \tag{6}\\
& \text { (5) }  \tag{7}\\
& \text { (6) } \\
& x_{n f}^{\text {max }}=x_{n f}^{\text {mid }}+r_{f} \\
& \text { (7) }
\end{align*} x_{n f}^{\text {min }}=x_{n f}^{\text {mid }}-r_{f},
$$

Compute extreme points of face $f$ in $x$ and $y$ directions

Determine if points fall on the physical part of rack (per Table 2).
Compare the feasible extreme points to determine $x^{\max }, x^{\min }, y^{\max }$, and $y^{\text {min }}$

$$
\begin{align*}
& x^{\max }=\max \left\{x_{n f}^{\max } \forall f, n\right\}, x^{\min }=\min \left\{x_{n f}^{\min } \forall f, n\right\}  \tag{10}\\
& y^{\max }=\max \left\{y_{n f}^{\max } \forall f, n\right\}, y^{\min }=\min \left\{y_{n f}^{\min } \forall f, n\right\}
\end{align*}
$$

Compute $\boldsymbol{L e}=y^{\max }-y^{\min }$ and $\boldsymbol{W i}=x^{\max }-x^{\min }$
Figure A1. Exact procedure to estimate $\boldsymbol{L e}$ and $\boldsymbol{W} \boldsymbol{i}$ of the layout


Figure A2. Dimensions of $L e$ and $W i$ for example layouts with $N=3$
Using the above procedure, we can compute the floor space for any combination of $\theta$ and $\alpha$. Figure A3 presents the floor space (right side of the layout only, both sides are identical) for 133 combinations of $\theta$ and $\alpha$ with values of $\theta$ ranging from $0^{\circ}$ to $180^{\circ}$ in steps of $10^{\circ}$ and values of $\alpha$ ranging from $0^{\circ}$ to $180^{\circ}$ in steps of $30^{\circ}$. Design parameters are identical to those in Table 5. For this specific configuration, the values of $\theta$ that provide maximum floor space are in the ranges $\left(\theta=40^{\circ}, \theta=50^{\circ}\right)$ and $\left(\theta=130^{\circ}, \theta=140^{\circ}\right)$; this is consistent for all values of $\alpha$. Minimum floor space meanwhile occurs at both $\theta=0^{\circ}$ and $\theta=180^{\circ}$ (again, for all values of $\alpha$ ). The value of $\alpha$ however that maximizes floor space changes with respect to $\theta$. When $\theta=0^{\circ}$, $90^{\circ}$, and $180^{\circ}, \alpha=150^{\circ}$ provides maximum floor space; when $\theta=40^{\circ}$ or $140^{\circ}, \alpha=90^{\circ}$ provides maximum floor space. A curvature of $\alpha=0^{\circ}$ meanwhile results in the minimum floor space for all values of $\theta$. While it would seem intuitive that space would always increase as $\alpha$ increases from $0^{\circ}$ to $180^{\circ}$, because our layouts maintain a fixed shelf-area for any value of $\alpha$, the chord length (i.e., aisle length) rapidly decreases as $\alpha$ approaches $180^{\circ}$. This reduces $W i$, thereby, lowering the total floor space.


Figure A3. Change in floor space $\left(\mathrm{ft}^{2}\right)$ for various rack orientations and curvatures

Aspect ratio $(r)$ is illustrated in Figure A4. While $\theta=0^{\circ}$ and $\theta=180^{\circ}$ (both at $\alpha=0^{\circ}$ ) required the lowest space, they both resulted in a long and skinny section; $r=29.77$. The rack layout of $\theta=90^{\circ}$ and $\alpha=0^{\circ}$ meanwhile resulted in layouts that were short and wide; $r=0.7$. Notice here for $\theta$ values $0^{\circ}$ and $10^{\circ}$ (alternately, $180^{\circ}$ and $170^{\circ}$ ) the aspect ratio decreases as $\alpha$ increases to $180^{\circ}$. This trend, however, steadily shifts to increasing for an increasing $\alpha$ as $\theta$ approaches $90^{\circ}$. This phenomenon occurs because of the nature of curved racks; increase either Wi or Le values relative to $\alpha=0^{\circ}$. That is, for $\theta$ values closer to $0^{\circ}$ or $180^{\circ}$, the curved racks cause an increase in $W i$, thereby lowering the aspect ratio (as $L e$ is much higher). In contrast, for $\theta$ values closer to $90^{\circ}$, the curved racks cause $L e$ to increase, subsequently increasing the aspect ratio (as $W i$ is much higher in this case).


