Supplemental material of "Higher-order bias corrections for kernel type density estimators on the unit or semi-infinite interval"

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We keep the notation in the main text.

S1. Supplemental results for Examples 1–5

Example 1 Using the Stirling number of the first kind, $s(\cdot,\cdot)$, we can prove that, for any $j \in \mathbb{N}$,

$$\mu_j(K^{(G)}_{x/\beta+1,\beta}(\cdot)) = \sum_{k=\lceil j/2 \rceil}^{j} \zeta_{j,k} x^{j-k}\beta^k,\ x \geq 0,$$

where

$$\zeta_{j,k} = \sum_{\ell=k}^{j} (-1)^{j+k-\ell}{}_jC_\ell\, s(\ell+1, \ell+1-k).$$

Hence, Assumption A4[p] holds for any $p \in \mathbb{N}$. For example,

$$\mu_1(K^{(G)}_{x/\beta+1,\beta}(\cdot)) = \beta, \quad \mu_2(K^{(G)}_{x/\beta+1,\beta}(\cdot)) = \beta x + 2\beta^2, \quad \mu_3(K^{(G)}_{x/\beta+1,\beta}(\cdot)) = 5\beta^2 x + 6\beta^3,$$
$$\mu_4(K^{(G)}_{x/\beta+1,\beta}(\cdot)) = 3\beta^2x^2 + 26\beta^3 x + 24\beta^4, \quad \mu_5(K^{(G)}_{x/\beta+1,\beta}(\cdot)) = 35\beta^3x^2 + 154\beta^4 x + 120\beta^5,$$
$$\mu_6(K^{(G)}_{x/\beta+1,\beta}(\cdot)) = 15\beta^3x^3 + 340\beta^4x^2 + 1044\beta^5 x + 720\beta^6, \quad \mu_8(K^{(G)}_{x/\beta+1,\beta}(\cdot)) = O(\beta^4(x+\beta)^4),$$

can be derived using a computer algebra system (e.g., Maple).

Example 2 Given $j \in \mathbb{N}$,

$$\mu_j(K^{(MIG_\varepsilon)}_{x/\beta+1,x+\beta}(\cdot)) = \sum_{k=\lceil j/2 \rceil}^{j} \zeta_{j,k} x^{j-k}\beta^k,\ x \geq 0,$$

for some constants $\zeta_{j,k}$'s, independent of β and x (we used a computer algebra system; Maple). For example,

$$\mu_1(K^{(MIG_\varepsilon)}_{x/\beta+1,x+\beta}(\cdot)) = (\varepsilon+1)\beta, \quad \mu_2(K^{(MIG_\varepsilon)}_{x/\beta+1,x+\beta}(\cdot)) = \beta x + (5\varepsilon+2)\beta^2,$$
$$\mu_3(K^{(MIG_\varepsilon)}_{x/\beta+1,x+\beta}(\cdot)) = 3(\varepsilon+2)\beta^2 x + (30\varepsilon+7)\beta^3,$$
$$\mu_4(K^{(MIG_\varepsilon)}_{x/\beta+1,x+\beta}(\cdot)) = 3\beta^2x^2 + 3(14\varepsilon+13)\beta^3 x + (229\varepsilon+37)\beta^4,$$
$$\mu_5(K^{(MIG_\varepsilon)}_{x/\beta+1,x+\beta}(\cdot)) = 15(\varepsilon+3)\beta^3x^2 + 5(105\varepsilon+62)\beta^4 x + (2165\varepsilon+266)\beta^5,$$
$$\mu_6(K^{(MIG_\varepsilon)}_{x/\beta+1,x+\beta}(\cdot)) = 15\beta^3x^3 + 45(9\varepsilon+13)\beta^4x^2 + 30(233\varepsilon+100)\beta^5 x + (24576\varepsilon+2431)\beta^6,$$
$$\mu_8(K^{(MIG_\varepsilon)}_{x/\beta+1,x+\beta}(\cdot)) = O(\beta^4(x+\beta)^4).$$

The moments of the following examples may be rational functions in x:

$$\text{M1}[p^*].\ \text{for } j \in \mathbb{N},\ \mu_j(K(\cdot;x,\beta)) = \sum_{k=\lceil j/2 \rceil}^{j} \zeta_{j,k}\, x^{j-k}\beta^k + \chi_{\{j\geq p^*\}}\beta^j Q_j\Big(\frac{x}{\beta}+1\Big),\ x \geq 0$$

for some $p^* \in \mathbb{N}$ and constants $\zeta_{j,k}$'s, independent of β and x. Here, Q_j is the rational function (independent of β and x) satisfying $\rho|Q_j(\rho)| \leq \overline{Q}_j < \infty$. For this, we further distinguish two types (they are referred to as M1[p^*] of type I or II), as follows:

$$Q_j(\rho) = \sum_{\ell_1=1}^{m_{1,j}} \sum_{\ell_2=1}^{m_{2,j}} \frac{\zeta_{j,\ell_1,\ell_2}}{(\rho + d_{\ell_2})^{\ell_1}} \text{ (type I) or } Q_j(\rho) = \frac{(\ell^*-1)\text{th polynomial in } \rho}{\ell^*\text{th polynomial in } \rho} \text{ (type II)}$$

for some $m_{1,j}, m_{2,j}, \ell^* \in \mathbb{N}$ and constants ζ_{j,ℓ_1,ℓ_2}'s and d_{ℓ_2}'s.

Example 3 The LN_ν KDE, $\nu = -1/2$ or $1/2$, satisfies M1[p^*] of type I for $p^* = 4$ or $p^* = 2$, respectively (we used a computer algebra system; Maple). For example,

$$\begin{aligned}
&\mu_1(K^{(LN)}_{\mu_\beta(x),\sigma^2_\beta(x),-1/2}(\cdot)) = \beta, \quad \mu_2(K^{(LN)}_{\mu_\beta(x),\sigma^2_\beta(x),-1/2}(\cdot)) = \beta x + 2\beta^2,\\
&\mu_3(K^{(LN)}_{\mu_\beta(x),\sigma^2_\beta(x),-1/2}(\cdot)) = 6\beta^2 x + 8\beta^3,\\
&\mu_4(K^{(LN)}_{\mu_\beta(x),\sigma^2_\beta(x),-1/2}(\cdot)) = 3\beta^2x^2 + 40\beta^3 x + 57\beta^4 + \frac{6\beta^5}{x+\beta} + \frac{\beta^6}{(x+\beta)^2},\\
&\mu_1(K^{(LN)}_{\mu_\beta(x),\sigma^2_\beta(x),1/2}(\cdot)) = 2\beta, \quad \mu_2(K^{(LN)}_{\mu_\beta(x),\sigma^2_\beta(x),1/2}(\cdot)) = \beta x + 7\beta^2 + \frac{\beta^3}{x+\beta}.
\end{aligned}$$

The $\text{LN}_{\pm(J+1/2)}$ KDEs, where $J \in \mathbb{N}$, satisfy M1[1] of type I. On the other hand, the LN_ν KDE, when ν is not an half-integer, satisfies M2.

