**Supplementary File**

**Table 1.** The variances, biases, and the REs of estimators with respect to $\hat{σ}\_{2}$ when the underlying population distribution is the standard normal distribution.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | $$L=2$$ | $$L=5$$ |
| $$(m,n)$$ | $$w$$ | **Estimators** | **Variance** | **Bias** | **RE** | **Variance** | **Bias** | **RE** |
| **(2, 2)** | **10** | $$\hat{σ}\_{H}$$ | 0.0077 | -0.0105 | 2.1590 | 0.0030 | -0.0066 | 8.2633 |
| $$\hat{σ}\_{1}$$ | 0.0071 | -0.0083 | 1.9732 | 0.0007 | -0.0017 | 1.9498 |
| $$\hat{σ}\_{2}$$ | 0.0036 | -0.0042 | 1.0000 | 0.0004 | -0.0009 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0018 | -0.1126 | 4.0131 | 0.0003 | -0.0258 | 2.5989 |
| $$\hat{σ}\_{4}$$ | 0.0027 | -0.0544 | 1.5616 | 0.0003 | -0.0130 | 1.3694 |
| **30** | $$\hat{σ}\_{H}$$ | 0.0025 | -0.0035 | 1.9899 | 0.0010 | -0.0019 | 8.1012 |
| $$\hat{σ}\_{1}$$ | 0.0024 | -0.0033 | 1.9161 | 0.0002 | -0.0007 | 2.0008 |
| $$\hat{σ}\_{2}$$ | 0.0012 | -0.0016 | 1.0000 | 0.0001 | -0.0004 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0006 | -0.1107 | 10.3669 | 0.0001 | -0.0254 | 6.1837 |
| $$\hat{σ}\_{4}$$ | 0.0009 | -0.0515 | 2.8645 | 0.0001 | -0.0124 | 2.1865 |
| **(4,2)** | **10** | $$\hat{σ}\_{H}$$ | 0.0066 | -0.0108 | 2.0958 | 0.0026 | -0.0057 | 7.7882 |
| $$\hat{σ}\_{1}$$ | 0.0060 | -0.0085 | 1.8987 | 0.0006 | -0.0013 | 1.8545 |
| $$\hat{σ}\_{2}$$ | 0.0032 | -0.0044 | 1.0000 | 0.0003 | -0.0008 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0016 | -0.1039 | 3.8512 | 0.0003 | -0.0237 | 2.4377 |
| $$\hat{σ}\_{4}$$ | 0.0024 | -0.0507 | 1.5363 | 0.0003 | -0.0120 | 1.3050 |
| **30** | $$\hat{σ}\_{H}$$ | 0.0021 | -0.0037 | 2.0348 | 0.0008 | -0.0019 | 7.4824 |
| $$\hat{σ}\_{1}$$ | 0.0020 | -0.0032 | 1.9176 | 0.0002 | -0.0004 | 1.9093 |
| $$\hat{σ}\_{2}$$ | 0.0011 | -0.0019 | 1.0000 | 0.0001 | -0.0003 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0005 | -0.1022 | 10.4075 | 0.0001 | -0.0232 | 5.7122 |
| $$\hat{σ}\_{4}$$ | 0.0008 | -0.0474 | 2.8824 | 0.0001 | -0.0114 | 2.0977 |
| **(2,4)** | **10** | $$\hat{σ}\_{H}$$ | 0.0021 | -0.0048 | 3.7554 | 0.0008 | 0.0000 | 13.4836 |
| $$\hat{σ}\_{1}$$ | 0.0019 | -0.0053 | 3.4928 | 0.0002 | -0.0008 | 3.1724 |
| $$\hat{σ}\_{2}$$ | 0.0006 | -0.0014 | 1.0000 | 0.0001 | -0.0003 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0003 | -0.0581 | 6.5947 | 0.0001 | -0.0133 | 3.7548 |
| $$\hat{σ}\_{4}$$ | 0.0003 | -0.0418 | 3.7832 | 0.0001 | -0.0099 | 2.4507 |