Figure A4. Change in Aspect Ratio with varying values of $\theta$ and $\alpha$

## Appendix B. Example for Deriving $\boldsymbol{v}_{\boldsymbol{l}}$

To describe our procedure for deriving $v_{l y}$ (i.e., the probability that a shopper's focal point will fall on location $l$ at step $y$ ), we present step-by-step mathematical expressions considering a single step $y$ and a single location $l$; see Figure B1. From Section 3.2.2, this procedure is divided into 3 stages (i.e., i, ii, and iii). For (i), we first compute the angular coordinates (i.e., $\Psi_{l i}, \Gamma_{l i}$ ) from the shopper to each vertex $i$ of each location $l$. Figure B2 (from Section 3.2.2) details all the coordinates and angles we use to derive these coordinates.


Figure B1. View of shopper and location $l$
Figure B2. Determining angular coordinates of location $l$
The following are values corresponding to position $y$ of shopper and location $l$ loation $l$ with $1-4$ as indices for each vertex):
$S_{E}=5 \mathrm{ft}, S_{x}=0 \mathrm{ft}, S_{y}=65 \mathrm{ft}, z_{1,1}=7 \mathrm{ft}, z_{1,2}=7 \mathrm{ft}, z_{1,3}=6 \mathrm{ft}, z_{1,4}=6 \mathrm{ft}, L h=1 \mathrm{ft}$
$x_{1,1}=6.77 \mathrm{ft}, y_{1,1}=74.42 \mathrm{ft}, x_{1,4}=6.77 \mathrm{ft}, y_{1,4}=74.42 \mathrm{ft}, x_{1,2}=7.53 \mathrm{ft}, y_{1,2}=75.15 \mathrm{ft}, x_{1,3}=7.53 \mathrm{ft}, y_{1,3}=75.15 \mathrm{ft}$
Using these, we can determine the unknown distances and angles from the shopper to each vertex of location $l$ using equations presented in Section 3.2.2:

$$
\begin{aligned}
& B_{l i}^{\mathrm{S}_{\mathrm{E}}}=\sqrt{\left(y_{l i}-S_{y}\right)^{2}+\left(x_{l i}-S_{x}\right)^{2}} ; \text { i.e., } B_{1,1}^{\mathrm{S}_{\mathrm{E}}}=\sqrt{(74.42-65)^{2}+(6.77-0)^{2}}=11.60 \mathrm{ft} \\
& B_{l i}=\sqrt{\left(B_{l i}^{S_{E}}\right)^{2}+\left(\left|z_{l i}-S_{E}\right|^{*} L h\right)^{2}} \text {; i.e., } B_{1,1}=\sqrt{(11.60)^{2}+(|7-5| * 1)^{2}}=11.77 \mathrm{ft} \\
& \Psi_{l i}=\sin ^{-1}\left(\frac{x_{l i}-S_{x}}{B_{l i}}\right) ; \text { i.e., } \Psi_{1,1}=\sin ^{-1}\left(\frac{6.77-0}{11.60}\right)=35.69^{\circ} \\
& \Gamma_{l i}=\sin ^{-1}\left(\frac{\left|z_{l i}-S_{E}\right|^{*} L h}{B_{l i}}\right) ; \text { i.e., } \Gamma_{1,1}=\sin ^{-1}\left(\frac{|7-5|^{* 1}}{11.77}\right)=9.78^{\circ}
\end{aligned}
$$

