Appendix

Derivation of Equation (26): Multiplying expression (24) by $n_1 p_1$ and using $\phi \equiv \tau^{1-\sigma}$

$$n_1 p_1 x_1^* = \mu_1 n_1 p_1^{1-\sigma} \left(\frac{Y_1^d}{P_1^{1-\sigma}} + \frac{\phi Y_2^d}{P_2^{1-\sigma}} \right)$$

replacing the left hand side for the corresponding expressions (11), (13) and (15)

$$\frac{L_{E_1} w_1}{\sigma - 1} + L_{E_1} w_1 = \mu_1 n_1 p_1^{1-\sigma} \left(\frac{Y_1^d}{P_1^{1-\sigma}} + \frac{\phi Y_2^d}{P_2^{1-\sigma}} \right)$$

using expressions (16) and (23), and by adding and subtracting $(1 - \mu_1) Y_1^d$,

$$H_1 w_{H_1} + L_1 w_1 - Y_1^d - \tfrac{1-\mu}{2} \left(\tfrac{p_{A_1}}{P_A} \right)^{1-\sigma} \left(Y_1^d + Y_2^d \right) + (1 - \mu) Y_1^d + \mu_1 Y_1^d = \mu_1 n_1 p_1^{1-\sigma} \left(\tfrac{Y_1^d}{P_1^{1-\sigma}} + \tfrac{\phi Y_2^d}{P_2^{1-\sigma}} \right)$$

where $\mu \equiv \mu_1 + \mu_2$. Considering the agricultural price index (9) and $Y_1^d = (1 - t) Y_1$, and after some manipulation, it yields

$$tY_1 + \tfrac{1-\mu}{2P_A^{1-\sigma}} \left(\tfrac{Y_1^d}{p_{A_2}^{\sigma-1}} - \tfrac{Y_2^d}{p_{A_1}^{\sigma-1}} \right) + \mu_1 Y_1^d \left(1 - \tfrac{n_1 p_1^{1-\sigma}}{P_1^{1-\sigma}} \right) = \mu_1 n_1 p_1^{1-\sigma} \tfrac{\phi Y_2^d}{P_2^{1-\sigma}}$$

finally, by using the expression of industrial price index (8), equation (26) it is obtained.

Proof of Proposition 1: First, and hereinafter, labor in region 2 is taken as numerarie, then, $w_2 = p_{s_2} = p_{A_2} = 1$ and $p_2 = \beta \frac{\sigma}{\sigma - 1}$, and $\mu \equiv \mu_1 + \mu_2$. Furthermore,

$$w \equiv \frac{p_1}{p_2} = \frac{p_{A_1}}{p_{A_2}} = \frac{p_{s_1}}{p_{s_2}} = \frac{w_1}{w_2} = w_1 \tag{36}$$

Using expressions (16), (20)-(23), (29) and replacing these in (27) and (28) it is obtained,

$$\begin{aligned} Y_1 &= \frac{\sigma}{\sigma-\mu_1}\left[Lw+\frac{t\mu_2 Lw}{\sigma-1+\mu_2(1-t)}+\frac{1-\mu}{2P_A^{1-\sigma}}\frac{Lw-(1\text{-}L)\,p_A^{1-\sigma}}{\sigma-1+\mu_2(1-t)}\right] && (37)\\ Y_2 &= \frac{\sigma}{\sigma-\mu_1}\left[(1\text{-}L)-\frac{t\mu_2 Lw}{\sigma-1+\mu_2(1-t)}+\frac{1-\mu}{2P_A^{1-\sigma}}\frac{(1\text{-}L)\,p_A^{1-\sigma}-Lw}{\sigma-1+\mu_2(1-t)}\right] && (38)\\ Y^w &= Y_1+Y_2=Y_1^d+Y_2^d \quad \text{and} \quad Y^w=\frac{\sigma}{(\sigma-\mu_1)}\left[Lw+(1-L)\right] && (39) \end{aligned}$$

Using (8)-(9), (36), (15), and (37)-(39) the current account equation (26) can be rewritten as

$$\begin{aligned} CA_2(H,w)\equiv s_y(1-t) \quad & \left\{\mu_1\phi\left[\frac{Hw^{1-\sigma}}{H\phi w^{1-\sigma}+1-H}+\frac{1-H}{Hw^{1-\sigma}+(1-H)\phi}\right]+(1-\mu)\right\}\\ +ts_y- \quad & \left[\mu_1\phi\frac{Hw^{1-\sigma}}{H\phi w^{1-\sigma}+1-H}+(1-\mu)\frac{w^{1-\sigma}}{1+w^{1-\sigma}}\right]=0 \qquad (40) \end{aligned}$$

where $s_y\equiv Y_1(w)/Y^w(w)$. Implicit differentiation of (40) leads to

$$\left.\frac{dw}{dH}\right|_{CA_2=0}=-\frac{\frac{\partial CA_2}{\partial H}}{\frac{\partial CA_2}{\partial w}} \qquad (41)$$

where,

$$\frac{\partial CA_2}{\partial H}=-\frac{\mu_1\phi w^{1-\sigma}}{(Hw^{1-\sigma}\phi+1-H)^2}\left\{1-s_y(1-t)\frac{(1-\phi^2)\left[\left(Hw^{1-\sigma}\right)^2-(1-H)^2\right]}{\left[Hw^{1-\sigma}+(1-H)\phi\right]^2}\right\}<0 \qquad (42)$$

the second term in curly brackets could be negative or positive, but in the last case, it will always be lower than one, so the expression is always negative. Additionally,

$$\begin{aligned} \frac{\partial CA_2}{\partial w} &= \frac{\mu_1\phi(\sigma-1)H(1-H)}{w^{\sigma}(Hw^{1-\sigma}\phi+1-H)^2}\left\{1-s_y(1-t)\frac{(1-\phi^2)\left[\left(Hw^{1-\sigma}\right)^2-(1-H)^2\right]}{\left[Hw^{1-\sigma}+(1-H)\phi\right]^2}\right\}+\frac{(1-\mu)(\sigma-1)w^{-\sigma}}{(1+w^{1-\sigma})^2}\\ &\quad+\frac{\partial s_y}{\partial w}\left\{1-\mu(1-t)+\mu_1\phi(1-t)\left[\frac{Hw^{1-\sigma}}{Hw^{1-\sigma}\phi+1-H}+\frac{1-H}{Hw^{1-\sigma}+(1-H)\phi}\right]\right\} \qquad (43) \end{aligned}$$