| **30** | $$\hat{σ}\_{H}$$ | 0.0007 | -0.0017 | 3.6787 | 0.0003 | -0.0009 | 13.0286 |
| $$\hat{σ}\_{1}$$ | 0.0006 | -0.0015 | 3.5097 | 0.0001 | -0.0004 | 3.5037 |
| $$\hat{σ}\_{2}$$ | 0.0002 | -0.0004 | 1.0000 | 0.0000 | -0.0002 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0001 | -0.0574 | 18.7603 | 0.0000 | -0.0131 | 9.6222 |
| $$\hat{σ}\_{4}$$ | 0.0001 | -0.0411 | 9.9767 | 0.0000 | -0.0097 | 5.6787 |
| **(4,4)** | **10** | $$\hat{σ}\_{H}$$ | 0.0016 | -0.0050 | 3.3795 | 0.0006 | -0.0029 | 12.5384 |
| $$\hat{σ}\_{1}$$ | 0.0014 | -0.0041 | 3.0659 | 0.0002 | -0.0008 | 3.0070 |
| $$\hat{σ}\_{2}$$ | 0.0005 | -0.0014 | 1.0000 | 0.0001 | -0.0003 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0002 | -0.0504 | 5.8928 | 0.0000 | -0.0115 | 3.4568 |
| $$\hat{σ}\_{4}$$ | 0.0003 | -0.0364 | 3.4249 | 0.0000 | -0.0086 | 2.2886 |
| **30** | $$\hat{σ}\_{H}$$ | 0.0005 | -0.0016 | 3.2006 | 0.0002 | -0.0009 | 10.0373 |
| $$\hat{σ}\_{1}$$ | 0.0005 | -0.0014 | 3.0093 | 0.0001 | -0.0003 | 2.5024 |
| $$\hat{σ}\_{2}$$ | 0.0002 | -0.0004 | 1.0000 | 0.0000 | -0.0001 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0001 | -0.0497 | 15.9120 | 0.0000 | -0.0114 | 6.9383 |
| $$\hat{σ}\_{4}$$ | 0.0001 | -0.0357 | 8.5788 | 0.0000 | -0.0084 | 4.0432 |

**Table 2.** The variances, biases, and the REs of estimators with respect to $\hat{σ}\_{2}$ when the ongoing process has a gamma distribution with unit parameters.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | $$L=2$$ | $$L=5$$ |
| $$(m,n)$$ | $$w$$ | **Estimators** | **Variance** | **Bias** | **RE** | **Variance** | **Bias** | **RE** |
| **(2, 2)** | **10** | $$\hat{σ}\_{H}$$ | 0.0138 | -0.0161 | 1.1733 | 0.0044 | -0.0084 | 2.6969 |
| $$\hat{σ}\_{1}$$ | 0.0133 | -0.0164 | 1.1309 | 0.0018 | -0.0038 | 1.1026 |
| $$\hat{σ}\_{2}$$ | 0.0118 | -0.0141 | 1.0000 | 0.0017 | -0.0036 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0059 | -0.1289 | 1.8779 | 0.0013 | -0.0303 | 1.3440 |
| $$\hat{σ}\_{4}$$ | 0.0087 | -0.0682 | 1.1099 | 0.0015 | -0.0160 | 1.0610 |
| **30** | $$\hat{σ}\_{H}$$ | 0.0049 | -0.0057 | 1.0972 | 0.0015 | -0.0035 | 2.4645 |
| $$\hat{σ}\_{1}$$ | 0.0050 | -0.0052 | 1.1250 | 0.0007 | -0.0013 | 1.0836 |
| $$\hat{σ}\_{2}$$ | 0.0044 | -0.0044 | 1.0000 | 0.0006 | -0.0011 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0022 | -0.1221 | 3.8389 | 0.0005 | -0.0281 | 2.1152 |
| $$\hat{σ}\_{4}$$ | 0.0033 | -0.0583 | 1.4933 | 0.0005 | -0.0141 | 1.2139 |