Similarly, using the above equations we can also find:

| $B_{1,2}^{S_{E}}=12.64 \mathrm{ft}$ | $B_{1,3}^{S_{E}}=12.64 \mathrm{ft}$ | $B_{1,4}^{S_{E}}=11.60 \mathrm{ft}$ |
| :--- | :--- | :--- |
| $B_{1,2}=12.79 \mathrm{ft}$ | $B_{1,3}=12.68 \mathrm{ft}$ | $B_{1,4}=11.64 \mathrm{ft}$ |
| $\Psi_{1,2}=36.57^{\circ}$ | $\Psi_{1,3}=36.57^{\circ}$ | $\Psi_{1,4}=35.69^{\circ}$ |
| $\Gamma_{1,2}=8.99^{\circ}$ | $\Gamma_{1,3}=4.52^{\circ}$ | $\Gamma_{1,4}=4.93^{\circ}$ |

For (ii), recall from Section 3.2.2 that the second step to compute $v_{l y}$ is to generate the probability distribution for shopper focal point position (i.e., $\Xi_{\Omega_{H}, \Omega_{V}}$ ). Table B1 provides an outline of the data we collected from 18 participants. For each participant, and at time steps of 0.016 seconds (at 60 Hz sampling rate), we recorded the horizontal $\left(\Omega_{H}\right)$ and vertical $\left(\Omega_{V}\right)$ angles of their head position.

Table B1. Outline of participant data

| Participant | Time (s) | $\Omega_{H}$ | $\Omega_{v}$ |
| :---: | :---: | :---: | :---: |
| 1 | 0 | 1.34 | 4.32 |
| 1 | 0.016 | 2.56 | 3.44 |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |

We then computed the frequency of $\Omega_{H}$ and $\Omega_{V}$ that fell in intervals of $\zeta_{H}=\zeta_{V}=0.25^{\circ}$ (i.e., $F_{\Omega_{H}, \Omega_{V}}$ ); for this example, we use intervals of $\zeta_{H}=\zeta_{V}=1^{\circ}$. Figure B3 shows $F_{\Omega_{H}, \Omega_{V}}$ for ranges near the values of $\Psi_{\mathrm{li}}$ and $\Gamma_{\mathrm{li}}$ for location 1 in this example. Figure B4 meanwhile shows the result of our smoothing process, which accounts for eye-movement of $15^{\circ}$ surrounding each bin. Finally, we compute the probability of a shopper's focal point falling within each interval as $\mathrm{p}\left(\Xi_{\Omega_{H}, \Omega_{V}}\right)=\frac{\mathrm{F}_{\Omega_{H}}, \Omega_{V}}{\sum \mathrm{~F}_{\Omega_{H}}, \Omega_{V}}$; see Figure B5.


Figure B3. Raw data, $F_{\Omega_{H}, \Omega_{V}}$

|  | $\Omega_{H}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $F_{\Omega_{H}, \Omega_{V}}$ | $34^{\circ}$ | $35^{\circ}$ | $36^{\circ}$ | $37^{\circ}$ |  | $3^{\circ} 8^{\circ}$.

Figure B4. Smooth data with eye movement


Figure B5. Probability mass function of shopper's eye fixation, $\Xi_{\Omega_{H}, \Omega_{V}}$

The final step (iii) for deriving $v_{l y}$ is to sum all of the intervals in $\Xi_{\Omega_{H}, \Omega_{V}}$ that fall over the effective area of location l. Figure B6 illustrates this process by showing the effective area of location $l$ broken down in a grid with intervals of $\zeta=1^{\circ}$ as used in deriving $\Xi_{\Omega_{H}, \Omega_{V}}$ (see Figure B5). We find these intervals by first computing the outer limits of both vertical and horizontal angles (i.e., $\Psi_{l i}$ and $\Gamma_{l i}$ ) to the vertices $i$ of location $l$. For the location $l$ shown in Figure B6, we estimate these outer limits as follows:


Figure B6. Overlapping of $\Xi_{\Omega_{H}, \Omega_{V}}$ over location $l$
$\zeta_{H}\left\lfloor\frac{\min \left\{\Psi_{1,1}, \Psi_{1,4}\right\}}{\zeta_{H}}\right\rfloor \leq \Omega_{H} \leq \zeta_{H}\left\lceil\frac{\max \left\{\Psi_{1,2}, \Psi_{1,3}\right\}}{\zeta_{H}}\right\rceil$ and $\zeta_{V}\left\lfloor\frac{\min \left\{\Gamma_{1,3,}, \Gamma_{1,4}\right\}}{\zeta_{V}}\right] \leq \Omega_{V} \leq \zeta_{V}\left[\frac{\max \left\{\Gamma_{1,1}, \Gamma_{1,2}\right\}}{\zeta_{V}}\right\rceil$; i.e., $1\left[\frac{\min \left\{35.69^{\circ}, 35.69^{\circ}\right\}}{1}\right] \leq \Omega_{H} \leq 1\left\lceil\frac{\max \left\{36.57^{\circ}, 36.57^{\circ}\right\}}{1}\right\rceil$ and $1\left[\frac{\min \left\{4.53^{\circ}, 4.93^{\circ}\right\}}{1}\right] \leq \Omega_{V} \leq 1\left\lceil\frac{\max \left\{9.78^{\circ}, 8.99^{\circ}\right\}}{1}\right\rceil ;$
i.e., $35^{\circ} \leq \Omega_{H} \leq 37^{\circ}$ and $4^{\circ} \leq \Omega_{H} \leq 9^{\circ}$.

Following this, we then sum all intervals in $\Xi_{\Omega_{H}, \Omega_{V}}$ that fall within both of the above two ranges. These ranges are surrounded by the dark border in Figure B7. Thus,
$v_{l y}=E_{l y} * \sum_{\Omega_{H}, \Omega_{V}}\left(E_{\Omega_{H}, \Omega_{V}}\right)$ for $35^{\circ} \leq \Omega_{H} \leq 37^{\circ}$ and $4^{\circ} \leq \Omega_{H} \leq 9^{\circ}$

$$
\begin{aligned}
= & 1 \times(0.00006+0.00006+0.00006+0.00008+0.00008+0.00008+0.00011+0.00011+0.00011 \\
& +0.00015+0.00015+0.00015+0.00020+0.00021+0.00021+0.00027+0.00027+0.00027+ \\
& 0.00033+0.00033+0.00032)
\end{aligned}
$$

## i.e., $v_{l y}=0.00358$

|  |  | $\Omega_{H}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Xi_{\Omega_{H}, \Omega_{V}}$ | $34^{\circ}$ | $35^{\circ}$ | $36^{\circ}$ | $37^{\circ}$ | $38^{\circ}$ |
| $\Omega_{V}$ | $10^{\circ}$ | 0.005\% | 0.006\% | 0.006\% | 0.006\% | 0.006\% |
|  | $9^{\circ}$ | 0.007\% | 0.008\% | 0.008\% | 0.008\% | 0.008\% |
|  | $8^{\circ}$ | 0.010\% | 0.011\% | 0.011\% | 0.011\% | 0.011\% |
|  | $7^{\circ}$ | 0.015\% | 0.015\% | 0.015\% | 0.015\% | 0.015\% |
|  | $6^{\circ}$ | 0.020\% | 0.020\% | 0.021\% | 0.021\% | 0.020\% |
|  | $5^{\circ}$ | 0.027\% | 0.027\% | 0.027\% | 0.027\% | 0.026\% |
|  | $4^{\circ}$ | 0.033\% | 0.033\% | 0.033\% | 0.032\% | 0.032\% |

Figure B7. Combinations of $\Omega_{H}$ and $\Omega_{V}$ that fall over location $l$

## Appendix C. Product Category Assortment Data

Table C1 contains product data from a Midwest Retailer. Demand is in terms of units. Profits are categorized by category (i.e., low, medium high). Similarly, impulse rates are categorized by category; actual rates used in experiments are also shown. Allocated area is the derived allocated shelf space of that product category at the Midwest retailer, while $1 \mathrm{ft} \times 1 \mathrm{ft}$ facings are the number of locations used in our experimental study. Further, our assigned aisle and side of the main aisle are given for each product category.