Example 4 The case $\nu = -1/2$ or $1/2$ corresponds to the MIG_ε KDE, with $\varepsilon = 0$ or 1, respectively (see Example 2). The $\text{MB}_{\pm(J+1/2)}$ KDEs, where $J \in \mathbb{N}$, satisfy M1[1] of type II. On the other hand, the MB_ν KDE, when ν is not an half-integer, satisfies M2.

Example 5 The case $\gamma = 1$ corresponds to the gamma KDE (see Example 1). The $\text{A}_{1/J}$ KDE, where $J \in \mathbb{N}\backslash\{1\}$, and $\text{A}_{-1/J}$ KDE, where $J \in \mathbb{N}$, satisfy M1[2] of type I. On the other hand, the A_γ KDE, except for $\gamma \in \{1/m \mid m \in \mathbb{Z}, m \neq 0\}$, satisfies M3.

S2. Supplemental simulation results in Section 5

In Table 1 of the main text, we reported the results for $n = 200$ and $a = 0.9$ only. The complete results (including the standard deviations over 1000 replications) for $n = 100, 200$ and $a = 0.1, 0.5, 0.9$ are given in Tables S1–S4. We observe that the average ISEs, $\sum_{k=1}^{1000} ISE^{[k]}/1000$, decreased, as the sample size n increased, and that, for the cases A and B, the average ISE of $\widehat{f}_{\beta,\#^3_{(1,a,1/a)}}$ decreased, as a was close to one.

As discussed in the main text of Section 5, overall, our bias corrections worked well, in the sense of the average ISE, which is in agreement with our theoretical findings in Section 3.

Table S1: Case A. The average ISEs$\times 10^6$ of the bias-uncorrected/corrected estimators, where A_γ, IG, BS, RIG, and $LN_{-1/2}$ stand for the asymmetric KDEs (see Examples 1, 2, 3, and 5). The number in the parentheses stands for the standard deviation$\times 10^6$ of the ISEs.

$n = 100$

	p	a	A_2	$A_{1.5}$	A_1	$A_{0.5}$	$A_{-0.5}$	A_{-1}	$A_{-1.5}$	A_{-2}	IG	BS	RIG	$LN_{-1/2}$
uncorrected			3389	3263	3006	2914	3201	3392	3735	4135	3316	3539	3751	3288
			(2946)	(3216)	(3144)	(3206)	(3235)	(2898)	(2674)	(2941)	(3346)	(3411)	(3468)	(3317)
ADD	2	0.1	2715	2607	2498	2595	2782	2750	3099	3419	2627	2944	3071	2564
			(3023)	(2994)	(3129)	(3061)	(2931)	(2606)	(2595)	(2666)	(2843)	(3396)	(3312)	(2756)
		0.5	2369	2356	2255	2382	2531	2519	2905	3344	2352	2574	2738	2328
			(3005)	(3185)	(3023)	(2826)	(2405)	(2279)	(2362)	(2572)	(2615)	(2992)	(3271)	(2649)
		0.9	2268	2326	2207	2372	2511	2565	2956	3476	2336	2564	2713	2327
			(2627)	(3185)	(2856)	(2781)	(2262)	(2432)	(2442)	(2700)	(2587)	(3042)	(3265)	(2682)
ADD	3	0.1	3533	3182	2866	2773	3193	3675	4237	4863	3336	2975	2954	3332
			(3758)	(3619)	(3584)	(3630)	(3633)	(3771)	(4052)	(4380)	(3809)	(3585)	(3533)	(3841)
		0.5	2221	2162	2000	2298	2502	2626	3311	4118	2281	2413	2513	2241
			(3074)	(3374)	(2786)	(2878)	(2710)	(2794)	(3047)	(3622)	(3047)	(3052)	(3258)	(2994)
		0.9	2090	2034	2004	2314	2446	2568	3455	4465	2176	2299	2392	2138
			(2814)	(2971)	(2784)	(2890)	(2411)	(2450)	(2996)	(3494)	(2661)	(2858)	(3142)	(2752)
TS	2	0.1	2434	2396	2334	2600	2653	2329	2500	2752	2314	2600	2717	2295
			(3047)	(3023)	(3016)	(3101)	(2465)	(2270)	(2542)	(2945)	(2616)	(2988)	(3162)	(2695)
		0.5	2061	2164	2071	2407	2557	2298	2767	3376	2202	2386	2477	2121
			(2459)	(3016)	(2690)	(2808)	(2242)	(2327)	(2630)	(2963)	(2478)	(2912)	(3096)	(2549)
		0.9	2026	2091	2066	2411	2606	2390	2967	3665	2165	2369	2470	2117
			(2383)	(2789)	(2702)	(2882)	(2303)	(2462)	(2782)	(3082)	(2581)	(2934)	(3163)	(2602)
TS	3	0.1	3864	3426	2997	2832	3307	3905	4687	5457	3545	3143	2990	3551
			(3911)	(3769)	(3637)	(3658)	(3696)	(3826)	(4282)	(4742)	(3929)	(3825)	(3672)	(3946)
		0.5	2593	2303	<u>1989</u>	2321	2631	2898	3777	4862	2456	2310	2355	2459
			(3315)	(3409)	(2969)	(2862)	(2739)	(2879)	(3218)	(3788)	(3040)	(3218)	(3172)	(3105)
		0.9	2574	2146	<u>1933</u>	2342	2614	2820	3941	5242	2380	2259	2313	2375
			(3246)	(2979)	(2919)	(2862)	(2690)	(2639)	(3251)	(3842)	(2979)	(3106)	(2998)	(3041)
JF	2	0.1	2692	2600	2483	2591	2753	2738	3088	3364	2615	2949	3044	2556
			(3010)	(3009)	(3054)	(3063)	(2823)	(2617)	(2610)	(2585)	(2842)	(3413)	(3291)	(2766)
		0.5	2226	2272	2150	2388	2531	2464	2838	3279	2266	2501	2643	2243
			(2795)	(3141)	(2849)	(2819)	(2272)	(2317)	(2436)	(2629)	(2492)	(2986)	(3182)	(2618)
		0.9	2029	2140	2069	2401	2581	2384	2898	3571	2198	2413	2533	2164
			(2377)	(3009)	(2680)	(2839)	(2249)	(2440)	(2624)	(3034)	(2574)	(2989)	(3202)	(2619)
JF	3	0.1	3435	3147	2807	2634	2748	3215	3805	4262	3013	3400	3659	3045
			(2975)	(2966)	(3219)	(3446)	(3380)	(2995)	(2942)	(2912)	(3228)	(3149)	(3232)	(3182)
		0.5	2368	2328	2154	2343	2455	2663	3136	3617	2396	2537	2694	2401
			(2960)	(3325)	(2877)	(2856)	(2520)	(2590)	(2600)	(2751)	(2833)	(3082)	(3205)	(2849)
		0.9	2139	2174	2112	2332	2464	2592	3124	3667	2297	2445	2622	2325
			(2615)	(3166)	(2933)	(2876)	(2467)	(2411)	(2527)	(2774)	(2527)	(3033)	(3412)	(2754)

