## Supplementary Material

The supplementary material consists of six sections. In Section A1, we show the relationship between our and McConnell’s (1992) model and the formal derivation of Equations (4) – (6) in the article. In Section A2, we make an interpretation of the first-order conditions associated with the maximization problem given by Equation (3) in the article. In Section A3, we describe the dual problem to the problem given by Equation (3), and derive the associated compensated demand functions. In Section A4, we derive our welfare estimates. Specifically, we derive Equations (9) and (10) in our econometric model (Section 3), and Equations (14), (15), (16), and (19) in our empirical specification (Section 4). In Section A5, we show that the substitution effects in our compensated demand functions for total duration of recreation and the number of visits are symmetric and negative semidefinite, while this is not necessarily the case for the compensated demand functions for time spent on-site. In Section A6, our implementation Cholesky factorization and Gauss-Hermite integration and how we calculate the probability of trips being greater than zero are described in more detail. We also describe how the variance of the WTP estimates are calculated using the delta method.

## A1. The Relationship Between Our and McConnell’s Model

In McConnell (1992), the utility function is defined as , however, we assume that the individual’s maximization problem is given by Equation (3) in the text, or:

|  |  |  |
| --- | --- | --- |
|  |  | (A1) |

with the associated Lagrangian function:

|  |  |  |
| --- | --- | --- |
|  |  |  |

The first-order conditions (FOCs) to the problem in Equation(A1) are given by:

|  |  |  |
| --- | --- | --- |
|  |  | (A2) |
|  |  | (A3) |
|  |  | (A4) |

and

|  |  |  |
| --- | --- | --- |
|  |  | (A5) |

From Equation (A4), we have:

|  |  |  |
| --- | --- | --- |
|  |  | (A6) |

Using Equation (A4), Equations (A2) and (A3) we have:

|  |  |  |
| --- | --- | --- |
|  |  | (A7) |

and

|  |  |  |
| --- | --- | --- |
|  |  | (A8) |

Combining Equations (A7) and (A8) results in the marginal rate of substitution:

|  |  |  |
| --- | --- | --- |
|  |  | (A9) |

The FOCs given by Equations (A2) – (A5) are solved to obtain the demand functions and the solution to problem (A1) is the indirect utility function:

|  |  |  |
| --- | --- | --- |
|  |  | (A10) |

To simplify the notation, we define:

|  |  |  |
| --- | --- | --- |
|  | and | (A11) |

At the optimum, the budget constraint in problem (A1), , can be written as:

|  |  |  |
| --- | --- | --- |
|  | . | (A12) |
|  |  |  |

*Roy’s Identity for the number of trips*  *– Equation (4)*

Differentiate the indirect utility function (A10) with respect to the price of a trip to obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (A13) |

From Equations (A6) - (A8), we write Equation (A13) as:

|  |  |  |
| --- | --- | --- |
|  |  | (A14) |

The budget constraint (A12) differentiated with respect to the price of a trip is:

|  |  |  |
| --- | --- | --- |
|  |  | (A15) |

From Equations (A14) and (A15), it follows:

|  |  |  |
| --- | --- | --- |
|  |  | (A16) |

The indirect utility function (A10) differentiated with respect to income gives:

|  |  |  |
| --- | --- | --- |
|  |  | (A17) |

From Equations (A6) - (A8), we write Equation (A17) as:

|  |  |  |
| --- | --- | --- |
|  |  | (A18) |

The budget constraint (A12) differentiated with respect to income is:

|  |  |  |
| --- | --- | --- |
|  |  | (A19) |

From Equations (A18) and (A19), it follows:

|  |  |  |
| --- | --- | --- |
|  |  | (A20) |

By using Roy’s identity, the Marshallian demand for the number of trips given by Equation (4) in the text is:

|  |  |  |
| --- | --- | --- |
|  |  | (A21) |
|  |  |  |

*Roy’s Identity for time spent on-site*  – Equations (5) and (6)

Differentiate the indirect utility function (A10) with respect to the price of time spent on-site to obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (A22) |

