

**Supplemental material for Tolouei K, *et al.* Application of an improved Lagrangian relaxation approach in the constrained long-term production scheduling problem under grade uncertainty. *Engineering Optimization*, 2020**

**Table S1.** Review of presented models since 1969.

| Authors (Year)                              | Models                                                                         | D <sup>1</sup> | U <sup>2</sup> | HMM <sup>3</sup> |
|---------------------------------------------|--------------------------------------------------------------------------------|----------------|----------------|------------------|
| Johnson (1969)                              | Linear Programming                                                             | *              |                |                  |
| Williams (1974)                             | Dynamic Programming, Integer Programming, Network Flow, Parametric Programming | *              |                |                  |
| Gershon (1983)                              | Linear Programming , Mixed Integer Programming                                 | *              |                |                  |
| Dagdelen and Johnson (1986)                 | Lagrangian Relaxation Method                                                   | *              |                |                  |
| Ravenscroft (1992)                          | Conditional Simulation                                                         |                | *              |                  |
| Dowd (1994)                                 | Geostatistical simulation                                                      |                | *              |                  |
| Elevli (1995)                               | Operation Research, Artificial Intelligence                                    | *              |                |                  |
| Denby and Schofield (1995)                  | Genetic Algorithm                                                              |                | *              | *                |
| Tolwinski (1998)                            | Dynamic Programming                                                            | *              |                |                  |
| Akaike and Dagdelen (1999)                  | 4D Network Relaxation                                                          | *              |                |                  |
| Whittle (2000)                              | Milawa Algorithm                                                               | *              |                | *                |
| Johnson, Dagdelen, and Ramazan (2002)       | Mixed Integer Programming                                                      | *              |                |                  |
| Dimitrakopoulos, Farrelly, and Godoy (2002) | Generalized Sequential Gaussian Simulation, Direct Block Simulation            |                | *              |                  |
| Godoy and Dimitrakopoulos (2004)            | Simulated Annealing Algorithm                                                  |                | *              | *                |
| Dimitrakopoulos and Ramazan (2004)          | Linear Programming                                                             | *              |                |                  |
| Ramazan and Dimitrakopoulos (2004a)         | Mixed Integer Programming                                                      | *              |                |                  |
| Ramazan and Dimitrakopoulos (2004b)         | Mixed Integer Programming                                                      |                | *              |                  |

<sup>1</sup> Deterministic, <sup>2</sup> Uncertainty, <sup>3</sup> Heuristic and Meta-heuristic Methods

**Table S1.** (Continued).

| Authors (Year)                               | Models                                                    | D <sup>1</sup> | U <sup>2</sup> | HMM <sup>3</sup> |
|----------------------------------------------|-----------------------------------------------------------|----------------|----------------|------------------|
| Gholamnejad, Osanloo, and Karimi (2006)      | Chance Constrained Programming                            |                | *              |                  |
| Gholamnejad and Osanloo (2007)               | Chance Constrained Integer Programming                    |                | *              |                  |
| Ramazan and Dimitrakopoulos (2007)           | Stochastic Integer Programming                            | *              |                |                  |
| Boland et al. (2009)                         | Mixed Integer Programming                                 | *              |                |                  |
| Bley et al. (2010)                           | Integer Programming                                       | *              |                |                  |
| Kumral (2010)                                | Robust stochastic optimization                            |                | *              |                  |
| Lamghari and Dimitrakopoulos (2012)          | Tabu Search                                               |                | *              | *                |
| Gholamnejad and Moosavi (2012)               | Binary Integer Programming                                |                | *              |                  |
| Nanjari and Golosinski (2013)                | Dynamic Programming, Mining Heuristic                     | *              |                | *                |
| Sattarvand and Niemann-Delius (2013)         | Ant Colony Optimization                                   | *              |                | *                |
| Goodfellow and Dimitrakopoulos (2013)        | Simulated Annealing Algorithm                             |                | *              | *                |
| Dimitrakopoulos and Jewbali (2013)           | Stochastic Integer programming                            | *              |                |                  |
| Leite and Dimitrakopoulos (2014)             | Stochastic Integer Programming                            |                | *              |                  |
| Moosavi et al. (2014a)                       | Lagrangian Relaxation Method, Genetic Algorithm           | *              |                | *                |
| Moosavi et al. (2014b)                       | Augmented Lagrangian Relaxation Method, Genetic Algorithm | *              |                | *                |
| Koushavand, Askari-Nasab, and Deutsch (2014) | Mixed Integer Linear Programming                          |                | *              |                  |
| Asad, Dimitrakopoulos, and Eldert (2014)     | Stochastic Network Flow, Lagrangian Relaxation Method     |                | *              | *                |
| Shishvan and Sattarvand (2015)               | Ant Colony Optimization                                   | *              |                | *                |
| Mokhtarian and Sattarvand (2016a)            | Imperialist Competitive Algorithm                         | *              |                | *                |

<sup>1</sup> Deterministic, <sup>2</sup> Uncertainty, <sup>3</sup> Heuristic and Meta-heuristic Methods

**Table S1.** (Continued).

| Authors (Year)                            | Models                                                                                                                                                                                               | D <sup>1</sup> | U <sup>2</sup> | HMM <sup>3</sup> |
|-------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|----------------|------------------|
| Mokhtarian and Sattarvand (2016b)         | Commodity Price Distribution function, Median Latin Hypercube sampling method, Integer Programming                                                                                                   | *              |                |                  |
| Goodfellow and Dimitrakopoulos (2016)     | Simulated Annealing Algorithm, Particle Swarm Optimization, Differential Evolution                                                                                                                   |                | *              | *                |
| Lamghari and Dimitrakopoulos (2016a)      | Rockafellar and Wets Progressive Hedging Algorithm                                                                                                                                                   |                | *              |                  |
| Lamghari and Dimitrakopoulos (2016b)      | Tabu Search Heuristic Incorporating a Diversification Strategy, Variable Neighborhood Descent Heuristic, Very Large Neighborhood Search Heuristic, Network Flow Techniques, Diversified Local Search |                | *              | *                |
| Mousavi, Kozan, and Liu (2016)            | Branch-and-bound, simulated annealing algorithm                                                                                                                                                      | *              |                | *                |
| Khan (2018)                               | Particle Swarm Optimization, Bat Algorithm                                                                                                                                                           |                | *              | *                |
| Rahimi et al. (2018)                      | Logical mathematical algorithm                                                                                                                                                                       | *              |                | *                |
| Tahernejad, Ataei, and Khalokakaie (2018) | Information Gap Decision Theory                                                                                                                                                                      |                | *              |                  |
| Jelvez, Morales, and Nancel-Penard (2018) | Expected Time Incremental Heuristic Algorithm                                                                                                                                                        | *              |                | *                |
| Khan and Asad (2018)                      | Mixed Integer Linear Programming                                                                                                                                                                     | *              |                |                  |
| Alipour et al. (2018)                     | Robust counterpart linear optimization, Genetic Algorithm                                                                                                                                            |                | *              | *                |
| Chatterjee and Dimitrakopoulos (2019)     | Lagrangian Relaxation Method, Sub-Gradient Method, Branch and Cut Algorithm                                                                                                                          |                | *              | *                |
| Dimitrakopoulos and Senécal (2019)        | Multi-Neighborhood Tabu Search                                                                                                                                                                       |                | *              | *                |

<sup>1</sup> Deterministic, <sup>2</sup> Uncertainty, <sup>3</sup> Heuristic and Meta-heuristic Methods

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