Note that,

$$\begin{aligned}
\frac{\partial Y_1}{\partial w} &= \frac{\sigma}{\sigma-\mu_1}\left\{L+\tfrac{\mu_2 tL}{\sigma-1+\mu_2(1-t)}+\tfrac{1-\mu}{\left(1+w^{1-\sigma}\right)^2}\tfrac{L\left(\sigma w^{1-\sigma}+1\right)+(1-L)(\sigma-1)w^{-\sigma}}{\sigma-1+\mu_2(1-t)}\right\}>0 \\
\frac{\partial Y^w}{\partial w} &= \frac{\sigma}{\sigma-\mu_1}L
\end{aligned}$$

On comparing these expressions it can be observed that, $\frac{\partial Y_1}{\partial w} > \frac{\partial Y^w}{\partial w}$, and $Y^w > Y_1$, thus,

$$\frac{\partial s_y}{\partial w} = \frac{\frac{\partial Y_1}{\partial w}Y^w - \frac{\partial Y^w}{\partial w}Y_1}{\left(Y^w\right)^2} > 0 \tag{44}$$

Then,

$$\frac{\partial CA_2}{\partial w} > 0 \tag{45}$$

Considering the signs of (42) and (45), (41) must always be positive.

Proof of Proposition 2: From expressions (37) and (39) it can be obtained that

$$\frac{\partial s_y}{\partial t} = \frac{\partial Y_1/\partial t}{Y^w} = \frac{\mu_2}{\sigma-1+\mu_2\left(1-t\right)}s_y \tag{46}$$

Then, deriving the current account equation (40) with respect to t,

$$\tfrac{\partial CA_2}{\partial t}=\tfrac{s_y}{\sigma-1+\mu_2(1-t)}\left\{(\sigma-1+\mu_2)-(\sigma-1)\left[\tfrac{\mu_1 H w^{1-\sigma}\phi}{Hw^{1-\sigma}\phi+1-H}+\tfrac{\mu_1(1-H)\phi}{Hw^{1-\sigma}+(1-H)\phi}+(1-\mu)\right]\right\} \tag{47}$$

The sum in the square brackets is equal to or lower than 1, and $[(\sigma-1+\mu_2)-(\sigma-1)(\mu_1+1-\mu)]>0$. Thus, the expression is always positive. Then:

$$\left.\frac{dw}{dt}\right|_{CA_2=0} = -\frac{\frac{\partial CA_2}{\partial t}}{\frac{\partial CA_2}{\partial w}} < 0$$

For the second part of the proposition, equation (19) is divided by (39), such that

$$\frac{T_2}{Y^w} = ts_y \quad \text{and} \quad \frac{Y^w - T_2}{Y^w} = (1 - ts_y)$$

Deriving this expressions and expression (39) with respect to t,

$$\begin{aligned}
\frac{d\,(ts_y)}{dt} &= t\tfrac{\partial s_y}{\partial t} + s_y + t\tfrac{\partial s_y}{\partial w}\tfrac{dw}{dt}\Big|_{CA_2=0} = \tfrac{\sigma-1+\mu_2}{\sigma-1+\mu_2(1-t)}s_y + t\tfrac{\partial s_y}{\partial w}\tfrac{dw}{dt}\Big|_{CA_2=0} > 0 \\
\frac{d\,(1-ts_y)}{dt} &= -\frac{d\,(ts_y)}{dt} < 0 \\
\frac{dY^w}{dt} &= \frac{\sigma}{\sigma-\mu_1}L\left.\frac{dw}{dt}\right|_{CA_2=0} < 0
\end{aligned}$$

On looking at equations (43)-(47), it is clear that the first expression is always positive, while the last two are always negative. Thus, if T_2/Y^w increases and $(Y^w - T_2)/Y^w$ decreases as t rises, dT_2/dt must be positive.

Proceeding in the same way for the disposable incomes,

$$\frac{Y_1^d}{Y^w} = \frac{(1-t)\,Y_1}{Y^w} = (1-t)\,s_y \quad \text{and} \quad \frac{Y_2^d}{Y^w} = 1 - (1-t)\,s_y$$

by differentiating these expressions with respect to t it is obtained that

$$\begin{aligned}
\frac{d\,[(1-t)\,s_y]}{dt} &= -\frac{\sigma-1}{\sigma-1+\mu_2\,(1-t)}s_y + (1-t)\,\frac{\partial s_y}{\partial w}\left.\frac{dw}{dt}\right|_{CA_2=0} < 0 \\
\frac{d\,[1-(1-t)\,s_y]}{dt} &= -\frac{d\,[(1-t)\,s_y]}{dt} > 0
\end{aligned}$$

Then, taking into account that $dY^w/dt < 0$, the last two expressions imply that

$$\frac{dY_1^d}{dt} < 0 \quad \text{and} \quad \frac{dY_2^d}{dt} > 0$$

Proof of Proposition 3: The change in the industrial sector as a proportion of the labor force in the sector is:

$$\frac{dL_{E_j}/dt}{L_{E_j}} = \frac{\partial L_{E_j}/\partial t}{L_{E_j}} + \frac{\partial L_{E_j}/\partial w}{L_{E_j}} \left.\frac{dw}{dt}\right|_{CA_2=0} \gtrless 0$$