From Equations (A6) - (A8), we write Equation (A22) as:

|  |  |  |
| --- | --- | --- |
|  |  | (A23) |

The budget constraint (A12) differentiated with respect to the price of time spent on-site is:

|  |  |  |
| --- | --- | --- |
|  |  | (A24) |

From Equations (A23) and (A24), it follows:

|  |  |  |
| --- | --- | --- |
|  |  | (A25) |

By using Roy’s identity, we derive Equation (5) in the text:

|  |  |  |
| --- | --- | --- |
|  |  | (A26) |

i.e., Roy’s identity does not give the optimal value for time spent on-site directly. However, as shown in McConnell (1992), by Equations (A16) and (A25) we derive Equation (6) in the text:

|  |  |  |
| --- | --- | --- |
|  |  | (A27) |
|  |  |  |

## A2. The Interpretation of the First-Order Conditions of Equation (3)

Larson (1993) defined recreational experience as:

|  |  |  |
| --- | --- | --- |
|  |  | (A28) |

which can be interpreted as the total duration of recreation. The maximization problem in Equation (3) in the text given by Equation (A1) becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (A29) |

with the associated Lagrangian function:

|  |  |  |
| --- | --- | --- |
|  |  |  |

The FOCs of Equation (A29) are given by:

|  |  |  |
| --- | --- | --- |
|  |  | (A30) |
|  |  | (A31) |
|  |  | (A32) |

and

|  |  |  |
| --- | --- | --- |
|  |  | (A33) |

From Equations (A31) and (A32), we have:

|  |  |  |
| --- | --- | --- |
|  |  | (A34) |

and

|  |  |  |
| --- | --- | --- |
|  |  | (A35) |

By inserting Equation (A34) into Equation (A30), we obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (A36) |

It follows from Equations (A34), (A35), and (A36), and the assumption that is quaisconcave in its variables that the necessary and sufficient conditions for maximum are given by:

|  |  |  |
| --- | --- | --- |
|  |  | (A37) |
|  |  | (A38) |

and

|  |  |  |
| --- | --- | --- |
|  |  | (A39) |

i.e., identical to the result of the standard utility maximization problem.

## A3. The Dual Problem to Equation (3)

The expenditure minimization problem corresponding to the maximization problem in Equation (3) in the article is to minimize expenditure given a certain level of utility, , or:

|  |  |  |
| --- | --- | --- |
|  |  | (A40) |

with the associated Lagrangian function:

|  |  |  |
| --- | --- | --- |
|  |  |  |

The FOCs associated with the problem in Equation (A40) are given by:

|  |  |  |
| --- | --- | --- |
|  |  | (A41) |
|  |  | (A42) |
|  |  | (A43) |

and

|  |  |  |
| --- | --- | --- |
|  |  | (A44) |

From Equations (A42) and (A43), we have:

|  |  |  |
| --- | --- | --- |
|  |  | (A45) |

and

|  |  |  |
| --- | --- | --- |
|  |  | (A46) |

By inserting Equation (A45) into Equation (A41), we obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (A47) |

and

|  |  |  |
| --- | --- | --- |
|  |  |  |

i.e., identical to the solution of the utility maximization problem in Equation (A39). The solution to the problem given in Equation (A40) is the expenditure function:

|  |  |  |
| --- | --- | --- |
|  |  | (A48) |

Hicksian demand functions are derived by Shephard’s lemma, and the associated indirect utility function is given by:

|  |  |  |
| --- | --- | --- |
|  |  | (A49) |
|  |  |  |

*Compensated demand for the number of trips,*

Differentiate the expenditure function (A48) with respect to the price of a trip to obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (A50) |

From the FOCs given by Equations (A41) - (A43), we have:

|  |  |  |
| --- | --- | --- |
|  |  | (A51) |

Differentiate the indirect utility function (A49) with respect to the price of a trip, to obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (A52) |

or:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Substituting Equation (A52) into Equation (A51), the compensated demand functions for the number of trips becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (A53) |
|  |  |  |

*Compensated demand for time spent on-site,*

Differentiate the expenditure function given by Equation (A48) with respect to the price of time spent on-site to obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (A54) |

From the FOCs given by Equations (A41) - (A43), we have:

|  |  |  |
| --- | --- | --- |
|  |  | (A55) |

Differentiate the indirect utility function given by Equation (A49) with respect to the price of time spent on-site to obtain:

|  |  |  |
| --- | --- | --- |
|  |  | (A56) |

or:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Substituting Equation (A56) into Equation (A55), we observe that Shephard’s lemma does not directly provide the compensated demand functions for time spent on-site:

|  |  |  |
| --- | --- | --- |
|  |  | (A57) |

However, Equation (A57) provides the compensated demand for total duration of recreation, .

