## **Supplementary Material**

## Stereo Digital Image Correlation (StereoDIC) With and Without Refraction Correction

StereoDIC is a vision-based measurement method that has gained worldwide acceptance for use in non-contacting shape, displacement, and strain measurements. The underlying principle for StereoDIC is based on the use of a pinhole perspective model as shown in Figure S1.



Figure S1: Traditional stereo digital image correlation pinhole camera model diagram

In Figure S1, the object point P is imaged by Cameras 1 and 2 as shown for a typical air environment. The Camera 1 image position of point P in the sensor plane is designated  $p_1$  and the corresponding Camera 2 image position of point P in the sensor plane is designated  $p_2$ . The sensor plane coordinates of points  $p_1$  and  $p_2$  are typically given in pixels, where coordinates ( $p_{1x}$ ,  $p_{1y}$ ) and (p<sub>2x</sub>, p<sub>2y</sub>) refer to locations within the light-sensing elements in the Camera 1 and Camera 2 sensor arrays, respectively. To determine the corresponding sensor points by triangulation of both cameras to the 3-D coordinates of P, several "parameters" must be determined. For each camera, these include: (i) the intrinsic parameters which contain the 2-D coordinates of the sensor plane center, C; the focal length, f; and the physical dimensions of the pixels in both directions, (ii) the extrinsic camera parameters which include the orientation and position of each camera coordinate system, and (iii) distortion correction parameters accounting for system deviations relative to a pinhole model. Without considering distortion, there are eleven parameters for each camera. To determine the camera parameters, a process known as camera calibration is performed typically using a calibration target consisting of several dots or squares with known spacing between them. These targets provide a distance connecting pixel coordinates to physical dimensions.



Figure S2. Schematic illustrating light refraction and ray path tracing using the singlecamera VRO model. The light ray path from object P without refraction (blue line) intersects point p on the image plane. The actual light ray path with optical interfaces (black line) is offset from the pinhole point as a function O(x,q) used to model the VRO path (brown line) to the point p\* in the image plane. The glass thickness is exaggerated here for illustrative purposes.

For a situation where an object is viewed through a medium denser than air (e.g., through glass and water), the pinhole model requires corrections to accurately estimate the true object motions using the matching stereo pixel positions p<sub>1</sub> and p<sub>2</sub>. Schematically, the refraction effect is shown for a single camera in Figure S2 with an exaggerated glass thickness.<sup>1</sup> One approach to account for refraction is to explicitly model the interface(s) using Snell's law. This approach was implemented effectively for planar glass interfaces in previous studies (Ke et al. 2008). While it removes measurement bias, the calibration methods are somewhat inconvenient and not easily implemented within modern bundle adjustment calibration procedures.

Another approach is the Variable Ray Origin (VRO) method (Schreier et al. 2017; Correlated Solutions 2018) developed by Correlated Solutions and has been incorporated and adopted into existing commercial software. The methods are briefly outlined here. As shown in Figure S2, if one traces a ray without refraction from an object point P to the sensor plane, the ray will pass through the pinhole before being imaged at point p. If one traces the actual optical ray from P, it will experience two refractions (e.g., fluid-glass, glass-air) before being imaged on the sensor plane at point  $p^*$ . Although the "actual" and "air only" rays are shown, the latter is not physically relevant for this case because the refracted rays forming the entire image no longer go through a single pinhole point, known as the focal point, which they otherwise would for an ideal pin-hole camera. VRO ray accounts for the actual image offset and is defined at the pinhole location by a function O(x,q), where x is the 2-D position in the sensor plane, and q is a vector set of parameters. By selecting a functional form for **O**, such as a two-dimensional polynomial expression, the additional parameters q are incorporated into a modified calibration process and the true positions of points can be determined.

<sup>&</sup>lt;sup>1</sup> In practice, the VRO model is effective and accurate when the thickness of the glass is small relative to the distances involved in imaging the specimen. Exaggerated glass thicknesses in Figure S1 are for illustrative purposes.

## **Supplement References**

- Correlated Solutions I. VIC-3D Image correlation software.
- Correlated Solutions I. 2018. Application Note AN -1802 Variable Ray Origin Calibration.
- Ke X, Sutton MA, Lessner SM, Yost M. 2008. Robust stereo vision and calibration methodology for accurate three-dimensional digital image correlation measurements on submerged objects. J Strain Anal Eng Des. 43:689–704.
- Schreier HW, Balabokhin A, Correlated Solutions I. 2017. Variable Ray Origin Camera Models for Accurate DIC Measurements Through Glass and Glass/ Water Interfaces. In: Annu Int DIC Soc Conf. Barcelona.