Using equations (23), (29), (37), (38), (43) and (47), the previous expression for region 1 at the symmetric equilibrium (30) is equal to

$$\left.\frac{dL_{E_1}/dt}{L_{E1}}\right|_{sym} = \frac{\sigma U(\phi)}{Z(\phi)} \gtrless 0 \tag{48}$$

where $U(\phi)$, and $Z(\phi) > 0$ for $\phi \geq 0$ ($dZ(\phi)/d\phi > 0$), are polynomials,

$$U(\phi) = [2\mu_2+\sigma(1-\mu)]\,\phi^2+2\mu_2(2\sigma-1)\,\phi - \sigma(1-\mu) \gtrless 0 \tag{49}$$

$$Z(\phi) = (\sigma - 1 + \mu_2)\left[4\mu_1(\sigma-1)\,\phi+(1-\mu)(\sigma-1)(1+\phi)^2\right] \tag{50}$$

$$+(\sigma-1+\mu_2)\left[1+\tfrac{\sigma(1-\mu)}{\sigma-1+\mu_2}\right]\left[(1-\mu_2)(1+\phi)^2 - \mu_1\left(1-\phi^2\right)\right] > 0 \tag{51}$$

where $Z(\phi) > 0$ for all $\phi \in [0,1]$. Then, the sign of expression (48) depends only on the numerator. The polynomial (49) has a unique positive root: $P(\phi = \phi^{sr}) = 0$ with $\phi^{sr} \in (0,1)$, and

$$\phi^{sr} = \frac{-\mu_2(2\sigma-1)+\sqrt{[\mu_2(2\sigma-1)]^2+\sigma(1-\mu)[2\mu_2+\sigma(1-\mu)]}}{[2\mu_2+\sigma(1-\mu)]} \tag{52}$$

Moreover, evaluating expression (48) for the extreme cases of $\phi = 0$ and $\phi = 1$ yields

$$\left.\frac{dL_{E_1}/dt}{L_{E1}}(\phi=0)\right|_{sym} = -\frac{\sigma}{(\sigma-\mu_1)} < 0$$

$$\left.\frac{dL_{E_1}/dt}{L_{E1}}(\phi=1)\right|_{sym} = \frac{\mu_2\sigma}{(1-\mu_2)(\sigma-\mu_1)} > 0$$

Then, expression (48) is negative for $0 \leq \phi < \phi^{sr}$ and positive for $\phi^{sr} < \phi \leq 1$. Proceeding in the same way for region 2, (and by symmetry) it is obtained that

$$\left.\frac{dL_{E_2}/dt}{L_{E2}}\right|_{sym} = -\left.\frac{dL_{E_1}/dt}{L_{E1}}\right|_{sym} = -\frac{\sigma U(\phi)}{Z(\phi)} \gtrless 0$$

Proof of Proposition 4: From equation $U(\phi) = 0$ (polynomial (49)) and the implicit differentiation, it is obtained that

$$\frac{\partial \phi^{sr}}{\partial \mu_2} = -\frac{2\phi^{sr}\left[\phi^{sr} + (2\sigma - 1)\right] + \sigma\left[1 - (\phi^{sr})^2\right]}{2\left[2\mu_2 + \sigma(1-\mu)\right]\phi^{sr} + 2\mu_2(2\sigma - 1)} < 0 \tag{53}$$

$$\frac{\partial \phi^{sr}}{\partial \sigma} = -\frac{(1-\mu)(\phi^{sr})^2 + 4\mu_2\phi^{sr} - (1-\mu)}{2\left[2\mu_2 + \sigma(1-\mu)\right]\phi^{sr} + 2\mu_2(2\sigma - 1)} < 0 \tag{54}$$

While expression (53) is clearly negative, expression (54) is also negative since $\mu_2 > 0$ and

$$\frac{\partial \phi^{sr}}{\partial \sigma} < 0 \longleftrightarrow \phi^{sr} > \phi^*$$

where ϕ^* is the unique positive root of the numerator of (54):

$$\phi^* = \frac{-2\mu_2}{1-\mu} + \sqrt{\left(\frac{2\mu_2}{1-\mu}\right)^2 + 1} \tag{55}$$

Proof of Proposition 5: The proof is divided in two parts. The first part proves the existence of the thresholds ϕ^b and ϕ^r that determine the stability/instability of the symmetric equilibrium. The second part derives the analytical expression for these thresholds.

Part 1: By differentiating $V(H, w)$ from equation (34) with respect to H,

$$\frac{dV}{dH} = \frac{\partial V}{\partial H} - \frac{\partial V}{\partial w}\frac{\partial CA_2/\partial H}{\partial CA_2/\partial w} \gtrless 0 \tag{56}$$

If this expression is negative, the equilibrium is stable, and if it is positive the equilibrium is unstable. Evaluating expression (56) at the interior symmetric equilibrium (30) it is obtained that

$$\left.\frac{dV}{dH}\right|_{sym} = -4\tfrac{[1-d+\phi(1+d)]}{1+\phi} + \frac{4\mu_1 \frac{\phi}{(1+\phi)^2}\left[\frac{\sigma^2(1-\mu)}{\mu_1(\sigma-1+\mu_2)}+1-\mu_2-\frac{\mu_1(1-\phi)}{(1+\phi)}\right]}{\frac{\mu_1(\sigma-1)\phi}{(1+\phi)^2}+\frac{(1-\mu)(\sigma-1)}{4}+\frac{1}{4}\left[1+\frac{\sigma(1-\mu)}{\sigma-1+\mu_2}\right]\left(1+\mu_2+\mu_1\frac{1-\phi}{1+\phi}\right)} \tag{57}$$

where $d \equiv \frac{\mu_1}{\sigma-1}$. Evaluating (57) at $\phi = 1$ yields,

$$\left.\frac{dV}{dH}\right|_{sym}(\phi = 1) = -4\frac{\mu_1(\sigma-1+\mu_2)^2}{\sigma(\sigma-\mu_1)(1-\mu_2)} < 0 \tag{58}$$