Table C1. Data from Midwest Retailer

| Category | Product Category | Monthly Demand | Profit Category | Impulse Category | Impulse <br> Rate | Allocated Area | 1*1 <br> Facings | Aisle | Side |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Gravy Mix | 6449 | M | M | 0.33 | 23.86 | 21 | 1 | Right |
| 2 | Kraft Spreads | 3644 | M | M | 0.29 | 20.08 | 14 | 1 | Right |
| 3 | Italian Supplies | 404 | M | M | 0.22 | 7.02 | 7 | 1 | Right |
| 4 | Chili | 8393 | L | H | 0.41 | 29.46 | 14 | 1 | Right |
| 5 | Tuna | 12379 | L | M | 0.30 | 37.94 | 35 | 1 | Right |
| 6 | Pickles | 5967 | M | M | 0.21 | 75.63 | 56 | 1 | Right |
| 7 | Japanese Food | 2698 | L | M | 0.24 | 41.82 | 42 | 1 | Right |
| 8 | Croutons | 2778 | M | L | 0.09 | 35.65 | 28 | 1 | Right |
| 9 | Dressing | 1823 | L | M | 0.17 | 23.70 | 7 | 1 | Right |
| 10 | Condiments | 10425 | L | M | 0.32 | 76.94 | 63 | 1 | Right |
| 11 | Macaroni | 11237 | L | L | 0.03 | 70.43 | 56 | 1 | Right |
| 12 | Gatorade | 5248 | M | M | 0.23 | 86.02 | 42 | 2 | Right |
| 13 | Pizza Supplies | 936 | L | M | 0.32 | 10.59 | 14 | 2 | Right |
| 14 | Baking/Chocolate | 4480 | M | M | 0.28 | 52.25 | 98 | 2 | Right |
| 15 | Pasta Sauce | 8821 | L | M | 0.30 | 73.26 | 84 | 2 | Right |
| 16 | Jell-O | 2775 | L | M | 0.27 | 25.42 | 14 | 2 | Right |
| 17 | Canned Fruit | 4607 | L | H | 0.47 | 27.66 | 21 | 2 | Right |
| 18 | Tonic Water | 7944 | M | L | 0.15 | 113.66 | 112 | 2 | Right |
| 19 | Baking Supplies | 946 | L | M | 0.32 | 10.76 | 7 | 2 | Right |
| 20 | Rice | 7519 | M | L | 0.07 | 58.67 | 56 | 2 | Right |
| 21 | Oil | 2158 | L | L | 0.08 | 24.56 | 7 | 2 | Right |
| 22 | Seasonings | 5267 | M | L | 0.04 | 25.67 | 21 | 2 | Right |
| 23 | Seasonings/Spices | 2145 | L | L | 0.07 | 5.22 | 7 | 2 | Right |
| 24 | Brita Water | 3291 | M | L | 0.03 | 25.34 | 14 | 2 | Right |
| 25 | Beans | 8456 | L | M | 0.31 | 40.83 | 35 | 2 | Right |
| 26 | Canned Vegetables | 14262 | L | M | 0.32 | 52.51 | 84 | 2 | Right |
| 27 | Tomato Sauce | 12676 | L | L | 0.08 | 18.87 | 14 | 2 | Right |
| 28 | Sugar | 3430 | L | L | 0.02 | 121.09 | 56 | 2 | Right |
| 29 | Chocolate Syrup | 1555 | M | H | 0.49 | 25.75 | 56 | 3 | Right |
| 30 | Dried Fruit | 2069 | M | H | 0.42 | 25.04 | 28 | 3 | Right |
| 31 | Coffee | 10344 | L | M | 0.25 | 104.36 | 126 | 3 | Right |
| 32 | Peanut Butter | 8931 | M | L | 0.15 | 54.21 | 56 | 3 | Right |
| 33 | Health Bars | 313 | L | M | 0.30 | 3.63 | 7 | 3 | Right |
| 34 | Cereal | 19257 | L | M | 0.30 | 260.00 | 252 | 3 | Right |
| 35 | Cereal Bars | 10049 | L | M | 0.29 | 51.03 | 84 | 3 | Right |
| 36 | Oatmeal | 3627 | L | L | 0.14 | 48.72 | 56 | 3 | Right |
| 37 | Filter | 628 | L | L | 0.02 | 10.19 | 7 | 3 | Right |
| 38 | Tea Leaves | 1739 | L | L | 0.01 | 27.95 | 14 | 3 | Right |
| 39 | Nuts | 7631 | M | H | 0.47 | 79.71 | 133 | 4 | Right |
| 40 | Air fresheners | 2834 | M | H | 0.46 | 32.48 | 21 | 4 | Right |
| 41 | Pretzels/Chips | 6426 | L | H | 0.49 | 41.24 | 133 | 4 | Right |
| 42 | Popcorn | 3757 | L | H | 0.50 | 132.76 | 56 | 4 | Right |
| 43 | Cleaning Wipes | 5555 | M | M | 0.28 | 80.66 | 56 | 1 | Left |
| 44 | Seeds | 74 | H | L | 0.10 | 1.39 | 14 | 1 | Left |
| 45 | Dish Soap | 4012 | M | L | 0.11 | 41.46 | 28 | 1 | Left |
| 46 | Vinegar | 1119 | L | L | 0.11 | 21.21 | 7 | 1 | Left |
| 47 | Toilet Paper | 5614 | L | L | 0.07 | 110.49 | 168 | 1 | Left |
| 48 | Juice | 12328 | L | M | 0.23 | 74.16 | 56 | 1 | Left |
| 49 | Rice Snacks | 1188 | L | L | 0.06 | 36.37 | 14 | 1 | Left |
| 50 | Lotion | 2187 | H | M | 0.32 | 45.63 | 56 | 2 | Left |
| 51 | Insect Repellent | 47 | M | M | 0.27 | 7.89 | 7 | 2 | Left |