| **(4,2)** | **10** | $$\hat{σ}\_{H}$$ | 0.0134 | -0.0175 | 1.1344 | 0.0041 | -0.0079 | 2.3549 |
| $$\hat{σ}\_{1}$$ | 0.0129 | -0.0168 | 1.0894 | 0.0019 | -0.0037 | 1.0634 |
| $$\hat{σ}\_{2}$$ | 0.0119 | -0.0155 | 1.0000 | 0.0018 | -0.0034 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0059 | -0.1242 | 1.7648 | 0.0014 | -0.0288 | 1.2655 |
| $$\hat{σ}\_{4}$$ | 0.0088 | -0.0649 | 1.0772 | 0.0016 | -0.0157 | 1.0143 |
| **30** | $$\hat{σ}\_{H}$$ | 0.0048 | -0.0068 | 1.0907 | 0.0014 | -0.0028 | 2.2676 |
| $$\hat{σ}\_{1}$$ | 0.0048 | -0.0062 | 1.0844 | 0.0007 | -0.0016 | 1.0659 |
| $$\hat{σ}\_{2}$$ | 0.0044 | -0.0057 | 1.0000 | 0.0006 | -0.0014 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0022 | -0.1172 | 3.5816 | 0.0005 | -0.0271 | 2.0004 |
| $$\hat{σ}\_{4}$$ | 0.0033 | -0.0565 | 1.4650 | 0.0005 | -0.0136 | 1.1862 |
|  **(2,4)** | **10** | $$\hat{σ}\_{H}$$ | 0.0038 | -0.0089 | 1.4426 | 0.0013 | -0.0046 | 3.5363 |
| $$\hat{σ}\_{1}$$ | 0.0036 | -0.0080 | 1.3770 | 0.0005 | -0.0019 | 1.2520 |
| $$\hat{σ}\_{2}$$ | 0.0026 | -0.0062 | 1.0000 | 0.0004 | -0.0015 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0013 | -0.0709 | 2.3796 | 0.0003 | -0.0166 | 1.5561 |
| $$\hat{σ}\_{4}$$ | 0.0016 | -0.0522 | 1.6336 | 0.0003 | -0.0124 | 1.3088 |
| **30** | $$\hat{σ}\_{H}$$ | 0.0013 | -0.0030 | 1.3684 | 0.0004 | -0.0017 | 3.2457 |
| $$\hat{σ}\_{1}$$ | 0.0013 | -0.0028 | 1.3362 | 0.0002 | -0.0007 | 1.2319 |
| $$\hat{σ}\_{2}$$ | 0.0010 | -0.0019 | 1.0000 | 0.0001 | -0.0005 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0005 | -0.0679 | 5.2824 | 0.0001 | -0.0156 | 2.6451 |
| $$\hat{σ}\_{4}$$ | 0.0006 | -0.0491 | 3.1223 | 0.0001 | -0.0116 | 1.8754 |
|  **(4,4)** | **10** | $$\hat{σ}\_{H}$$ | 0.0035 | -0.0080 | 1.2901 | 0.0011 | 0.0012 | 2.8997 |
| $$\hat{σ}\_{1}$$ | 0.0033 | -0.0079 | 1.2211 | 0.0005 | 0.0005 | 1.1539 |
| $$\hat{σ}\_{2}$$ | 0.0027 | -0.0063 | 1.0000 | 0.0004 | 0.0004 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0014 | -0.0653 | 2.0335 | 0.0003 | 0.0005 | 0.7953 |
| $$\hat{σ}\_{4}$$ | 0.0017 | -0.0485 | 1.4698 | 0.0003 | 0.0005 | 0.8720 |
| **30** | $$\hat{σ}\_{H}$$ | 0.0012 | -0.0030 | 1.2221 | 0.0004 | -0.0015 | 2.6560 |
| $$\hat{σ}\_{1}$$ | 0.0012 | -0.0032 | 1.2133 | 0.0002 | -0.0005 | 1.1431 |
| $$\hat{σ}\_{2}$$ | 0.0010 | -0.0028 | 1.0000 | 0.0001 | -0.0004 | 1.0000 |
| $$\hat{σ}\_{3}$$ | 0.0005 | -0.0628 | 4.4396 | 0.0001 | -0.0142 | 2.2313 |
| $$\hat{σ}\_{4}$$ | 0.0007 | -0.0451 | 2.6933 | 0.0001 | -0.0107 | 1.6652 |