Thus, when $\phi = 1$, the symmetric equilibrium is always stable. Additionally, evaluating expression (57) at $\phi = 0$ yields,

$$\left.\frac{dV}{dH}\right|_{sym}(\phi = 0) = 4\left[\frac{\mu_1}{\sigma-1} - 1\right] \tag{59}$$

Which implies that, if the BHC holds, the symmetric equilibrium is unstable for $\phi = 0$, and stable otherwise. Furthermore, expression (57) can be rewritten as

$$\left.\frac{dV}{dH}\right|_{sym} = \frac{P(\phi)}{K(\phi)} = \frac{-A\phi^3 + B\phi^2 + C\phi + D}{K(\phi)} \gtrless 0 \tag{60}$$

where

$$A \equiv (1+d)\left[\left(1+\tfrac{\sigma(1-\mu)}{\sigma-1+\mu_2}\right)(1-\mu_2+\mu_1) + (1-\mu)(\sigma-1)\right] > 0 \tag{61}$$

$$B \equiv 4\mu_1\left[\tfrac{\sigma(1-\mu)}{\sigma-1+\mu_2}+1-\mu_2+\mu_1\right] - 2(1+d)\left[\tfrac{\sigma(1-\mu)(\sigma-\mu_1)}{\sigma-1+\mu_2}+\mu_1(\sigma-1)\right] \tag{62}$$

$$-(1-d)\left\{\left[\tfrac{\sigma(1-\mu)}{\sigma-1+\mu_2}+1\right](1-\mu_2+\mu_1) + (1-\mu)(\sigma-1)\right\}$$

$$C \equiv 4\mu_1\left[\tfrac{\sigma^2(1-\mu)}{\mu_1(\sigma-1+\mu_2)}+1-\mu\right] - (1+d)\tfrac{\sigma(1-\mu)(\sigma-\mu_1)}{\sigma-1+\mu_2} \tag{63}$$

$$-2(1-d)\left[\tfrac{\sigma(1-\mu_2)(\sigma-\mu_1)}{\sigma-1+\mu_2} + \mu_1(\sigma-1)\right] \tag{64}$$

$$D \equiv (d-1)\tfrac{\sigma(1-\mu)(\sigma-\mu_1)}{\sigma-1+\mu_2} \tag{65}$$

$$K(\phi) \equiv \frac{4\mu_1(\sigma-1)\phi+(1-\mu)(\sigma-1)(1+\phi)^2+\left(1+\frac{\sigma(1-\mu)}{(\sigma-1+\mu_2)}\right)\left[(1-\mu_2)(1+\phi)-\mu_1\left(1-\phi^2\right)\right]}{4(1+\phi)^{-1}} > 0 \tag{66}$$

Since expression (66) is positive for all values of $\phi \geq 0$, only $P(\phi)$ determines the sign of the expression (57). As $\phi \to \infty$, $P(\phi) \to -\infty$; and as $\phi \to -\infty$, $P(\phi) \to \infty$. Moreover, if $d \gtreqless 1$, then $D \gtreqless 0$. Also, when $d \geq 1$, $C > 0$, then there exists a threshold $\bar{\mu}_1(\sigma, \mu_2) \in (0, \min[1, \sigma - 1])$ for the parameter μ_1, which can be expressed as $\bar{d} \equiv \frac{\bar{\mu}_1(\sigma,\mu_2)}{\sigma-1}$, such that if $\bar{d} < d < 1$, then $C > 0$, and there exist two real positive roots of the polynomial $P(\phi)$. And whenever $C < 0$, $B < 0$, according to expression (67), there are, therefore, no real positive roots.

$$B - C = -2\mu_1 \frac{[(1+\mu_1)+2(1-\mu_2)]\sigma^2 - [(1+\mu_1)+(1+3\mu_1)(1-\mu_2)]\sigma + (1-\mu_2)(1+2\mu_1)}{(\sigma-1)(\sigma-1+\mu_2)} < 0 \tag{67}$$

Part 2: In order to obtain a closed form for the thresholds (ϕ^b and ϕ^r) it is taken into account that $\phi^* = -1$ is always a solution of $P(\phi) = 0$. Then, this polynomial can be rewritten as

$$P(\phi) = -(\phi + 1)\left[\phi^2 - (Tr)\,\phi + (Det)\right] \tag{68}$$

where, $Tr \equiv \frac{B}{A} + 1$ and $Det \equiv -\frac{D}{A}$. Thus, the other two roots of $P(\phi)$ are

$$\phi^b = \frac{Tr - \sqrt{(Tr)^2 - 4Det}}{2} \tag{69}$$

$$\phi^r = \frac{Tr + \sqrt{(Tr)^2 - 4Det}}{2} \tag{70}$$

If $(Tr)^2 - 4Det > 0$, there are three cases: 1) if $Tr > 0$ and $Det > 0$, then $0 < \phi^b < \phi^r < 1$; 2) if $Tr \lesseqgtr 0$ and $Det < 0$, then $\phi^b < 0 < \phi^r < 1$ and 3) if $Tr < 0$ and $Det > 0$, then $\phi^b < \phi^r < 0$. If $(Tr)^2 - 4Det = 0$, then $\phi^b = \phi^r \in [0, 1)$. If $(Tr)^2 - 4Det < 0$, then ϕ^b and ϕ^r are conjugated complexes.