| 52 | Hardware | 262 | M | M | 0.23 | 195.93 | 224 | 2 | Left |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 53 | Cat Supplies | 1224 | M | L | 0.06 | 97.62 | 126 | 2 | Left |
| 54 | Stain Remover | 1423 | M | L | 0.05 | 35.74 | 21 | 2 | Left |
| 55 | Laundry Soap | 4793 | M | L | 0.05 | 109.79 | 126 | 2 | Left |
| 56 | Plastic Utensils | 687 | L | L | 0.12 | 2.03 | 7 | 2 | Left |
| 57 | Napkins | 1956 | L | L | 0.08 | 50.62 | 56 | 2 | Left |
| 58 | Cups | 291 | M | L | 0.01 | 14.59 | 7 | 2 | Left |
| 59 | Dyer Sheets | 1723 | L | L | 0.01 | 38.68 | 56 | 2 | Left |
| 60 | Pet Food | 3641 | M | M | 0.34 | 22.95 | 56 | 3 | Left |
| 61 | Meal Replacements | 713 | H | L | 0.14 | 21.91 | 7 | 3 | Left |
| 62 | Dog Supplies | 541 | L | M | 0.30 | 25.39 | 21 | 3 | Left |
| 63 | Deodorant | 3009 | L | M | 0.26 | 34.44 | 56 | 3 | Left |
| 64 | Cat Food | 9615 | M | L | 0.12 | 66.22 | 77 | 3 | Left |
| 65 | Shaving Gel | 1626 | M | L | 0.06 | 25.85 | 14 | 3 | Left |
| 66 | Diapers | 1153 | L | L | 0.10 | 176.13 | 196 | 3 | Left |
| 67 | Baby Supplies | 571 | L | L | 0.10 | 8.52 | 7 | 3 | Left |
| 68 | Shampoo | 320 | M | L | 0.05 | 10.23 | 21 | 3 | Left |
| 69 | Dog Food | 1426 | M | L | 0.04 | 113.06 | 168 | 3 | Left |
| 70 | Hair Supplies | 283 | L | L | 0.03 | 7.11 | 7 | 3 | Left |
| 71 | Soap | 6404 | L | L | 0.07 | 72.27 | 56 | 3 | Left |
| 72 | Nutrition Bars | 1204 | H | M | 0.28 | 21.45 | 14 | 4 | Left |
| 73 | Vitamins | 1938 | H | M | 0.26 | 17.06 | 14 | 4 | Left |
| 74 | Toothpaste | 7025 | H | L | 0.07 | 68.18 | 63 | 4 | Left |
| 75 | Drugs | 1192 | H | L | 0.07 | 15.58 | 14 | 4 | Left |
| 76 | Eye Care | 604 | H | L | 0.07 | 11.31 | 7 | 4 | Left |
| 77 | Ointments | 1462 | M | L | 0.09 | 25.38 | 21 | 4 | Left |
| 78 | Personal Products | 163 | H | L | 0.04 | 4.40 | 7 | 4 | Left |
| 79 | Medicine | 135 | M | L | 0.04 | 0.89 | 7 | 4 | Left |
| 80 | Cough Drops | 4162 | H | L | 0.02 | 27.60 | 21 | 4 | Left |
| 81 | Antacid | 1286 | H | L | 0.02 | 16.67 | 14 | 4 | Left |
| 82 | Feminine Products | 2865 | L | L | 0.04 | 131.48 | 105 | 4 | Left |
| 83 | Pain Medication | 2158 | H | L | 0.01 | 19.30 | 14 | 4 | Left |
| 84 | Boost Drinks | 639 | H | L | 0.01 | 40.60 | 42 | 4 | Left |