## A4. Calculation of Welfare Estimates

Willingness to pay (WTP) for access is obtained by integrating the Marshallian demand curve given by Equation (7) in the article over a price change from the current price, , to the choke price, :

|  |  |  |
| --- | --- | --- |
|  |  | (A58) |

The price of the duration of recreation depends on the price per trip , the price per unit of time on-site , the time on-site , and the number of trips *n.* The total cost of recreation is given by Equation (8) in the article:

|  |  |  |
| --- | --- | --- |
|  |  | (A59) |

By utilizing the definition of demand in Equation (A28), Equation (A59) becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (A60) |

The total differential of Equation (A60) is:

|  |  |  |
| --- | --- | --- |
|  |  | (A61) |

and the WTP for access in Equation (A58) can be rewritten as Equation (9) in the article:

|  |  |  |
| --- | --- | --- |
|  |  | (A62) |

However, Equation (A62) still contains a total differential for , , which is an endogenous variable in the model. The total differential for is given by:

|  |  |  |
| --- | --- | --- |
|  |  | (A63) |

By plugging Equation (A63) into Equation (A61), we get:

|  |  |  |
| --- | --- | --- |
|  |  | (A64) |

Therefore, the WTP for access in Equation (A62) becomes:

|  |  |
| --- | --- |
|  (a) (b) (c) (d) (e) | (A65) |

Parts (c), (d) and (e) of Equation (A65) can be rewritten in terms of the uncompensated demand elasticities:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Own-Price Elasticity | Cross-Price Elasticity | Income Elasticity | (A66) |
|  |  |  |  |  |
|  |  |  |  |  |

Thus, part (c) of Equation (A65) becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (A67) |

part (d) becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (A68) |

and part (e) becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (A69) |

Therefore, Equation (A65) can be written as:

|  |  |  |
| --- | --- | --- |
|  |  | (A70) |

By collecting the parts with the same total differentials, Equation (A70) can be written as Equation (10) in the article:

|  |  |  |
| --- | --- | --- |
|  |  | (A71) |

*Our empirical specification*

In Section 4 of the article, the demand functions and have a semi-log functional form. This is functional form is typically used in the recreational demand literature. Furthermore, the price of time spent on-site is assumed to be proportional to income, i.e. This results in perfect multicollinearity between this price and income, and income is deleted from the demand functions. The deterministic parts of the demand functions (Equations (14) and (15) in the article) are specified as:

|  |  |  |
| --- | --- | --- |
|  |  | (A72) |

and

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

The associated demand for duration of recreation (deterministic part of Equation (16) in the article) is:

|  |  |  |
| --- | --- | --- |
|  |  | (A73) |

The associated own-price and cross-price elasticities are:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Own-Price Elasticity | Cross-Price Elasticity | (A74) |
|  |  |  |
|  |  |  |

Finally, we derive Equation (19) in the article. With the semi-log functional form for demand, Equation (A72), the choke price becomes infinity and Equation (A70) becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (A75) |

Integration by parts gives: and Equation (A75) becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (A76) |

Finally, if , and are all and then Equation (A76) becomes Equation (19) in the text:

|  |  |  |
| --- | --- | --- |
|  |  | (A77) |

## A5. Symmetric Substitution Effects

The matrix of compensated substitution effects for and are symmetric and negative semidefinite, but the same does not hold for the matrix of compensated substitution effect for , nor between and . First, we will show that is concave in and . For concavity, fix the utility level at , and let and for . Suppose , , , and are optimal solutions to the expenditure problem in Equation (A40), when prices are and . If so:

|  |  |  |
| --- | --- | --- |
|  |  | (A78) |
|  |  |  |
|  |  |  |

where the inequality follows from , and the definition of the expenditure function, which implies that

|  |  |  |
| --- | --- | --- |
|  |  | (A79) |

and

|  |  |
| --- | --- |
|  | (A80) |

Next, we will show that for all and , the compensated demand is the derivative vector of the expenditure function w.r.t. . That is . This follows directly from the Envelope theorem. Thus:

|  |  |  |
| --- | --- | --- |
|  |  | (A81) |

The second derivative of the expenditure function gives:

|  |  |
| --- | --- |
|  | (A82) |

The expenditure function is concave, and the matrix of compensated substitution effects for is symmetric and negative semidefinite. It follows that the matrix of compensated substitution effects for is not necessarily symmetric and negative semidefinite, nor the substitution effects between and , but are jointly symmetric through the identity and Equation (A82).