Additionally, from these relations, $\bar{\mu}_1(\sigma, \mu_2) \in (0, \min[1, \sigma - 1])$ can be implicitly defined as the value of μ_1 that ensures that the following conditions are fulfilled:

$$\begin{aligned} Tr^2 - 4Det &= 0 \text{ with } Tr > 0 \text{ and } Det > 0 \qquad (71) \\ \mu_1 - (\sigma - 1) &< 0 \qquad (72) \end{aligned}$$

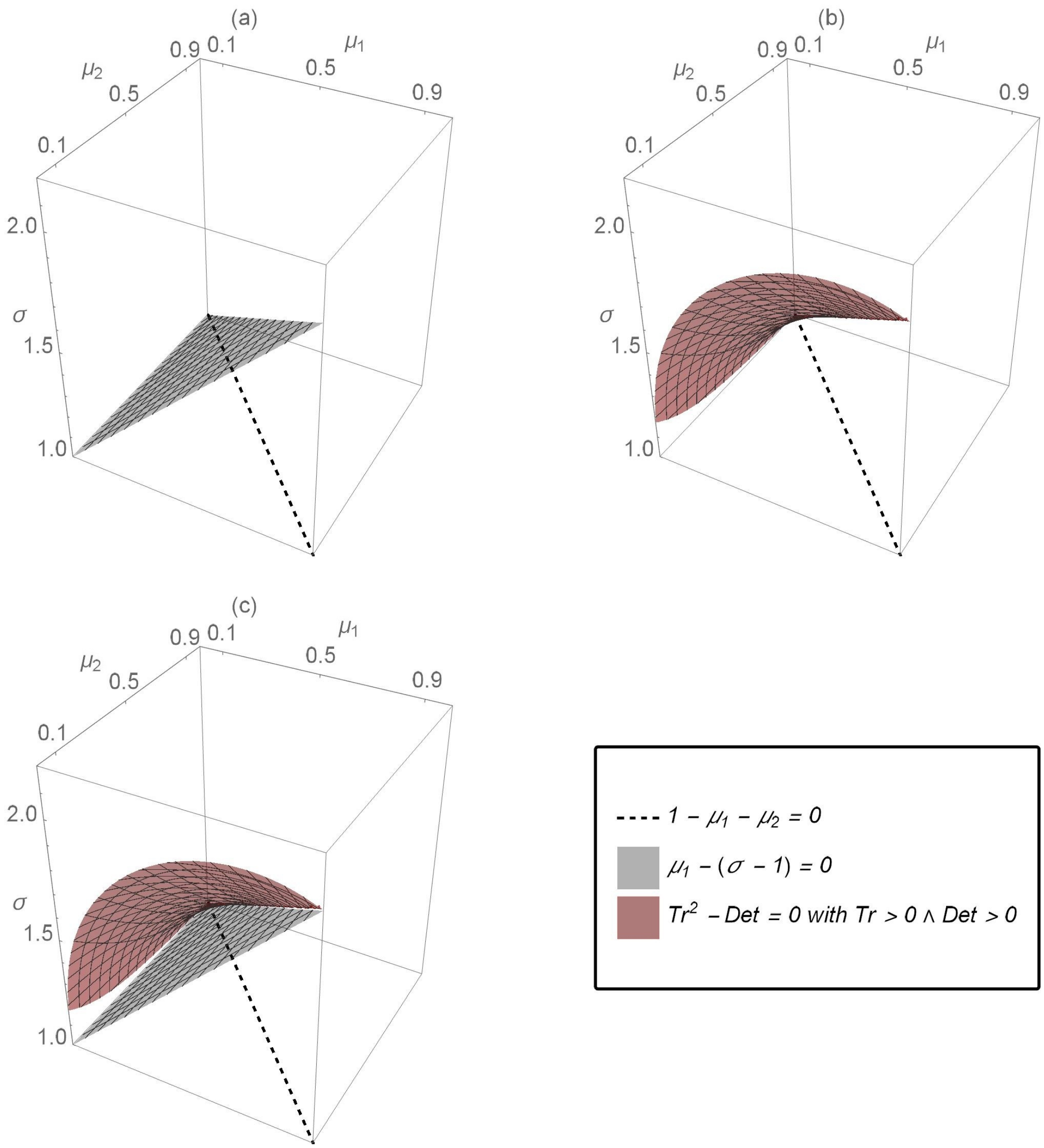

Figure 7: Regions of Bifurcation Points in the space (μ_1,μ_2,σ)

The region above the plane in Figure 7 (a) corresponds to $d < 1$ (condition (72)). Only the parameter values below the dashed line of Figure 7 in the plane (μ_1, μ_2) are feasible due to the parameter restriction: $\mu_1 + \mu_2 \equiv \mu \in (0, 1)$. The red surface in Figure 7 (b) depicts condition (71). Below this surface $Tr^2 - 4Det > 0$, and above $Tr^2 - 4Det < 0$. Thus, for each value of σ and μ_2, there exist a value $\mu_1 = \bar{\mu}_1(\sigma, \mu_2)$ such that $Tr^2 - 4Det = 0$. Moreover, Figure 7 (c) divides the space of parameters (μ_1, μ_2, σ) in three regions: 1) below the gray plane, $d > 1$ and the symmetric equilibrium has only one bifurcation point, ϕ^r; 2) above the gray plane and below the red surface, $\bar{d} < d < 1$ and the symmetric equilibrium has two bifurcation points, ϕ^b and ϕ^r; and 3) above the red surface, $d < \bar{d} < 1$ and the symmetric equilibrium is stable for all values of ϕ.