## Appendix D. Comparison of Impulse Profit Estimates from our Model with Estimates in the Literature

We present an approach to evaluate how the impulse profit estimates from our model (based on our sample dataset) compare to what has been reported in the literature. First, we show the steps of computing a range of impulse profit per shopper based on values found in literature and products used in our dataset. We then compare this range to low and high estimates from our experiments using a $\left\{90^{\circ}, 0^{\circ}\right\}$ layout as a baseline. Finally, we contextualize the potential improvements in marginal impulse profit from varying $\theta$ and $\alpha$ from a $\left\{90^{\circ}, 0^{\circ}\right\}$ layout.

From literature, the average shopper basket size at a typical grocery store ranges from \$40-\$60 (Stilley et al., 2010; Hui et al., 2013). We multiply each of these values by low and high estimates of impulse revenue as a $\%$ of total revenue; we use $\pm 10 \%$ of the average values found in the literature (Hui et al., 2013). These result in an estimated impulse revenue of \$12-\$30 (see Table D1). To then estimate impulse profit (the focus of our study), we first sort the profit margin among the actual products we had access to from a retailer and compute the $1^{\text {st }}$ and $3^{\text {rd }}$ quantiles ( $20 \%$ and $40 \%$, see table). Multiplying these by our estimated impulse revenue range results in an estimated impulse profit of $\$ 2.40-\$ 12.00$ per shopper.

Next, we derive the impulse profit per shopper resulting from our model. For this, we consider a baseline layout of $\left(90^{\circ}, 0^{\circ}\right)$ racks, using the Demand Ordering Rule and Impulse Ordering Rule (both using the Distance Location Rule) as our low and high estimates respectively. Table 6 in the manuscript provides the objective value (marginal profit, after discounting for space). We add the cost of floor space back to this objective value to derive the underlying impulse profit per shopper.

However, we must also consider that our layout represents only a section of a store. The product categories used in our experiments is a subset (see Appendix C) of the complete list of product categories obtained from a Midwest retailer. The number of allocations assigned to each product category is taken straight from this dataset, in addition to the height and width of each facing on the shelf. Thus, we can calculate the allocated shelf area, and then derive the number of 1 x 1 ft locations assigned to each product category (we round to the nearest 7 to create whole columns). That is, the product categories used in our experiments were pre-derived to fill a 6 -rack layout ( 3 racks on each side of the main shopper pathway), each with a perimeter of 110 ft and a height of 7 ft . Based on the allocated area for all product categories found in the complete dataset, the shelf space of product categories used in our experiments constitutes approximately $50 \%$. The complete list of product categories from the Midwest retailer, however, did not include "fresh" items usually found in grocery stores, such as fruit, produce, deli, and bakery; alcoholic beverages were also excluded. Given this, we estimate that our 6-rack layout makes up approximately $30 \%$ $40 \%$ of the total product shelf space (for our Midwest Retailer).