The underlined or double-underlined number indicates the smallest or second smallest average ISE, respectively.

Table S1: (continued).

$n = 200$

	p	a	A_2	$A_{1.5}$	A_1	$A_{0.5}$	$A_{-0.5}$	A_{-1}	$A_{-1.5}$	A_{-2}	IG	BS	RIG	$LN_{-1/2}$
uncorrected			2107	2005	1802	1690	1831	2048	2283	2489	1971	2157	2329	1942
			(2285)	(2353)	(2171)	(2077)	(2098)	(2044)	(2018)	(1913)	(2229)	(2372)	(2475)	(2166)
ADD	2	0.1	1579	1532	1458	1421	1607	1650	1779	1988	1566	1724	1763	1538
			(1884)	(2155)	(2185)	(1739)	(2011)	(1932)	(1508)	(1902)	(2153)	(2337)	(2235)	(2134)
		0.5	1323	1282	1256	1333	1473	1465	1624	1850	1361	1516	1599	1363
			(1799)	(1793)	(1805)	(1724)	(1787)	(1580)	(1259)	(1320)	(1884)	(2257)	(2315)	(1991)
		0.9	1301	1265	1232	1316	1468	1437	1639	1886	1336	1505	1589	1337
			(1717)	(1747)	(1734)	(1728)	(1664)	(1300)	(1252)	(1321)	(1848)	(2273)	(2330)	(1950)
ADD	3	0.1	1879	1723	1562	1460	1663	1924	2204	2538	1767	1662	1733	1768
			(2531)	(2418)	(2350)	(2194)	(2136)	(2370)	(2330)	(2617)	(2397)	(2406)	(2346)	(2428)
		0.5	1255	1176	1165	1267	1466	1421	1744	2083	1244	1355	1401	1239
			(1904)	(1798)	(1796)	(1622)	(1750)	(1442)	(1629)	(1856)	(1689)	(2061)	(2010)	(1769)
		0.9	1192	1122	1153	1232	1417	1401	1770	2282	1220	1296	1372	1220
			(1429)	(1733)	(1819)	(1455)	(1420)	(1380)	(1546)	(1869)	(1443)	(1812)	(1996)	(1745)
TS	2	0.1	1381	1384	1352	1432	1577	1404	1361	1445	1376	1560	1571	1331
			(1807)	(2093)	(2119)	(1798)	(1950)	(1813)	(1324)	(1382)	(1995)	(2280)	(2189)	(1923)
		0.5	1239	1217	1198	1333	1475	1296	1417	1678	1245	1378	1435	1221
			(1730)	(1760)	(1749)	(1711)	(1529)	(1238)	(1286)	(1470)	(1619)	(2006)	(2134)	(1629)
		0.9	1202	1182	1180	1313	1475	1320	1512	1826	1238	1374	1421	1238
			(1714)	(1717)	(1729)	(1555)	(1534)	(1318)	(1401)	(1533)	(1579)	(2025)	(2131)	(1683)
TS	3	0.1	2029	1814	1616	1495	1692	2005	2388	2749	1844	1673	1614	1853
			(2580)	(2422)	(2366)	(2216)	(2106)	(2326)	(2513)	(2579)	(2405)	(2437)	(2396)	(2434)
		0.5	1365	1219	1106	1304	1514	1490	1908	2403	1262	1299	1364	1264
			(1682)	(1904)	(1791)	(1612)	(1836)	(1473)	(1571)	(1884)	(1719)	(1858)	(2121)	(1713)
		0.9	1331	1173	1080	1257	1480	1478	1983	2659	1232	1272	1310	1229
			(1518)	(1823)	(1750)	(1413)	(1423)	(1367)	(1616)	(1996)	(1634)	(1811)	(1895)	(1619)
JF	2	0.1	1570	1512	1456	1419	1600	1641	1767	1963	1559	1728	1775	1533
			(1907)	(2131)	(2180)	(1739)	(1991)	(1931)	(1505)	(1873)	(2146)	(2357)	(2353)	(2128)
		0.5	1280	1244	1226	1330	1480	1382	1551	1796	1303	1465	1537	1302
			(1779)	(1757)	(1771)	(1715)	(1760)	(1285)	(1224)	(1398)	(1657)	(2198)	(2243)	(1751)
		0.9	1208	1185	1194	1311	1466	1343	1516	1802	1248	1389	1441	1247
			(1713)	(1704)	(1772)	(1556)	(1526)	(1312)	(1363)	(1509)	(1585)	(2036)	(2149)	(1687)
JF	3	0.1	1926	1855	1656	1484	1587	1891	2135	2351	1808	1993	2085	1793
			(2021)	(2420)	(2211)	(2139)	(2106)	(2064)	(1881)	(1792)	(2332)	(2464)	(2383)	(2201)
		0.5	1287	1225	1205	1291	1447	1489	1694	1951	1341	1462	1493	1338
			(1819)	(1789)	(1809)	(1773)	(1693)	(1590)	(1298)	(1521)	(1921)	(2292)	(2044)	(1978)
		0.9	1226	1157	1182	1238	1397	1466	1705	1997	1245	1400	1416	1266
			(1798)	(1726)	(1827)	(1457)	(1387)	(1552)	(1253)	(1499)	(1379)	(2168)	(1983)	(1716)

Table S2: Case B. The average ISEs$\times 10^6$ of the bias-uncorrected/corrected estimators, where A_γ, IG, BS, RIG, and $LN_{-1/2}$ stand for the asymmetric KDEs (see Examples 1, 2, 3, and 5). The number in the parentheses stands for the standard deviation$\times 10^6$ of the ISEs.