Proof of Proposition 6: By fully differentiating the system (40)-(33) with respect to t, it is obtained that

$$\begin{pmatrix} \frac{\partial CA_2}{\partial w} & \frac{\partial CA_2}{\partial H} \\ \frac{\partial V}{\partial w} & \frac{\partial V}{\partial H} \end{pmatrix} \begin{pmatrix} \frac{dw}{dt} \\ \frac{dH}{dt} \end{pmatrix} = \begin{pmatrix} -\frac{\partial CA_2}{\partial t} \\ -\frac{\partial V}{\partial t} \end{pmatrix}$$

Then, the change in the number of firms is

$$\frac{dH}{dt} = \frac{\left(\frac{\partial CA_2}{\partial t}\frac{\partial V}{\partial w} - \frac{\partial V}{\partial t}\frac{\partial CA_2}{\partial w}\right)}{\left(\frac{\partial CA_2}{\partial w}\frac{\partial V}{\partial H} - \frac{\partial CA_2}{\partial H}\frac{\partial V}{\partial w}\right)}$$

After some manipulation,

$$\frac{dH}{dt} = -\frac{\frac{\partial V}{\partial t} + \frac{\partial V}{\partial w}\left(-\frac{\partial CA_2/\partial t}{\partial CA_2/\partial w}\right)}{\frac{\partial V}{\partial H} + \frac{\partial V}{\partial w}\left(-\frac{\partial CA_2/\partial H}{\partial CA_2/\partial w}\right)} \tag{73}$$

The denominator is equal to the stability condition (57) in Proposition 5, while the numerator is the effect of a change in the rate of transfers (t) over the ratio of indirect utilities (V_1/V_2). Additionally, using (16) and (32), the numerator of (73) can be rewritten as

$$\frac{dV}{dt}=\frac{V_1}{V_2}\left\{\left[\frac{\frac{dL_{E_1}}{dt}}{L_{E_1}}-\frac{\frac{dL_{E_2}}{dt}}{L_{E_2}}\right]+\left[\frac{\frac{dw}{dt}\Big|_{CA_2=0}}{w}\right]-\left[\frac{\mu_2}{w}+\mu_1\frac{\frac{\partial(P_1/P_2)}{\partial w}}{P_1/P_2}\right]\frac{dw}{dt}\Big|_{CA_2=0}\right\}$$

which is equal to expression (35). Evaluating at the symmetric equilibrium,

$$\frac{dV}{dt}\Big|_{sym}=\frac{2}{Z(\phi)}\left[\sigma U\left(\phi\right)-J\left(\phi\right)\right]\gtrless 0 \tag{74}$$

where

$$\begin{aligned} J\left(\phi\right) &\equiv \left(1-\mu_2+\mu_1\right)\left[\mu_2\sigma-\mu_1\left(\sigma-1\right)\right]\phi^2 \\ &+2\left[\mu_2(1-\mu_2)\sigma+\mu_1^2\left(\sigma-1\right)\right]\phi+\left(1-\mu\right)\left[\mu_2\sigma+\mu_1\left(\sigma-1\right)\right] \end{aligned} \tag{75}$$

$U\left(\phi\right)$ and $Z\left(\phi\right)$ are defined in (49) and (50), and $J\left(\phi\right)>0$. Thus, the sign is determined by the numerator. After some manipulations it is obtained that

$$\sigma U\left(\phi\right)-J\left(\phi\right)=a\phi^2+b\phi+c\gtrless 0 \tag{76}$$

where

$$a \equiv 2\mu_2\sigma-\left(1-\mu_2+\mu_1\right)\left[\mu_2\sigma-\mu_1\left(\sigma-1\right)\right]+\sigma^2\left(1-\mu\right)>0 \tag{77}$$

$$b \equiv 2\left[2\mu_2\sigma\left(\sigma-1\right)+\mu_2^2\sigma-\mu_1^2\left(\sigma-1\right)\right]\gtrless 0 \tag{78}$$

$$c \equiv -\left(1-\mu\right)\left[\sigma^2+\mu_2\sigma+\mu_1\left(\sigma-1\right)\right]<0 \tag{79}$$

Additionally, evaluating (74) at the extreme cases $\phi=0$ and $\phi=1$,

$$\frac{dV}{dt}\Big|_{sym}\left(\phi\ =\ 0\right)=-2\frac{\mu_1\left(\sigma-1\right)+\sigma\left(\sigma+\mu_2\right)}{\sigma\left(\sigma-1\right)}<0 \tag{80}$$

$$\frac{dV}{dt}\Big|_{sym}\left(\phi\ =\ 1\right)=2\mu_2\frac{\sigma-1+\mu_2}{\left(\sigma-\mu_1\right)\left(1-\mu_2\right)}>0 \tag{81}$$

Thus, the polynomial (76) has only one positive root,

$$\phi^{lr} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \in (0, 1) \tag{82}$$

Furthermore, because $J(\phi) > 0$ for all $\phi \geq 0$, then the following relation must hold:

$$0 \;<\; \phi^{sr} < \phi^{lr} < 1 \quad \text{when} \quad \mu_2 \in (0, 1 - \mu_1) \tag{83}$$

$$\phi^{sr} = \phi^{lr} \quad \text{when} \quad \mu_2 = 0, 1 - \mu_1 \tag{84}$$

Combining these results with those from Proposition 5 properties *i)* and *ii)* of Proposition 6 are derived. Additionally, from polynomial (49) and the implicit differentiation, it is obtained that

$$\frac{\partial \phi^{lr}}{\partial \sigma} = -\frac{\frac{\partial a}{\partial \sigma}\left(\phi^{lr}\right)^2 + \frac{\partial b}{\partial \sigma}\phi^{lr} + \frac{\partial c}{\partial \sigma}}{2a\phi^{lr} + b} \tag{85}$$

The denominator is positive since $\phi^{lr} > -b/(2a)$. Hence, the sign of (85) depends on the numerator. Figure 8 depicts the region for which $\partial \phi^{lr}/\partial \sigma > 0$.

