**Appendix**

**Proof for Proposition 2**

 We prove this proposition in two cases: $θ\_{A}<d$ in case 1 (Figure 3.1) and $θ\_{A}>d$ in case 2 (Figure 3.2).

 In case 1, the consumers in segments $M'A^{\*}$, $A^{\*}K$ and $KM$ have utility functions as $θ\_{A}\left(1-y-z\right)-zd-p$, $θ\_{A}\left(1-y+w\right)-wd-p$, and $θ\_{A}(1-\left(w-y\right))-wd-p$, respectively, where $z$ and $w$ represent the distance (demand) of $KM$ and $M^{'}K$, respectively. If the firm sets the price too high (like $p\_{1}$ in Figure 3.1), there is no demand. When the price is $p\_{2}$ ($p\_{3})$, the left-end consumer falls in segment $A^{\*}K$ ($M'A^{\*}$). Hence, we can set corresponding utility functions above to zero, solving $z$ and $w$ accordingly, and obtain the total demand that is $z+w$. When the price is $p\_{2} (p\_{3})$, we obtain $z=\frac{θ\_{A}\left(1-y\right)-p}{θ\_{A}+d}$, $w=\frac{θ\_{A}\left(1-y\right)-p}{d-θ\_{A}}$ ($w=\frac{θ\_{A}\left(1+y\right)-p}{d+θ\_{A}}$), and the total demand is $\frac{2d\left[θ\_{A}\left(1-y\right)-p\right]}{d^{2}-θ\_{A}^{2}}$ ($\frac{2(θ\_{A}-p)}{θ\_{A}+d}$). The profit function equals the profit margin $(p-v\_{A})$ times the corresponding demand $(z+w)$ with price $(p)$ and product position $(y)$ as decision variables.

 When the price is $p\_{2}$, the profit function is strictly decreasing in $y$ as $\frac{dπ}{dy}=\frac{2dθ\_{A}\left(p-v\_{A}\right)}{θ\_{A}^{2}-d^{2}}<0$ and $\frac{d^{2}π}{dy^{2}}=0,$ , and we can set $y=0$ to optimize the profit. When the prices is set low like $p\_{3}$, the profit function is invariant in $y$ as $\frac{dπ}{dy}=\frac{d^{2}π}{dy^{2}}=0$. In both situations, the profit function is strictly concave in $p,,$ , and the optimal price is $\frac{θ\_{A}+v\_{A}}{2}$. However, $\left(p^{\*},y^{\*}\right)=(\frac{θ\_{A}+v\_{A}}{2}, 0)$ is not the only optimal solution. We find that as long as $p^{\*}=\frac{θ\_{A}+v\_{A}}{2}$ and $y\leq min⁡\{\frac{θ\_{A}-v\_{A}}{2θ\_{A}},\frac{θ\_{A}-v\_{A}}{2d}\}$, the demand is $\frac{θ\_{A}-v\_{A}}{(θ\_{A}+d)}$ and the total profit under these conditions is $\frac{\left(θ\_{A}-v\_{A}\right)^{2}}{2(θ\_{A}+d)}$.

 We can conduct similar algebra analysis for case 2 by splitting the left and right demand on point $A^{\*}$ (instead of K). Since the optimal solutions are the same as in case 1, the proof is omitted here. $∎$

**Proof for Proposition 3**

 We prove this proposition in two cases: $θ\_{A}-θ\_{B}<d$ in case 1 (Figure 4.1) and $θ\_{A}-θ\_{B}>d$ in case 2 (Figure 4.2). The demand derivation and total profit analysis are similar to Proposition 1.

In case 1, the consumers in segment $M'A^{\*}$, $A^{\*}K $, and $KM$ have utility functions as $θ\_{A}\left(1-y-z\right)+θ\_{B}(z+y)-dz-p$, $θ\_{A}\left(1-y+w\right)+θ\_{B}(y-w)-dw-p$, and $θ\_{A}(1-w+y)+θ\_{B}(w-y)-dw-p$, respectively, where $z$ and $w$ represent $KM$ and $M^{'}K$, respectively. When the price is $p\_{2} (p\_{3})$, we obtain $z=\frac{θ\_{A}\left(1-y\right)+θ\_{B}y-p}{θ\_{A}-θ\_{B}+d}$, $w=\frac{θ\_{A}\left(1-y\right)+θ\_{B}y-p}{d-θ\_{A}+θ\_{B}}$ ($w=\frac{θ\_{A}\left(1+y\right)-θ\_{B}y-p}{d+θ\_{A}-θ\_{B}}$), and the total demand $(z+w)$ is $\frac{2d\left[θ\_{A}\left(1-y\right)+θ\_{B}y-p\right]}{d^{2}-\left(θ\_{A}-θ\_{B}\right)^{2}}$ ($\frac{2(θ\_{A}-p)}{θ\_{A}-θ\_{B}+d}$).

