Supplemental material for Alfouneh, Ji, and Luo, "Damping design of harmonically excited flexible structures with graded materials to minimize sound pressure and radiation", *Engineering Optimization*, 2020.

The formulations of $\bar{p}_{\mathrm{Re}}$, $\bar{p}_{\mathrm{Im}}$, A_1 and A_2 in Eq. (13) are given by:

$$
\begin{aligned}
\bar{p}_{\mathrm{Re}} &= \frac{1}{\omega^2}\left(\bar{\alpha}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}}G_{\mathrm{Re}} - \bar{\alpha}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}}G_{\mathrm{Im}} - \bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}}G_{\mathrm{Im}} - \bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Im}}G_{\mathrm{Re}}\right) + \\
&\quad \left(\bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}}G_{\mathrm{Re}} - \bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Im}}G_{\mathrm{Im}} - \bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Re}}G_{\mathrm{Im}} - \bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Im}}G_{\mathrm{Re}}\right) \\
\bar{p}_{\mathrm{Im}} &= \frac{1}{\omega^2}\left(\bar{\alpha}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}}G_{\mathrm{Re}} + \bar{\alpha}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}}G_{\mathrm{Im}} + \bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}}G_{\mathrm{Re}} - \bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Im}}G_{\mathrm{Im}}\right) + \\
&\quad \left(\bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Im}}G_{\mathrm{Re}} + \bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}}G_{\mathrm{Im}} + \bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Re}}G_{\mathrm{Re}} - \bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Im}}G_{\mathrm{Im}}\right)
\end{aligned}
\tag{S1}
$$

$$
A_1 = \frac{r_1}{\sqrt{r_1^2 + s_1^2}},\; A_2 = \frac{s_1}{\sqrt{r_1^2 + s_1^2}} \tag{S2}
$$

in which

$$
\begin{aligned}
r_1 &= \sum_{e=1}^{nel} \frac{1}{\omega^2}\left(\bar{\alpha}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}(e)}G_{\mathrm{Re}(e)} - \bar{\alpha}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}(e)}G_{\mathrm{Im}(e)} - \bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}(e)}G_{\mathrm{Im}(e)} - \bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Im}(e)}G_{\mathrm{Re}(e)}\right) + \\
&\quad \left(\bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}(e)}G_{\mathrm{Re}(e)} - \bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Im}(e)}G_{\mathrm{Im}(e)} - \bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Re}(e)}G_{\mathrm{Im}(e)} - \bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Im}(e)}G_{\mathrm{Re}(e)}\right) \\
s_1 &= \sum_{e=1}^{nel} \frac{1}{\omega^2}\left(\bar{\alpha}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}(e)}G_{\mathrm{Im}(e)} + \bar{\alpha}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}(e)}G_{\mathrm{Im}(e)} + \bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}(e)}G_{\mathrm{Re}(e)} - \bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Im}(e)}G_{\mathrm{Im}(e)}\right) + \\
&\quad \left(\bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Im}(e)}G_{\mathrm{Re}(e)} + \bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}(e)}G_{\mathrm{Im}(e)} + \bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Re}(e)}G_{\mathrm{Re}(e)} - \bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Im}(e)}G_{\mathrm{Im}(e)}\right)
\end{aligned}
\tag{S3}
$$

It is obvious that in Eq. S1-S3, *Re* and *Im* denote the real and imaginary part of the complex values.

The formulations of Π_{Re}, Π_{Im}, B_1 and B_2 in Eq. (15) are:

$$
\begin{aligned}
\Pi_{\mathrm{Re}} &= \frac{1}{\omega^4}(\bar{\alpha}_{\mathrm{Re}}^2\bar{T}_{\mathrm{Re}}^2 - \bar{\alpha}_{\mathrm{Re}}^2\bar{T}_{\mathrm{Im}}^2 - \bar{\alpha}_{\mathrm{Im}}^2\bar{T}_{\mathrm{Re}}^2 + \bar{\alpha}_{\mathrm{Im}}^2\bar{T}_{\mathrm{Im}}^2 - 4\bar{\alpha}_{\mathrm{Re}}\bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}}) + \\
&(\bar{\beta}_{\mathrm{Re}}^2\bar{U}_{\mathrm{Re}}^2 - \bar{\beta}_{\mathrm{Re}}^2\bar{U}_{\mathrm{Im}}^2 - \bar{\beta}_{\mathrm{Im}}^2\bar{U}_{\mathrm{Re}}^2 + \bar{U}_{\mathrm{Im}}^2\bar{\beta}_{\mathrm{Im}}^2 - 4\bar{U}_{\mathrm{Re}}\bar{U}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}) + \\
&\frac{2}{\omega^2}(\bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}} - \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}}\bar{U}_{\mathrm{Im}} - \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}}\bar{U}_{\mathrm{Im}} - \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}} - \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}}\bar{U}_{\mathrm{Im}} - \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}} \\
&- \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}} + \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Im}}\bar{U}_{\mathrm{Im}})
\end{aligned}
\tag{S4}
$$

$$
\begin{aligned}
\Pi_{\mathrm{Im}} = {} & \frac{1}{\omega^4}(2\bar{\alpha}_{\mathrm{Re}}^2\bar{T}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}} + 2\bar{\alpha}_{\mathrm{Im}}^2\bar{T}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}} + 2\bar{\alpha}_{\mathrm{Re}}\bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}}^2 - 2\bar{\alpha}_{\mathrm{Re}}\bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Im}}^2) + \\
& (2\bar{\beta}_{\mathrm{Re}}^2\bar{U}_{\mathrm{Re}}\bar{U}_{\mathrm{Im}} - 2\bar{\beta}_{\mathrm{Im}}^2\bar{U}_{\mathrm{Re}}\bar{U}_{\mathrm{Im}} + 2\bar{\beta}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Re}}^2 - 2\bar{U}_{\mathrm{Im}}^2\bar{\beta}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}) + \\
& \frac{1}{\omega^2}(\bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}}\bar{U}_{\mathrm{Im}} + \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}} + \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}} - \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Im}}\bar{U}_{\mathrm{Im}} \\
& + \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}} - \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}}\bar{U}_{\mathrm{Im}} - \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}}\bar{U}_{\mathrm{Im}} - \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}})
\end{aligned}
\tag{S5}
$$