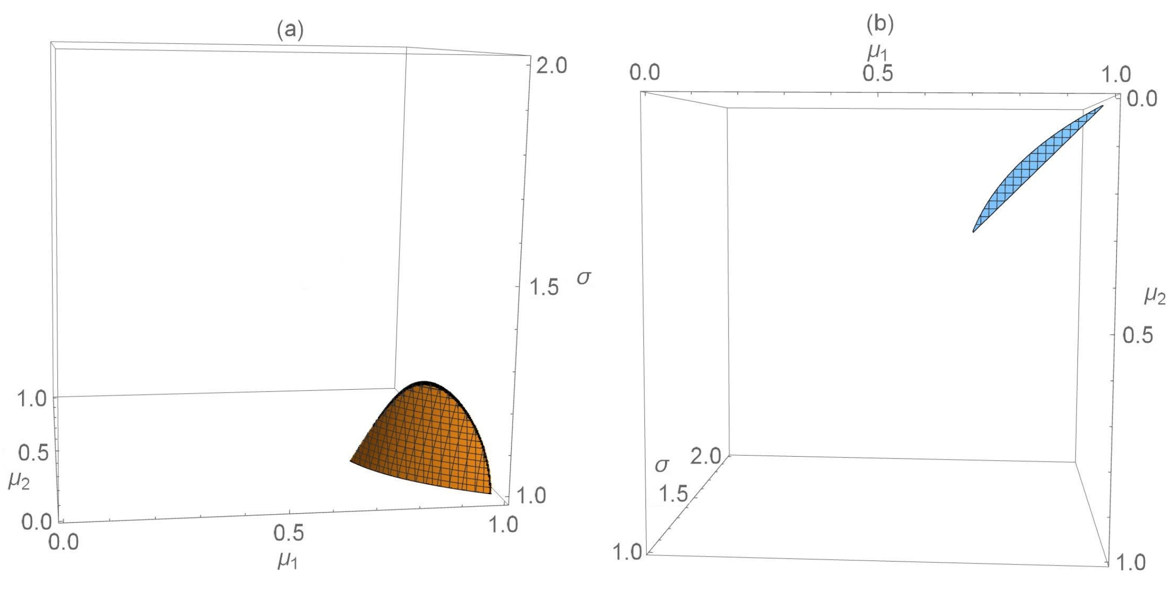

Figure 8: Region for $\partial \phi^{lr}/\partial \sigma > 0$ in the space (μ_1, μ_2, σ)

Figure 8 (a) shows that only for a very narrow range of values of the parameters (μ_1,μ_2,σ) is the derivative (85) positive. Furthermore, Figure 8 (b) highlights that if the agricultural sector is not too small (approximately $1-\mu > 0.08$), derivative (85) will be negative.

Derivation of the Figures 3 (a) - (e): First the focus is putted on ϕ^{lr}, which presents the same shape for all values of d. Then, ϕ^b and ϕ^r are analyzed, by considering the different cases ($d < 1$, $d = 1$ and $d > 1$).

Differentiating of the polynomial (76) with respect to μ_2 yields

$$\frac{\partial(\sigma U(\phi)-J(\phi))}{\partial\mu_2} = \sigma\left\{2\mu_2\phi^2 + \left(\mu\text{-}\mu_1\phi^2\right) + (\sigma\text{-}1)\,\phi\,(4\text{-}\phi) + 4\mu_2\phi + \sigma\text{-}1 + \mu_2\right\} + \mu_1(\sigma\text{-}1) > 0$$

And the differential with respect to ϕ is

$$\frac{\partial\left(\sigma U(\phi) - J(\phi)\right)}{\partial\phi} = 2a\phi + b > 0$$

which is positive because $\phi^{lr} > \frac{-b}{2a}$ (see expression (82)). Then, the implicit differentiation gives

$$\frac{\partial\phi^{lr}}{\partial\mu_2} = -\frac{\partial\left(\sigma U(\phi) - J(\phi)\right)/\partial\mu_2}{\partial\left(\sigma U(\phi) - J(\phi)\right)/\partial\phi} < 0$$

Additionally, evaluating the polynomial (76) at $\mu_2 = 0$ and $\mu_2 = 1-\mu_1$,

$$\phi^{lr}(\mu_2 = 0) = 1 \text{ and } \phi^{lr}(\mu_2 = 1-\mu_1) = 0$$

For ϕ^b and ϕ^r the simplest case, $d = 1$ ($\sigma - 1 = \mu_1$) is studied first. In this special case it is obtained that

$$\phi^b(\sigma - 1 = \mu_1) = 0 \text{ and } \phi^r(\sigma - 1 = \mu_1) = \frac{1-\mu_2-\mu_1\left[4\mu_1^2+\mu_1(6\mu_2-1)+2\mu_2^2+\mu_2-2\right]}{1-\mu_2-\mu_1\left[2\mu_1^2+\mu_1(2\mu_2-1)+\mu_2-2\right]}$$

Thus, only ϕ^r needs to be analyzed. Differentiating $\phi^r\left(\sigma-1=\mu_1\right)$ with respect to μ_2,

$$\frac{\partial\phi^r(\sigma-1=\mu_1)}{\partial\mu_2}=\frac{\mu_1^2\left(5\text{-}2\mu_1^2\right)+\mu_2\left(2\text{-}\mu_2\text{-}\mu_1^3\right)+3\mu_1^3(1\text{-}\mu_2)+2\mu_1[1+\mu_2(2\text{-}\mu_2)]+2\mu_1^2\mu_2(1\text{-}\mu_2)}{-(2\mu_1)^{-1}\left\{1-\mu_2-\mu_1\left[2\mu_1^2+\mu_1(2\mu_2-1)+\mu_2-2\right]\right\}^2}<0$$