$n = 100$

	p	a	A_2	$A_{1.5}$	A_1	$A_{0.5}$	$A_{-0.5}$	A_{-1}	$A_{-1.5}$	A_{-2}	IG	BS	RIG	$LN_{-1/2}$
uncorrected			6452	6480	6049	5578	5924	6652	7505	7983	6565	7184	7466	6478
			(5871)	(7440)	(7483)	(7131)	(6366)	(6154)	(6561)	(6147)	(7350)	(8495)	(7694)	(7071)
ADD	2	0.1	5247	5315	4989	5022	5215	5615	6042	6552	5220	5827	5942	5239
			(5829)	(7027)	(6508)	(7021)	(6105)	(5991)	(5760)	(5512)	(6176)	(7052)	(6380)	(6481)
		0.5	4142	4365	4490	4686	4684	4707	5340	5912	4537	5046	5400	4419
			(4704)	(5369)	(5935)	(6591)	(5231)	(4215)	(4569)	(4675)	(4995)	(5758)	(6548)	(5051)
		0.9	4069	4312	4368	4659	4693	4662	5337	5940	4505	5013	5208	4337
			(4699)	(5469)	(5789)	(6653)	(5309)	(4154)	(4556)	(4668)	(5644)	(5811)	(6185)	(4881)
ADD	3	0.1	5417	5211	4748	4816	5292	5966	6757	7402	5431	5794	5874	5393
			(6988)	(7563)	(6885)	(7329)	(6916)	(7284)	(7439)	(7606)	(7545)	(7618)	(6431)	(7726)
		0.5	3697	3819	4109	4532	4613	4450	5132	5926	4129	4543	4495	4039
			(5053)	(5180)	(5736)	(6727)	(5319)	(4371)	(5057)	(5721)	(4401)	(5325)	(5297)	(4667)
		0.9	3234	3627	3893	4369	4589	4258	5012	6105	4007	4447	4324	3805
			(3697)	(5093)	(5439)	(6268)	(5429)	(4240)	(4902)	(5671)	(4310)	(5362)	(5313)	(4336)
TS	2	0.1	4577	4593	4681	4980	4983	4496	4619	4870	4627	5312	5344	4638
			(5600)	(6078)	(6526)	(7108)	(5656)	(4617)	(4379)	(4458)	(5460)	(6822)	(6464)	(6346)
		0.5	3583	3869	4081	4529	4674	4069	4447	5130	3926	4588	4490	3753
			(4768)	(5368)	(5697)	(6471)	(5223)	(4352)	(3999)	(4223)	(4352)	(5425)	(5480)	(4394)
		0.9	3498	3785	4013	4503	4714	4124	4765	5547	3906	4501	4432	3730
			(4686)	(5342)	(5594)	(6483)	(5304)	(4172)	(4793)	(5000)	(4330)	(5404)	(5530)	(4375)
TS	3	0.1	5308	4990	4584	4747	5362	6117	7263	8087	5488	5298	5144	5548
			(7213)	(7508)	(7008)	(7387)	(6899)	(7029)	(7961)	(8237)	(7766)	(7936)	(6567)	(8052)
		0.5	3057	3326	3732	4434	4785	4618	5877	7406	3855	4157	4036	3708
			(4216)	(5215)	(5687)	(6652)	(5835)	(4666)	(5427)	(6395)	(4611)	(5016)	(5222)	(4714)
		0.9	<u>2874</u>	<u>2987</u>	3583	4353	4738	4560	6292	8495	3813	4056	3931	3570
			(3934)	(4640)	(5510)	(6700)	(5585)	(4569)	(5859)	(6950)	(4414)	(4929)	(5309)	(4658)
JF	2	0.1	5175	5307	4974	5014	5221	5587	6058	6421	5224	5804	5926	5267
			(5740)	(7057)	(6524)	(7018)	(6112)	(6018)	(5957)	(4998)	(6320)	(7074)	(6431)	(6608)
		0.5	3961	4195	4416	4610	4682	4554	5065	5614	4265	4964	5038	4150
			(4743)	(5412)	(6560)	(6549)	(5257)	(4441)	(4511)	(4498)	(4603)	(5891)	(6210)	(4706)
		0.9	3548	3887	4070	4506	4748	4155	4777	5468	3980	4656	4554	3817
			(4640)	(5438)	(5609)	(6477)	(5474)	(4146)	(4683)	(4878)	(4351)	(5579)	(5625)	(4403)
JF	3	0.1	6808	6348	6139	5570	5760	6905	7568	8463	6715	7146	7315	6572
			(6137)	(5891)	(6817)	(7097)	(5966)	(6381)	(5599)	(6085)	(7364)	(8813)	(6553)	(6831)
		0.5	4303	4329	4393	4627	4554	4711	5400	6201	4540	4919	5137	4511
			(5092)	(5433)	(5794)	(6618)	(4591)	(4243)	(4661)	(5190)	(4965)	(5559)	(6004)	(5007)
		0.9	3877	4000	4195	4439	4520	4579	5265	6262	4301	4689	4793	4257
			(4514)	(5052)	(5623)	(6062)	(5266)	(4248)	(4574)	(5216)	(4557)	(5280)	(5434)	(4767)

The underlined or double-underlined number indicates the smallest or second smallest average ISE, respectively.

Table S2: (continued).