Therefore, we scale our estimated impulse profit per shopper. Table D1 shows that the range for impulse profit per shopper derived from the literature ( $\$ 2.40-\$ 12.00$ ) is smaller than, but still overlapping the range produced by our model ( $\$ 6.05-\$ 28.87$ ). One likely explanation for this discrepancy, more particular with the high estimate, is that our experiments assumed the impulse ordering rule, which placed the product categories with highest impulse potential (unit profit * impulse purchase rate) in the most visible locations. In reality however, product groups are often located in aisles based on affinity and/or attributes (e.g., medicine, cough drops, and antacid product groups would likely be placed next to each other, as opposed to perhaps vitamins being located in between these groups). In other words, it is unlikely a true impulse ordering rule is used in in a real store; thus, justification for the upper values in the ranges being a bit high in our case.

Now considering the best improvements we generated over our baseline layout (i.e., cases 3 and 4 in Table 6), we show the impulse profits per shopper for $\left\{23.3^{\circ}, 0^{\circ}\right\}$ and $\left\{27.4^{\circ}, 60.5^{\circ}\right\}$ layouts (cases 5 and 6), which equate to improvements of $243 \%$ and $58 \%$, respectively. While on the surface the improvement of $243 \%$ seems too high to be realistic, we reiterate our discussion from the paper, which argues that the Demand Ordering Rule in combination with a $\left\{90^{\circ}, 0^{\circ}\right\}$ layouts essentially hides a majority of the products with high impulse potential from the shopper; varying the layout to expose these products to
shoppers passing by dramatically increases the expected impulse profit. That being said, as brought up earlier, grocery stores usually do not order products solely on demand, nor impulse potential; there usually is some middle ground (i.e., product affinity). Thus, the $243 \%$ increase can be seen as an upper bound on the impulse revenue increase.

Table D1. Steps for deriving impulse profit per shopper.

| Case | Basket Size | \% of Revenue as Impulse | Impulse Basket Size | \% of Revenue as Profit | Impulse <br> Profit Per <br> Shopper | From |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | \$ 40.00 | 30\% | \$ 12.00 | 20\% | \$ 2.40 |  |
| 2 | \$ 60.00 | 50\% | \$ 30.00 | 40\% | \$ 12.00 |  |
|  |  |  | Impulse Profit Per Shopper (Store Section) | \% of Store | Impulse Profit Per Shopper (Whole Store) | Our <br> Model |
| 3 | Demand Ordering Rule | $\begin{aligned} & \hline \text { Baseline } \\ & \left(90^{\circ}, 0^{\circ}\right) \\ & \hline \end{aligned}$ | \$ 2.42 | 40\% | \$ 6.05 |  |
| 4 | Impulse Ordering Rule | $\begin{aligned} & \text { Baseline } \\ & \left(90^{\circ}, 0^{\circ}\right) \end{aligned}$ | \$ 8.66 | 30\% | \$ 28.87 |  |
| 5 | Demand Ordering Rule | $\begin{gathered} \text { Best } \\ \left(23.3^{\circ}, 0^{\circ}\right) \\ \hline \end{gathered}$ | \$ 8.30 | 40\% | $\begin{array}{r} \$ 20.75 \\ (+243 \%) \\ \hline \end{array}$ |  |
| 6 | Impulse Ordering Rule | $\begin{gathered} \hline \text { Best } \\ \left(27.4^{\circ}, 60.5^{\circ}\right) \end{gathered}$ | \$ 13.67 | 30\% | $\begin{aligned} & \$ 45.57 \\ & (+58 \%) \end{aligned}$ |  |