 When the price is $p\_{2}$, the profit function strictly decreases in $y$ as $\frac{dπ}{dy}=-\frac{2d\left(θ\_{A}-θ\_{B}\right)\left(p-v\_{A}-v\_{B}\right)}{d^{2}-\left(θ\_{A}-θ\_{B}\right)^{2}}<0$ and $\frac{d^{2}π}{dy^{2}}=0$. If we set $y=0$, then the profit function becomes $\frac{2d\left(θ\_{A}-p\right)\left(p-v\_{A}-v\_{B}\right)}{d^{2}-\left(θ\_{A}-θ\_{B}\right)^{2}}$. When the price is set low like $p\_{3}$, the profit function is invariant in $y$ as $\frac{dπ}{dy}=\frac{d^{2}π}{dy^{2}}=0$. In both situations, the profit function is strictly concave in p, and the optimal price is $\frac{θ\_{A}+v\_{A}+v\_{B}}{2}$. Substituting $p^{\*}=\frac{θ\_{A}+v\_{A}+v\_{B}}{2}$ back to $z=\frac{θ\_{A}\left(1-y\right)+θ\_{B}y-p}{θ\_{A}-θ\_{B}+d}$ and $w=\frac{θ\_{A}\left(1+y\right)-θ\_{B}y-p}{d+θ\_{A}-θ\_{B}}$, we find that the optimality stays when $p^{\*}=\frac{θ\_{A}+v\_{A}+v\_{B}}{2}$ and $y\leq min⁡\{\frac{θ\_{A}-v\_{A}-v\_{B}}{2\left(θ\_{A}-θ\_{B}\right)},\frac{θ\_{A}-v\_{A}-v\_{b}}{2d}\}$. When $\frac{θ\_{A}-v\_{A}-v\_{B}}{θ\_{A}-θ\_{A}+d}<2,$ the derived demand is $\frac{θ\_{A}-v\_{A}-v\_{B}}{θ\_{A}-θ\_{B}+d} $total profit , and the total profit is $\frac{\left(θ\_{A}-v\_{A}-v\_{B}\right)^{2}}{2\left(θ\_{A}-θ\_{B}+d\right)}$. However, when $θ\_{A}-θ\_{B}$ and $d$ are both small, it is possible that $\frac{θ\_{A}-v\_{A}-v\_{B}}{θ\_{A}-θ\_{B}+d}\geq 2$. Since the market size cannot exceed 2, the optimal profit will be $\left(\frac{θ\_{A}+v\_{A}+v\_{B}}{2}-v\_{A}-v\_{B}\right)\*2=θ\_{A}-v\_{A}-v\_{B}$.

 We can conduct similar algebra analysis for case 2 by splitting the left and right demand on point $A^{\*}$ (instead of K). Since the optimal solutions are the same as in case 1, the proof is omitted here.$ ∎$

**Proof for Lemma 1.**

The consumer at $A^{\*}$ is the one with highest total valuation as $θ\_{A}>θ\_{B}$. We assume that an optimal solution (optimal price and optimal product location) exists such that its capture demand (say D1) does not include $A^{\*}$. We know that the demand captured by the optimal solution will have both ends at zero net utility. We can propose another solution that has the same optimal price but a different product location, which is slightly closer to $A^{\*}$. Since D1 does not include $A^{\*}$, moving product closer to $A^{\*}$ will reduce distance disutility in D1 in both ends; as a result, the new capture demand (say D2) will be greater than the original capture demand D1, which contradicts the disclamation that the original solution is optimal, since the price remains the same. As a result, the optimal solution should always have the final captured demand that includes the consumer with the highest total valuation. $∎$

**Proof for Proposition 4**

 From propositions above, it is clear that pricing and product position are not two totally independent decisions. Mathematically, decision variables $p$, $y$, and the left $(w)$ and right $(z)$ demands have implicit connections with one another. For Proposition 4, we have to solve this problem in an indirect way. Our approach is through several iterations of variable substitution, constructing the profit function as a function of price, solving the optimal price, and then obtaining other values accordingly. Since point $A^{\*}$ must be included in the captured demand (Lemma 1), we set $w$ and $z$ as the demand captured on the left and right side of $A^{\*}$.