$n = 200$

	p	a	A_2	$A_{1.5}$	A_1	$A_{0.5}$	$A_{-0.5}$	A_{-1}	$A_{-1.5}$	A_{-2}	IG	BS	RIG	$LN_{-1/2}$
uncorrected			3726	3531	3222	2882	3084	3701	4098	4515	3470	3749	4063	3456
			(2917)	(3335)	(3304)	(2891)	(2702)	(3209)	(2766)	(2800)	(3329)	(3342)	(3441)	(3299)
ADD	2	0.1	2917	2798	2620	2502	2656	3020	3342	3711	2839	3083	3248	2792
			(2771)	(2849)	(2843)	(2772)	(2581)	(3253)	(2720)	(2883)	(3334)	(3403)	(3163)	(3223)
		0.5	2601	2515	2342	2357	2482	2655	3028	3463	2476	2772	2948	2502
			(2983)	(2955)	(2724)	(2720)	(2472)	(2291)	(2244)	(2400)	(2676)	(2931)	(3172)	(2786)
		0.9	2464	2403	2321	2336	2477	2627	3010	3557	2430	2753	2893	2420
			(2704)	(2782)	(2757)	(2730)	(2500)	(2259)	(2208)	(2492)	(2627)	(3084)	(3116)	(2682)
ADD	3	0.1	2859	2676	2488	2433	2591	3024	3448	3886	2750	2990	3161	2721
			(2899)	(2870)	(2855)	(2804)	(2673)	(3426)	(3219)	(3471)	(3431)	(3034)	(3043)	(3438)
		0.5	2155	2195	2122	2240	2388	2398	2806	3350	2226	2501	2592	2209
			(2666)	(2975)	(2729)	(2663)	(2476)	(2083)	(2245)	(2633)	(2530)	(2955)	(3032)	(2584)
		0.9	2015	2075	2074	2206	2322	2318	2863	3425	2138	2414	2532	2125
			(2362)	(2889)	(2786)	(2633)	(2416)	(2079)	(2311)	(2560)	(2483)	(2902)	(3113)	(2509)
TS	2	0.1	2566	2473	2398	2437	2553	2627	2742	2920	2544	2823	2880	2503
			(2827)	(2779)	(2799)	(2720)	(2414)	(3028)	(2415)	(2331)	(3289)	(3418)	(3107)	(3209)
		0.5	2095	2164	2137	2297	2445	2302	2600	2934	2199	2512	2624	2173
			(2532)	(2817)	(2717)	(2669)	(2382)	(2090)	(2273)	(2304)	(2511)	(2907)	(3070)	(2612)
		0.9	2015	2131	2103	2289	2443	2279	2602	3083	2191	2467	2592	2101
			(2276)	(2834)	(2722)	(2676)	(2387)	(2035)	(2153)	(2376)	(2548)	(2862)	(3097)	(2479)
TS	3	0.1	2612	2436	2268	2332	2559	2979	3472	3954	2612	2666	2860	2621
			(3095)	(3379)	(2936)	(2800)	(2593)	(3508)	(3455)	(3781)	(3435)	(3288)	(3531)	(3514)
		0.5	1740	1824	1943	2212	2430	2328	3022	3909	2057	2299	2371	1965
			(2207)	(2660)	(2761)	(2664)	(2382)	(2113)	(2558)	(3165)	(2451)	(2841)	(3233)	(2380)
		0.9	1580	1633	1901	2193	2363	2320	3321	4510	2023	2191	2259	1862
			(1841)	(2131)	(2772)	(2667)	(2263)	(2034)	(2739)	(3380)	(2408)	(2727)	(3040)	(2061)
JF	2	0.1	2896	2778	2604	2493	2655	3022	3326	3680	2827	3046	3240	2773
			(2777)	(2850)	(2829)	(2767)	(2582)	(3286)	(2738)	(2862)	(3338)	(3389)	(3201)	(3219)
		0.5	2404	2355	2290	2318	2453	2541	2954	3344	2384	2694	2819	2341
			(2710)	(2878)	(2822)	(2717)	(2432)	(2270)	(2335)	(2453)	(2656)	(2944)	(3134)	(2684)
		0.9	2078	2171	2126	2291	2445	2321	2699	3148	2213	2527	2632	2156
			(2337)	(2827)	(2730)	(2678)	(2397)	(2027)	(2162)	(2351)	(2502)	(2862)	(3075)	(2514)
JF	3	0.1	3492	3294	3035	2768	2937	3438	3869	4252	3252	3445	3750	3236
			(2868)	(2850)	(2771)	(2940)	(2696)	(2779)	(2915)	(2872)	(2887)	(2804)	(3116)	(2824)
		0.5	2468	2360	2274	2289	2410	2584	2925	3356	2408	2654	2840	2420
			(2790)	(2856)	(2785)	(2742)	(2548)	(2237)	(2286)	(2446)	(2675)	(2973)	(3197)	(2727)
		0.9	2240	2263	2187	2230	2346	2454	2925	3439	2295	2555	2699	2282
			(2357)	(2939)	(2782)	(2637)	(2537)	(2017)	(2306)	(2536)	(2499)	(2987)	(3044)	(2526)

Table S3: Case C. The average ISEs$\times 10^6$ of the bias-uncorrected/corrected estimators, where A_γ, IG, BS, RIG, and $LN_{-1/2}$ stand for the asymmetric KDEs (see Examples 1, 2, 3, and 5). The number in the parentheses stands for the standard deviation$\times 10^6$ of the ISEs.