Additionally, note that the previous derivative tends to $-\infty$ when $\mu_2=1-\mu_1$. Evaluating $\phi^r(\sigma-1=\mu_1)$ at $\mu_2=0$ and $\mu_2=1-\mu_1$:

$$\phi^r(\sigma-1=\mu_1,\mu_2=0)=\frac{1+2\mu_1+\mu_1^2-4\mu_1^3}{1+2\mu_1+\mu_1^2-2\mu_1^3}\text{ and }\phi^r(\sigma-1=\mu_1,\mu_2=1-\mu_1)=0$$

Bringing these results together, $d=1$ yields $\phi^r(\mu_2=0)<\phi^{lr}\left(\mu_2=0\right)$ and $\phi^r(\mu_2=1-\mu_1)=\phi^{lr}\left(\mu_2=1-\mu_1\right)=0$. Both thresholds diminish as μ_2 increases, and they cross at least once within the interval $\mu_2\in(0,1-\mu_1)$.

When $d>1$ $(\sigma-1<\mu_1)$, the BHC case, $\phi^b<0$. Then, again, only ϕ^r needs to be studied. By differentiating expression (70) with respect to μ_2,

$$\frac{\partial\phi^r}{\partial\mu_2}=\frac{1}{\sqrt{(Tr)^2-4Det}}\left[\frac{\partial Tr}{\partial\mu_2}\phi^r-\frac{\partial Det}{\partial\mu_2}\right]\tag{86}$$

where

$$\begin{aligned}\frac{\partial Det}{\partial\mu_2}&=\frac{-2\mu_1\sigma(\sigma-1-\mu_1)(\sigma-\mu_1)^2}{(\sigma\text{-}1+\mu_1)\left[\sigma\mu_1^2\text{-}\sigma^2(1\text{-}\mu_2)+\mu_1(\sigma\text{-}2)(\sigma\text{-}1+\mu_2)\right]^2}>0\text{ if }\sigma-1<\mu_1\\ \frac{\partial Tr}{\partial\mu_2}-\frac{\partial Det}{\partial\mu_2}&=\frac{-\left\{\sigma\left(\sigma^2\text{-}\mu_1^2\right)\text{-}\mu_1(1\text{-}\mu_2)+\sigma[(1\text{-}\mu_2)\sigma\text{-}\mu_1(\sigma\text{-}1)]+\mu_1[\sigma(2\text{-}\mu)\text{-}(1\text{-}\mu_2)]\right\}}{[4\mu_1(\sigma\text{-}1)(\sigma\text{-}1+\mu_2)]^{-1}(\sigma\text{-}1+\mu_1)\left[\mu_1^2\sigma\text{-}\sigma^2(1\text{-}\mu_2)+\mu_1(\sigma\text{-}2)(\sigma\text{-}1+\mu_2)\right]^2}<0\end{aligned}$$

Then, expression (86) must be negative whenever $d>1$. Now, evaluating ϕ^r at $\mu_2=0$ and $\mu_2=1-\mu_1$ yields that $\phi^r\in(0,1)$. Thus, when the BHC holds with inequality $(d>1)$, $\phi^r(\mu_2=0)<\phi^{lr}\left(\mu_2=0\right)$ and $\phi^r(\mu_2=1-\mu_1)>\phi^{lr}\left(\mu_2=1-\mu_1\right)$. As in the previous case, both thresholds diminish as μ_2 increases, and they cross at least once within the interval $\mu_2\in(0,1-\mu_1)$.

When $d<1$ $(\sigma-1>\mu_1)$, the analysis focuses on the case when the thresholds are real numbers ($0 < \phi^b \leq \phi^r < 1$), that is, when $\bar{d} \leq d < 1$. From Proposition 5 a value $\mu_2 = \mu_{2_0}$ (implicitly defined by $(Tr)^2 - 4Det = 0$) can be defined, such that $\phi_0 \equiv \phi^b = \phi^r$. Then, by differentiating the polynomial $\mathcal{O}_{(\phi,\mu_2)} \equiv \phi^2 - (Tr)\,\phi + Det = 0$ (see the polynomial (68)), and evaluating at (μ_{2_0},ϕ_0),

$$
\begin{aligned}
\frac{\partial \mathcal{O}}{\partial \phi}(\mu_{2_0}, \phi_0) &= 2\phi - Tr|_{\phi_0} = 2\phi_0 - (\phi_0 + \phi_0) = 0 \\
\frac{\partial \mathcal{O}}{\partial \mu_2}(\mu_{2_0}, \phi_0) &> 0 \\
\frac{\partial^2 \mathcal{O}}{\partial \phi^2}(\mu_{2_0}, \phi_0) &= 2
\end{aligned}
$$

Thus, for the function $\mu_2(\phi)$ implicitly defined by $\mathcal{O}_{(\phi,\mu_2)} = 0$, it is obtained that

$$
\frac{d\mu_2}{d\phi}(\mu_{2_0}, \phi_0) = 0 \quad \text{and} \quad \frac{d^2\mu_2}{d\phi^2}(\mu_{2_0}, \phi_0) < 0
$$

which implies that the function $\mu_2(\phi)$ (implicitly defined by $\mathcal{O}_{(\phi,\mu_2)} = 0$) has a maximum at ($\mu_{2_0}$,$\phi_0$). In a close neighborhood of μ_{2_0}, ϕ^b increases, and ϕ^r diminishes as μ_2 increases until $\mu_2 = \mu_{2_0}$. At this point, both thresholds converge to the value ϕ_0.