B_1, B_2 in Eq. (15) are calculated as follows:

$$
B_1 = \frac{r_2}{\sqrt{r_2^2 + s_2^2}}, \; B_2 = \frac{s_2}{\sqrt{r_2^2 + s_2^2}} \tag{S6}
$$

in which

$$
\begin{aligned}
r_2 = {} & \frac{1}{\omega^4}\sum_{e=1}^{nel}(\bar{\alpha}_{\mathrm{Re}}^2\bar{T}_{\mathrm{Re}(e)}^2 - \bar{\alpha}_{\mathrm{Re}}^2\bar{T}_{\mathrm{Im}(e)}^2 - \bar{\alpha}_{\mathrm{Im}}^2\bar{T}_{\mathrm{Re}(e)}^2 + \bar{\alpha}_{\mathrm{Im}}^2\bar{T}_{\mathrm{Im}(e)}^2 - 4\bar{\alpha}_{\mathrm{Re}}\bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}(e)}\bar{T}_{\mathrm{Im}(e)}) + \\
& \sum_{e=1}^{nel}(\bar{\beta}_{\mathrm{Re}}^2\bar{U}_{\mathrm{Re}(e)}^2 - \bar{\beta}_{\mathrm{Re}}^2\bar{U}_{\mathrm{Im}(e)}^2 - \bar{\beta}_{\mathrm{Im}}^2\bar{U}_{\mathrm{Re}(e)}^2 + \bar{U}_{\mathrm{Im}(e)}^2\bar{\beta}_{\mathrm{Im}}^2 - 4\bar{U}_{\mathrm{Re}(e)}\bar{U}_{\mathrm{Im}(e)}\bar{\beta}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}) + \\
& \frac{2}{\omega^2}\sum_{e=1}^{nel}(\bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}(e)}\bar{U}_{\mathrm{Re}(e)} - \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}(e)}\bar{U}_{\mathrm{Im}(e)} - \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}(e)}\bar{U}_{\mathrm{Im}(e)} - \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Re}(e)}\bar{T}_{\mathrm{Im}(e)} \\
& - \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}(e)}\bar{U}_{\mathrm{Im}(e)} - \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}(e)}\bar{T}_{\mathrm{Im}(e)} - \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}(e)}\bar{U}_{\mathrm{Re}(e)} + \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Im}(e)}\bar{U}_{\mathrm{Im}(e)})
\end{aligned}
\tag{S7}
$$

$$
\begin{aligned}
s_2 = {} & \frac{1}{\omega^4}\sum_{e=1}^{nel}(2\bar{\alpha}_{\mathrm{Re}}^2\bar{T}_{\mathrm{Re}(e)}\bar{T}_{\mathrm{Im}(e)} + 2\bar{\alpha}_{\mathrm{Im}}^2\bar{T}_{\mathrm{Re}(e)}\bar{T}_{\mathrm{Im}(e)} + 2\bar{\alpha}_{\mathrm{Re}}\bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}(e)}^2 - 2\bar{\alpha}_{\mathrm{Re}}\bar{\alpha}_{\mathrm{Im}}\bar{T}_{\mathrm{Im}(e)}^2) + \\
& \sum_{e=1}^{nel}(2\bar{\beta}_{\mathrm{Re}}^2\bar{U}_{\mathrm{Re}(e)}\bar{U}_{\mathrm{Im}(e)} - 2\bar{\beta}_{\mathrm{Im}}^2\bar{U}_{\mathrm{Re}(e)}\bar{U}_{\mathrm{Im}(e)} + 2\bar{\beta}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Re}(e)}^2 - 2\bar{U}_{\mathrm{Im}(e)}^2\bar{\beta}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}) + \\
& \frac{1}{\omega^2}\sum_{e=1}^{nel}(\bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}(e)}\bar{U}_{\mathrm{Im}(e)} + \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Re}}\bar{U}_{\mathrm{Re}(e)}\bar{T}_{\mathrm{Im}(e)} + \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}(e)}\bar{U}_{\mathrm{Re}(e)} - \bar{\alpha}_{\mathrm{Re}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Im}(e)}\bar{U}_{\mathrm{Im}(e)} \\
& + \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Re}(e)}\bar{U}_{\mathrm{Re}(e)} - \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Re}}\bar{T}_{\mathrm{Im}(e)}\bar{U}_{\mathrm{Im}(e)} - \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Im}}\bar{T}_{\mathrm{Re}(e)}\bar{U}_{\mathrm{Im}(e)} - \bar{\alpha}_{\mathrm{Im}}\bar{\beta}_{\mathrm{Im}}\bar{U}_{\mathrm{Re}(e)}\bar{T}_{\mathrm{Im}(e)})
\end{aligned}
\tag{S8}
$$