$n = 100$

	p	a	A_2	$A_{1.5}$	A_1	$A_{0.5}$	$A_{-0.5}$	A_{-1}	$A_{-1.5}$	A_{-2}	IG	BS	RIG	$LN_{-1/2}$
uncorrected			7178	6656	6045	5411	5763	6688	7454	8060	6462	6926	7545	6482
			(4902)	(4722)	(4604)	(4325)	(4565)	(4967)	(4944)	(4936)	(4970)	(5060)	(5310)	(4912)
ADD	2	0.1	7477	6978	6194	5339	5869	6845	7774	8532	6627	7148	7936	6707
			(4909)	(4915)	(4651)	(4249)	(4682)	(4964)	(4932)	(4979)	(4884)	(5261)	(5564)	(4897)
		0.5	7818	7199	6286	5335	5818	7126	8381	9492	6744	7294	8162	6814
			(5031)	(4894)	(4526)	(4440)	(4394)	(4465)	(4841)	(5337)	(4767)	(5267)	(5476)	(4653)
		0.9	7788	7200	6309	5299	5871	7269	8489	9430	6769	7310	8144	6814
			(4900)	(4911)	(4600)	(4395)	(4562)	(4511)	(4977)	(5501)	(4667)	(5238)	(5321)	(4677)
ADD	3	0.1	7358	6711	5920	<u><u>5164</u></u>	5561	6482	7569	8409	6311	6995	7948	6369
			(5172)	(4992)	(4618)	(4128)	(4685)	(5134)	(5715)	(5877)	(5081)	(5189)	(5554)	(5091)
		0.5	8374	7636	6468	5301	5588	6191	7015	8125	6483	7503	8523	6620
			(5196)	(5102)	(4652)	(4386)	(4253)	(4423)	(4727)	(5347)	(4598)	(5307)	(5645)	(4709)
		0.9	8534	7766	6495	5237	5600	6133	6967	7704	6527	7606	8770	6583
			(4637)	(5094)	(4742)	(4285)	(4238)	(4388)	(4840)	(5199)	(4582)	(5288)	(5678)	(4587)
TS	2	0.1	7898	7258	6314	5331	5891	7612	9214	10476	6882	7345	8292	6992
			(5059)	(4934)	(4687)	(4287)	(4530)	(4595)	(4846)	(5234)	(4659)	(5119)	(5531)	(4737)
		0.5	8570	7694	6543	5303	5961	7884	9762	11405	7341	7537	8715	7681
			(5066)	(5186)	(4768)	(4403)	(4506)	(4681)	(5929)	(7180)	(4621)	(5017)	(5451)	(4491)
		0.9	8522	7698	6582	5274	5797	6898	8032	9209	6972	7577	8757	7179
			(4612)	(5161)	(4980)	(4410)	(4402)	(4628)	(5375)	(6409)	(4654)	(5075)	(5430)	(4728)
TS	3	0.1	7808	6964	6009	<u>5117</u>	5524	6566	7881	8975	6408	7103	8213	6491
			(5548)	(5183)	(4798)	(4154)	(4735)	(5264)	(5942)	(6215)	(5214)	(5329)	(5694)	(5265)
		0.5	8803	7906	6719	5274	5527	5981	6672	7750	6328	7667	9289	6484
			(5285)	(5011)	(4807)	(4447)	(4431)	(4881)	(5048)	(5534)	(4828)	(5196)	(5335)	(5092)
		0.9	8880	8132	6809	5212	5430	5725	6468	7659	6291	7761	9706	6474
			(4900)	(5046)	(5038)	(4319)	(4166)	(4412)	(5121)	(5914)	(4647)	(5221)	(5382)	(5156)
JF	2	0.1	7493	6994	6202	5332	5872	6873	7797	8585	6645	7165	7966	6747
			(4922)	(4913)	(4652)	(4254)	(4727)	(4899)	(4931)	(4990)	(4860)	(5262)	(5583)	(5073)
		0.5	8130	7402	6439	5313	5903	7698	9224	10554	6942	7437	8329	6986
			(5080)	(5080)	(4748)	(4396)	(4431)	(4377)	(4948)	(5869)	(4652)	(5257)	(5394)	(4634)
		0.9	8416	7628	6521	5283	5837	7051	8131	9201	7006	7525	8678	7182
			(4791)	(5136)	(4763)	(4421)	(4440)	(4656)	(5287)	(6011)	(4645)	(5060)	(5417)	(4661)
JF	3	0.1	7367	6837	6222	5370	5685	6874	7657	8392	6591	6950	7519	6633
			(4911)	(4754)	(4617)	(4227)	(4240)	(5031)	(5011)	(5431)	(4805)	(4819)	(4898)	(4808)
		0.5	7739	7266	6365	5264	5770	6866	7692	8399	6741	7389	8264	6829
			(4941)	(4791)	(4594)	(4383)	(4389)	(4362)	(4609)	(4672)	(4499)	(5161)	(5440)	(4560)
		0.9	7797	7347	6425	5223	5792	6842	7576	8122	6845	7500	8471	6936
			(4804)	(5022)	(4746)	(4288)	(4305)	(4264)	(4179)	(4494)	(4614)	(5296)	(5572)	(4503)

The underlined or double-underlined number indicates the smallest or second smallest average ISE, respectively.

Table S3: (continued).

$n = 200$

	p	a	A_2	$A_{1.5}$	A_1	$A_{0.5}$	$A_{-0.5}$	A_{-1}	$A_{-1.5}$	A_{-2}	IG	BS	RIG	$LN_{-1/2}$
uncorrected			4241	3911	3492	3048	3224	3774	4248	4696	3646	3977	4359	3681
			(2773)	(2770)	(2659)	(2492)	(2634)	(2799)	(2842)	(2942)	(2777)	(2813)	(2895)	(2802)
ADD	2	0.1	4455	4042	3544	2933	3136	3779	4435	5108	3670	4055	4549	3701
			(2927)	(2823)	(2740)	(2504)	(2432)	(2612)	(3050)	(3308)	(2657)	(2887)	(3032)	(2689)
		0.5	4505	4049	3426	2835	3038	4127	5337	6288	3667	4058	4554	3695
			(2852)	(2712)	(2322)	(2041)	(2202)	(3139)	(3757)	(4285)	(2777)	(2893)	(3076)	(2816)
		0.9	4472	3979	3424	2818	3023	4313	5776	6805	3651	4057	4526	3666
			(2741)	(2572)	(2305)	(2018)	(2200)	(3153)	(3886)	(4392)	(3011)	(2917)	(3064)	(2819)
ADD	3	0.1	4466	4048	3542	2900	3083	3662	4271	4836	3599	4081	4578	3614
			(2965)	(2846)	(2749)	(2268)	(2371)	(2571)	(2847)	(3189)	(2587)	(2902)	(3048)	(2595)
		0.5	4676	4095	3484	2816	3013	3594	4338	5097	3513	4077	4657	3591
			(3053)	(2760)	(2436)	(2032)	(2162)	(2621)	(3058)	(3460)	(2557)	(2901)	(3272)	(2739)
		0.9	4877	4137	3433	2804	3035	3764	4633	5523	3593	4104	4760	3626
			(3125)	(2908)	(2415)	(2021)	(2194)	(2788)	(3332)	(4130)	(2697)	(2964)	(3400)	(2866)
TS	2	0.1	4691	4169	3588	2894	3115	4307	5705	6884	3773	4168	4706	3805
			(3013)	(2873)	(2584)	(2275)	(2435)	(3167)	(3759)	(4343)	(2745)	(2988)	(3120)	(2779)
		0.5	4851	4234	3560	2816	2983	5476	7348	8759	4146	4187	4780	4273
			(3053)	(2842)	(2471)	(2019)	(2127)	(3967)	(5067)	(5958)	(3164)	(2836)	(3179)	(3257)
		0.9	4791	4194	3546	2806	2995	4477	5837	6713	3942	4188	4825	4076
			(2936)	(2741)	(2474)	(2032)	(2183)	(3344)	(4336)	(4960)	(3068)	(2853)	(3263)	(3196)
TS	3	0.1	4651	4142	3522	2873	3052	3634	4308	4945	3586	4116	4734	3605
			(3083)	(2902)	(2471)	(2253)	(2358)	(2579)	(2884)	(3258)	(2601)	(2756)	(3154)	(2616)
		0.5	4948	4297	3560	2803	2946	3459	4065	4667	3541	4234	5141	3579
			(3240)	(2845)	(2409)	(2026)	(2074)	(2499)	(2923)	(3188)	(2591)	(2938)	(3497)	(2780)
		0.9	5367	4335	3548	2792	2979	3499	4109	4943	3509	4346	5557	3520
			(3543)	(2918)	(2440)	(2003)	(2131)	(2480)	(3061)	(4306)	(2511)	(3068)	(3774)	(2551)
JF	2	0.1	4471	4049	3550	2932	3132	3794	4449	5139	3677	4073	4561	3694
			(2923)	(2827)	(2748)	(2512)	(2433)	(2634)	(3048)	(3338)	(2662)	(2920)	(3060)	(2670)
		0.5	4688	4147	3450	2821	3036	4601	6325	7562	3744	4126	4635	3773
			(3021)	(3003)	(2278)	(2031)	(2237)	(3395)	(4163)	(4860)	(2844)	(2917)	(3086)	(2892)
		0.9	4730	4156	3525	2802	3006	4596	6092	6985	3948	4179	4759	4063
			(2821)	(2642)	(2382)	(2000)	(2194)	(3391)	(4346)	(4864)	(3075)	(2854)	(3207)	(3169)
JF	3	0.1	4214	3921	3511	2959	3158	3781	4290	4676	3649	3955	4323	3664
			(2922)	(2919)	(2812)	(2592)	(2704)	(2709)	(2954)	(2881)	(2680)	(2914)	(3036)	(2685)
		0.5	4431	4030	3444	2807	3031	3908	4639	5127	3577	4074	4561	3625
			(2859)	(2741)	(2364)	(2037)	(2214)	(2910)	(3279)	(3372)	(2565)	(2968)	(3134)	(2681)
		0.9	4414	3994	3472	2809	3064	4211	4984	5425	3656	4077	4656	3774
			(2877)	(2692)	(2665)	(2030)	(2284)	(3016)	(3257)	(3365)	(2687)	(3013)	(3301)	(3000)

