**Conditional and unconditional tests for the extended Stuart-Maxwell design**

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**Supplementary Material**

**1. Stuart’s Test Statistic**

Stuart gives the covariance matrix V of his test statistic as having elements

V*hh* =  and .

Since  = ,  =  and  = ,

V*hh* =  +  –  and .

We have that  = diag() – (). For *t* = 2 this gives, first,

 = 

=  (since *Nihj* = 0 or 1,  = *Nihj*)

=  = V*hh*/4,

and second,

 =  =



=  = ,

since : on the *j*th block the first (and also the second) treatment cannot be assigned to both the *h*th and th categories. Thus V = 4.

From section 1 with *t* = 2, 2*b* = () so that . From section 2,  = 0, so with *t* = 2, **1 = – **2. From section 3, again with *t* = 2,

*W*C =  = 

=  =  = ,

the Stuart test statistic.

**2. Examples**

In the following two further examples are given, along with their analysis using both conditional and unconditional tests. In both there is good agreement for the conditional test p-values using the asymptotic chi-squared distribution and a permutation test. However there is less agreement for the unconditional test p-values using the asymptotic chi-squared distribution and the bootstrap.

*Likely to purchase fries data*

Table 1 gives counts for three repeated responses to three hot ‘chips’ or ‘fries’. Each of 55 consumers rated their likelihood of purchase of the potato products A, B and C using three categories: would not buy (NB), undecided (U) or would buy (WB) with the repeated trinary responses shown in the three-way square contingency table below. This data is derived from Rayner et al. (2005, Table 6.14). Categories 1 and 2 as well as categories 4 and 5 were combined so as to avoid many zeroes and to make presentation easier. The products differed as they were prepared with different cooking oils.

We find *W*C = 1.888 with  p-value of 0.756 indicating similar marginal counts. The permutation test p-value is 0.770. For the unconditional test *W*U = 3\**W*C/2 = 2.832 with  p-value 0.586. The bootstrap p-value is 0.765. As section 5 suggests, the unconditional test  p-value is out of line with the other p-values. All tests indicate similar marginal counts.

*Kullback data*

The data in Table 2 is analysed in Kullback (1971). We find *W*C = 5.73 with  p-value 0.22 and permutation test p-value 0.22 also. For the unconditional test *W*U = 3\**W*C/2 = 8.60 with  p-value 0.072 and bootstrap p-value 0.22. While all tests conclude a lack of significance at the 0.05 level, it would seem that less reliance should be placed on the asymptotic chi-squared p-value for the unconditional test.

**Table 1**. Likely to purchase counts for three products

|  |  |  |  |
| --- | --- | --- | --- |
|  | A NB | A U | A WB |
| B | NB | U | WB | NB | U | WB | NB | U | WB |
| C | NB | 6 | 5 | 2 | 3 | 1 | 6 | 3 | 1 | 2 |
| C | U | 4 | 1 | 1 | 4 | 1 | 0 | 0 | 0 | 1 |
| C | WB | 5 | 2 | 0 | 0 | 2 | 1 | 2 | 1 | 1 |
| Sum B | 15 | 8 | 3 | 7 | 4 | 7 | 5 | 2 | 4 |
| Sum A | 26 | 18 | 11 |

**Table 2**. Kullback data *xijk*

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *j* |  | 1 |  |  | 2 |  |  | 3 |  |  |
| *k* | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |  |
| 1 | 223 | 24 | 6 | 40 | 42 | 2 | 19 | 4 | 12 | 372 |
| 2 | 28 | 6 | 9 | 25 | 218 | 6 | 3 | 13 | 9 | 317 |
| 3 | 26 | 3 | 18 | 18 | 30 | 24 | 12 | 16 | 164 | 311 |
| Sum | 277 | 33 | 33 | 83 | 290 | 32 | 34 | 33 | 185 | 100 |
|  |  | 343 |  |  | 405 |  |  | 252 |  |  |

 = 394, 356, 250 for *k* = 1, 2 and 3 respectively.

**References**

Kullback, S. (1971). Marginal homogeneity of multidimensional contingency tables. *Annals of Mathematical Statistics*, 42, 594-606.

Rayner, J.C.W., Best, D.J., Brockhoff, P.B. and Rayner, G.D. (2005). *Nonparametrics for Sensory Science - A More Informative Approach.* Ames : Blackwell.