**Online Supplement for “A flow picking system for order fulfillment in e-commerce warehouses” by “Peng Yang, Zhijie Zhao and Zuo-Jun Max Shen”**

# Appendix 1: List of notation

|  |  |  |
| --- | --- | --- |
| Notation | Description | Unit |
|  | average picking density | items/m |
|  | order arrival rate of entire picking zone | orders/s |
|  | expected number of items in an order | items/order |
|  | time picking an item | s/item |
|  | time gathering an item into an order carton | s/item |
|  | walking speed of order picker | m/s |
|  | average velocity of the conveyor-based equipment  | m/s |
|  | working period of order picker | s |
|  | number of picked items during *T* | item |
|  | number of items included in newly arrival orders during *T* | item |
|  | walking distance during *T* | m |
|  | walking time during *T* | s |
|  | picking time during *T* | s |
|  | turnover time of an order in a single order picker system | s |
|  | number of picked items during  | item |
|  | walking distance during | m |
|  | minimum of | m |
|  | maximum of | m |
|  | normalization of | / |
|  | walking time during  | s |
|  | picking time during  | s |
|  | sorting time during  | s |
|  | waiting time during | s |
|  | delivery time during | s |
|  | gathering time during | s |
| *L* | length of the shortest route to traverse all the aisles | m |
| *Lc* | expected distance for an item being delivered to a chute | m |
| *m* | number of aisles | aisle |
| *l* | length of an aisle | m |
| *w* | center-to-center distance between two adjacent aisles | m |
|  | time for filling a tote from the empty state | s |
| *N* | number of items included in an order | item |
| *M* | capacity of a tote | item |
|  | time proportion occupied by picking | % |
|  | time of a loop traversing all the aisles twice | s |
|  | turnover time of an order in the *ith* sub-area | s |
|  | order arrival rate of the *ith* sub-area | orders/s |
|  | picking time per item of *ith* order picker | s |
|  | walking speed of *ith* order picker | m/s |
|  | length of the shortest route to traverse the *ith* sub-area | m |
|  | area proportion of *ith* sub-area to entire picking zone | % |
|  | turnover time of an order in multiple order pickers system | s |

# Appendix 2: Derivation of

To pick all the items of an order, the order picker should visit all the storage locations of items included in an order. Therefore,is related to these locations. For example, if the items of an order are all at the order picker’s current location, then , which is the luckiest case. Of course, there exists the unluckiest case. The order, an item of which is located in the first storage location, arrives in the system just as the order picker leaves the first storage location. To complete the order, the order picker needs to traverse all the aisles twice before he returns to the first storage location again. In this worst case, we obtain

 

where *L* is the length of the shortest route to traverse all the aisles. Based on the forward rule and the return rule mentioned in section 1, the order picker returns once he reaches the last aisle. Let *m* denote the number of aisles. Then we obtain

 

where *l* and *w* are the length of an aisle, and the center-to-center distance between two adjacent aisles, respectively.

From the previous analysis, we knowand are easy to calculate. For the expectation of, we can also approximately estimate the value with some reasonable assumptions. **Assumption 1**: Each storage location is visited with the same probability; **Assumption 2**: The order picker moves within the normalized continuous interval [0,1], which represents the range of the route that traverses all the aisles from the first location to the last location by the “S-shaped” rule. Then each location can be mapped in the interval [0,1]; **Assumption 3:** Letrepresent the location of itemin an order,represent the number of items included in this order, andrepresent the location of the order picker when the order arrives. Based on assumption 1, when, we can assume that and are continuous, independent, and identically distributed random variables following a uniform distribution within interval [0,1]; **Assumption 4:**The order arrival rate is so high that the order picker should traverse all the aisles.

As shown in Figure S1, when an order arrives in the order picking system, the order picker is located in , and items are located in. The order picker is moving to the right (moving forward). The situation where the order picker moves to the left (returning) is the complete opposite. The calculation methods of the expected value of are the same in each of these two situations. Hence, we only focus on the situation of moving right as shown in Figure S1 and calculate the expected value of.



1. Two cases of the locations of the order picker and the items

In Figure S1, and. Letdenote the normalization of, and then. Based on the relationship between positions of and ,, the expression of is different in the two cases shown in Figure S1. To pick all the items in the order, in case (a), the order picker should walk to 1 and then turn back to; however, in case (b), the order picker only needs to walk to. Hence, we can easily obtain

 

Then the expected value of  can be given by

 

We divide two cases to discuss .

**Case 1: **

 

whereis the location of the only one item, so andare replaced by. Consider the independence betweenand, then

 

**Case 2: **

 

where is the joint conditional probability density function of ,and , under the condition . Because  is independent ofand, we get

 

where  is the joint conditional probability density function of and, is the conditional probability density function of , under the condition. Becauseis independent ofand follows a uniform distribution in interval [0,1],  can be easily shown as

 

We can get by calculating the partial derivative of the joint conditional probability mass function of  and. The following analysis is all under the condition . In order to facilitate writing and reading, we omitted the expression in all P(…).

 

Based on the independence among $X\_{i}, i=0,1,…,n$, we can easily get



 



We can obtain



According to the above probability distribution function,  is obtained by

 

Then



When, , which also satisfies the above equation, then in general

 

Then,is obtained by

 