Table S4: Case D. The average ISEs$\times 10^6$ of the bias-uncorrected/corrected estimators, where A_γ, IG, BS, RIG, and $LN_{-1/2}$ stand for the asymmetric KDEs (see Examples 1, 2, 3, and 5). The number in the parentheses stands for the standard deviation$\times 10^6$ of the ISEs.

$n = 100$

	p	a	A_2	$A_{1.5}$	A_1	$A_{0.5}$	$A_{-0.5}$	A_{-1}	$A_{-1.5}$	A_{-2}	IG	BS	RIG	$LN_{-1/2}$
uncorrected			4665	4230	3669	3168	3155	3800	4600	5371	3708	4170	4705	3721
			(2851)	(2744)	(2491)	(2295)	(2316)	(2659)	(3011)	(3326)	(2599)	(2736)	(2907)	(2586)
ADD	2	0.1	5011	4399	3736	3190	3120	4095	5443	6363	3699	4223	4890	3786
			(3046)	(2861)	(2592)	(2436)	(2405)	(3053)	(3537)	(3660)	(2681)	(2826)	(3113)	(2742)
		0.5	5191	4602	3818	3159	3046	4362	5559	6291	3763	4257	5065	3940
			(3076)	(3013)	(2689)	(2400)	(2372)	(3204)	(3318)	(3280)	(2895)	(2967)	(3359)	(3065)
		0.9	5209	4627	3848	3151	3071	3881	4744	5299	3662	4247	5103	3783
			(2817)	(3021)	(2751)	(2350)	(2460)	(2977)	(3083)	(3130)	(2795)	(2970)	(3430)	(2898)
ADD	3	0.1	4861	4301	3625	3129	3099	3702	4466	5138	3644	4180	4847	3681
			(3227)	(2952)	(2612)	(2403)	(2414)	(2698)	(2997)	(3230)	(2679)	(2827)	(3109)	(2691)
		0.5	5342	4708	3879	3146	<u>3023</u>	3442	4189	4861	3522	4217	5174	3585
			(3145)	(3087)	(2796)	(2267)	(2397)	(2648)	(2996)	(3208)	(2679)	(2984)	(3530)	(2744)
		0.9	5254	4766	3864	3155	<u>2984</u>	3422	4111	4800	3497	4243	5289	3561
			(2751)	(3094)	(2729)	(2294)	(2276)	(2684)	(3016)	(3238)	(2694)	(3021)	(3650)	(2722)
TS	2	0.1	5265	4583	3834	3233	3171	4589	5628	6328	4008	4368	5168	4211
			(3144)	(2986)	(2613)	(2392)	(2409)	(3346)	(3790)	(4061)	(2972)	(2901)	(3277)	(3157)
		0.5	5446	4875	4000	3240	3177	4241	5285	6080	3906	4572	5616	4052
			(2984)	(3064)	(2711)	(2371)	(2464)	(2984)	(3158)	(3268)	(2839)	(3079)	(3649)	(2988)
		0.9	5424	4854	4014	3236	3152	4004	4794	5312	3822	4541	5552	3922
			(2876)	(2987)	(2702)	(2341)	(2382)	(2969)	(3013)	(3071)	(2718)	(3019)	(3559)	(2783)
TS	3	0.1	5108	4493	3740	3183	3153	3804	4624	5372	3753	4313	5020	3795
			(3315)	(3073)	(2689)	(2422)	(2414)	(2724)	(3035)	(3292)	(2711)	(2887)	(3149)	(2722)
		0.5	5641	5008	4079	3284	3167	3692	4381	5003	3817	4537	5444	3853
			(3108)	(3114)	(2751)	(2394)	(2420)	(2666)	(2997)	(3180)	(2728)	(3017)	(3423)	(2727)
		0.9	5489	5033	4134	3273	3128	3675	4278	4838	3816	4587	5584	3882
			(2642)	(3024)	(2755)	(2353)	(2279)	(2689)	(3004)	(3161)	(2716)	(3034)	(3547)	(2782)
JF	2	0.1	5046	4415	3747	3196	3126	4139	5510	6479	3718	4240	4919	3804
			(3086)	(2864)	(2593)	(2442)	(2405)	(3048)	(3571)	(3699)	(2695)	(2831)	(3124)	(2747)
		0.5	5328	4751	3910	3200	3103	4353	5429	6107	3860	4403	5263	4047
			(2960)	(3006)	(2678)	(2395)	(2386)	(3103)	(3208)	(3170)	(2902)	(2979)	(3394)	(3060)
		0.9	5402	4836	3997	3227	3144	3979	4774	5282	3805	4502	5534	3907
			(2852)	(2987)	(2701)	(2340)	(2381)	(2946)	(3007)	(3054)	(2718)	(2992)	(3561)	(2791)
JF	3	0.1	4554	4102	3588	3155	3222	3852	4414	4855	3730	4141	4621	3757
			(2874)	(2727)	(2548)	(2343)	(2374)	(2693)	(2837)	(3027)	(2640)	(2748)	(2893)	(2644)
		0.5	5366	4751	3938	3151	3082	3590	4253	4887	3665	4347	5235	3738
			(3034)	(3007)	(2739)	(2235)	(2394)	(2716)	(2961)	(3183)	(2704)	(2999)	(3402)	(2758)
		0.9	5326	4819	3933	3153	3034	3521	4121	4706	3622	4371	5370	3698
			(2683)	(2972)	(2731)	(2209)	(2266)	(2710)	(2974)	(3088)	(2706)	(3040)	(3553)	(2794)