Restrictions on the matrix of compensated substitution effects for can be found using the same arguments as above. The compensated demand is the derivative vector of the expenditure function w.r.t. . That is , which follows directly from the Envelope theorem. Thus:

|  |  |  |
| --- | --- | --- |
|  |  | (A83) |

The second derivative of the expenditure function gives:

|  |  |
| --- | --- |
|  | (A84) |

The expenditure function is concave and thus the matrix of substitution effects for is symmetric and negative semidefinite. Q.E.D.

**A6. Maximum likelihood estimation and Gauss-Hermite integration**

## Our likelihood function is given by Equation (13) in the article:

|  |  |  |
| --- | --- | --- |
|  |  | (A85) |

where the joint density of our distributions are given by the following expression:

|  |  |  |
| --- | --- | --- |
|  |  | (A86) |

|  |  |  |
| --- | --- | --- |
|  |  |  |

|  |  |  |
| --- | --- | --- |
|  |  | (A87) |

The variance covariance matrix of is given by , where is the lower triangular Cholesky factor of see, for example, Cameron and Trivedi (2005). We transform such that , where and , The likelihood function can then be written as:

The Gauss-Hermite approximation of the likelihood function using *m* quadrature points for each dimension of , and following the notation in Min and Agresti (2005), is given by:

|  |  |  |
| --- | --- | --- |
|  |  | (A88) |

where are the nodes (i.e., points of evaluation) and are the weights of the Gauss-Hermite integration of order *m*. The nodes *c* and weights *w* have been calculated for the Gauss-Hermite integration, and can be found in tables in Abramowitz and Stegun (1971).

The approximation in Equation (A88) is then optimized using the Dual Quasi-Newton (DQN) method. The DQN method computes the gradient, but does not need to calculate the second derivatives, since the Hessian is approximated. The DQN updates the Cholesky factor of the approximated Hessian, instead of updating the approximate inverse Hessian as the standard Quasi-Newton method, and this method can save computation time.

## *Probability calculations*

The unconditional distribution of *n* is given by:

|  |  |
| --- | --- |
|   | (A89) |

The probability of zero trips is given by:

|  |  |  |
| --- | --- | --- |
|  |   | (A90) |
|  | =. |  |

There does not exist a closed form solution to the integral in Equation (A90), and it is approximated by using Gauss-Hermite integration. We do the following transformation, , where . Then, Equation (A90) is rewritten as:

|  |  |
| --- | --- |
|  | (A91) |
|  |  |

where *m* is the number of quadrature points, and and are the weights and the nodes of a Gauss-Hermite integration. Usually it is sufficient to use 4-6 quadrature points, since by then the approximation is not changing significantly, however, adding more points of evaluation only increases accuracy. In the following empirical analysis, we use 8 quadrature points. As was stated in the previous subsection, values for *w* and *c* can be found in tables in Abramowitz and Stegun (1971).

## *The variance of the WTP estimates*

To calculate the variance for the WTP estimate, we differentiate Equation (A77) by the price parameters:

|  |  |  |
| --- | --- | --- |
|  |  | (A92) |
|  |  |  |
|  |  |  |

and

|  |  |  |
| --- | --- | --- |
|  |  |  |

The vector of averages of these derivatives over all groups is . By using the delta method, the variance of the WTP estimate is given by:

|  |  |
| --- | --- |
|  | (A93) |

**Appendix References**

Abramowitz, M., and Stegun, I. A. 1971. *Handbook of Mathematical Functions*. New York: Dover Press.

Cameron, A. C., and Trivedi, P. K. 2005. *Microeconometrics: Methods and Applications*. New York: Cambridge University Press.