The underlined or double-underlined number indicates the smallest or second smallest average ISE, respectively.

Table S4: (continued).

$n = 200$

	p	a	A_2	$A_{1.5}$	A_1	$A_{0.5}$	$A_{-0.5}$	A_{-1}	$A_{-1.5}$	A_{-2}	IG	BS	RIG	$LN_{-1/2}$
uncorrected			2896	2615	2286	1988	1976	2311	2683	3055	2287	2579	2874	2293
			(1845)	(1780)	(1677)	(1564)	(1548)	(1724)	(1828)	(2007)	(1714)	(1775)	(1851)	(1717)
ADD	2	0.1	3016	2677	2273	1975	1924	2240	2870	3772	2195	2568	2913	2222
			(1957)	(1908)	(1762)	(1633)	(1593)	(1792)	(2245)	(2902)	(1726)	(1862)	(1963)	(1757)
		0.5	3155	2711	2273	1963	1891	2424	3998	5128	2136	2509	2971	2159
			(2089)	(1988)	(1775)	(1624)	(1593)	(2281)	(2988)	(3031)	(1732)	(1869)	(2127)	(1811)
		0.9	3175	2715	2257	1952	1877	2155	3056	3774	2145	2521	2948	2140
			(1950)	(1986)	(1754)	(1610)	(1593)	(1861)	(2492)	(2542)	(1755)	(1890)	(2103)	(1754)
ADD	3	0.1	3031	2693	2269	1968	1924	2222	2645	3034	2207	2567	2927	2226
			(2028)	(1930)	(1759)	(1613)	(1685)	(1771)	(1895)	(1998)	(1745)	(1868)	(1983)	(1766)
		0.5	3319	2834	2318	1944	1888	2038	2450	2908	2107	2552	2981	2133
			(2063)	(2067)	(1860)	(1389)	(1631)	(1746)	(1941)	(2109)	(1751)	(1982)	(2167)	(1802)
		0.9	3604	2876	2320	1945	1883	2023	2439	3022	2109	2573	3014	2098
			(1851)	(2135)	(1856)	(1388)	(1626)	(1740)	(1795)	(2351)	(1785)	(2013)	(2233)	(1764)
TS	2	0.1	3131	2771	2352	2012	1964	2590	3413	4009	2295	2667	3036	2329
			(2028)	(1919)	(1777)	(1638)	(1598)	(2297)	(2900)	(3286)	(1779)	(1886)	(1985)	(1814)
		0.5	3376	2912	2404	2020	1945	2536	3863	4928	2281	2722	3191	2308
			(2081)	(2017)	(1810)	(1635)	(1596)	(2173)	(2745)	(2872)	(1771)	(1937)	(2152)	(1842)
		0.9	3385	2914	2416	2010	1935	2281	3079	3780	2289	2729	3205	2285
			(1919)	(2014)	(1809)	(1610)	(1587)	(1812)	(2319)	(2411)	(1790)	(1919)	(2162)	(1793)
TS	3	0.1	3156	2795	2350	2002	1962	2314	2758	3164	2295	2681	3046	2313
			(2048)	(1940)	(1769)	(1603)	(1601)	(1793)	(1911)	(2025)	(1761)	(1904)	(1994)	(1782)
		0.5	3565	3042	2471	2029	1953	2229	2643	3060	2299	2777	3281	2309
			(1989)	(2068)	(1851)	(1636)	(1636)	(1809)	(1972)	(2100)	(1793)	(1998)	(2197)	(1848)
		0.9	3790	3103	2505	2017	1946	2216	2677	3140	2317	2807	3303	2305
			(1717)	(2048)	(1854)	(1441)	(1628)	(1801)	(2005)	(2266)	(1805)	(1992)	(2200)	(1821)
JF	2	0.1	3035	2693	2282	1979	1930	2266	2926	3860	2219	2582	2929	2238
			(2010)	(1910)	(1762)	(1634)	(1592)	(1810)	(2297)	(2950)	(1741)	(1863)	(1962)	(1756)
		0.5	3282	2833	2347	1997	1917	2499	3942	5001	2223	2636	3091	2229
			(2094)	(2009)	(1805)	(1637)	(1594)	(2229)	(2815)	(2852)	(1771)	(1914)	(2106)	(1805)
		0.9	3359	2900	2394	2003	1937	2269	3100	3762	2279	2718	3175	2283
			(1905)	(2012)	(1784)	(1608)	(1604)	(1814)	(2357)	(2399)	(1790)	(1920)	(2134)	(1814)
JF	3	0.1	2957	2693	2347	2009	2029	2433	2817	3101	2389	2666	2955	2402
			(1869)	(1796)	(1682)	(1521)	(1525)	(1730)	(1842)	(1907)	(1703)	(1803)	(1881)	(1706)
		0.5	3338	2889	2357	1997	1922	2147	2514	2960	2208	2633	3088	2218
			(1959)	(2037)	(1788)	(1640)	(1615)	(1805)	(1922)	(2146)	(1752)	(1934)	(2132)	(1793)
		0.9	3658	2930	2394	1960	1919	2085	2503	3031	2189	2659	3105	2178
			(1806)	(2096)	(1858)	(1390)	(1653)	(1725)	(1862)	(2286)	(1767)	(1988)	(2193)	(1783)