 To derive market demand, we must determine the end points of demand captured by the price. In Figure 5.3, prices $p\_{1}$, $p\_{2,, }$ , and $p\_{3}$ will cause the end points fall in regions I, II, and III, respectively. Region II is further divided into II.1 $(t>\frac{θ\_{A}+θ\_{B}}{2θ\_{A}})$ and II.2 $(t<\frac{θ\_{A}+θ\_{B}}{2θ\_{A}})$. In Region I, by equating $u\_{L}$ and $u\_{R}, ,$ , we obtain $y=\frac{w\left(θ\_{A}-θ\_{B}+d\right)-z\left(θ\_{A}+θ\_{B}+d\right)}{2d}$ and substitute it back to $u\_{L}$ and $u\_{R}$. The utility decreases in both sides of point $A^{\*}$ should be the same; as a result we can set $w\left(θ\_{A}-θ\_{B}\right)=z\left(θ\_{A}+θ\_{B}\right).$ Letting $z=\frac{w\left(θ\_{A}-θ\_{B}\right)}{\left(θ\_{A}+θ\_{B}\right)}$ and setting $u\_{L}$ and $u\_{R}$ to zero, we obtain $w=\frac{\left(θ\_{A}+θ\_{B}\right)\left(θ\_{A}+(1-t)θ\_{B}-p\right)}{θ\_{A}\left(θ\_{A}+d\right)-θ\_{B}^{2}}$. The captured demand is $w+z=\frac{2θ\_{A}\left(θ\_{A}+(1-t)θ\_{B}-p\right)}{θ\_{A}\left(θ\_{A}+d\right)-θ\_{B}^{2}}$. We can now derive the firm’s profit function as $π\_{I}=\left(p-v\_{A}+v\_{B}\right)\frac{2θ\_{A}\left(θ\_{A}+(1-t)θ\_{B}-p\right)}{θ\_{A}\left(θ\_{A}+d\right)-θ\_{B}^{2}}.$ The profit function is strictly concave in product price as $\frac{d^{2}π\_{I}}{dp^{2}}=\frac{-4θ\_{A}}{θ\_{A}\left(θ\_{A}+d\right)-θ\_{B}^{2}}<0$. The optimal price is $p^{\*}=\frac{\left(θ\_{A}+(1-t)θ\_{B}+v\_{A}+v\_{B}\right)}{2}$. Substituting $p^{\*}$ back to $w$ and $z$, we find the product should be positioned at $y^{\*}=\frac{θ\_{B}\left(θ\_{A}+\left(1-t\right)θ\_{B}-v\_{A}-v\_{B}\right)}{2θ\_{A}\left(θ\_{A}+d\right)-2θ\_{B}^{2}}$ on the left side of point $A^{\*}$. The derived demand is $\frac{θ\_{A}\left(θ\_{A}+\left(1-t\right)θ\_{B}-v\_{A}-v\_{B}\right)}{θ\_{A}\left(θ\_{A}+d\right)-θ\_{B}^{2}}$ , and the profit at $p^{\*}$ and $y^{\*}$ is $\frac{θ\_{A}\left(θ\_{A}+\left(1-t\right)θ\_{B}-v\_{A}-v\_{B}\right)^{2}}{θ\_{A}\left(θ\_{A}+d\right)-θ\_{B}^{2}}$.

 Similar algebra can be applied to other situations when the demand-end points fall in regions II.1 $(t>\frac{θ\_{A}+θ\_{B}}{2θ\_{A}})$, II.2 $(t<\frac{θ\_{A}+θ\_{B}}{2θ\_{A}})$ , and III. The mathematical derivations are similar to that in region I, and thus, have been omitted. We summarize the optimal solutions for four cases in Table 4.

Since the captured demand cannot exceed 2, cases II.1 and III need further discussion. In case II.1, when $d\leq \frac{2θ\_{B}-v\_{A}-v\_{B}}{2}$, the derived demand is greater than 2. In this situation, we can derive the optimal price by setting $w+z=\frac{2\left(θ\_{A}-p\right)}{θ\_{A}-θ\_{B}+d}=2$, and get the optimal price $p^{\*}$ to be $θ\_{B}-d$. The optimal solution is $(p^{\*},y^{\*})=\left(θ\_{B}-d,0\right),, $ , and the derived profit is $2(θ\_{B}-d-v\_{A}-v\_{B})$. This special situation only occurs when $θ\_{B}$ is very close to $θ\_{A}.$

In case III, when $d\leq \frac{θ\_{B}^{2}-θ\_{A}^{2}+θ\_{A}\left(tθ\_{B}-v\_{A}-v\_{B}\right)}{2θ\_{A}}$ (note that $\frac{θ\_{B}^{2}-θ\_{A}^{2}+θ\_{A}\left(tθ\_{B}-v\_{A}-v\_{B}\right)}{2θ\_{A}}$ is positive only when $θ\_{B}$ and $t$ are both fairly large in value), the derived demand is greater than 2. In this situation, we can, at best, capture the whole market demand; as a result, we can obtain the optimal price by setting $w+z=\frac{2\left(θ\_{A}\left(θ\_{A}+tθ\_{B}-p\right)-θ\_{B}^{2}\right)}{θ\_{A}\left(θ\_{A}+d\right)-θ\_{B}^{2}}=2$, in which $p^{\*}=tθ\_{B}-d$. The optimal solution is $(p^{\*},y^{\*})=\left(tθ\_{B}-d,0\right)$ , and the derived profit is $2(tθ\_{B}-d-v\_{A}-v\_{B})$. This special situation only occurs when $θ\_{B}$ is very close to $θ\_{A} , ,$, and where $t$ is relatively large.

 Comparing the results from our four cases above, we derive several conclusions. First, $π\_{I}>π\_{III}$ if $t<\frac{θ\_{A}+θ\_{B}}{2θ\_{A}}$ , and $π\_{I}<π\_{III}$ otherwise. Second, cases II.1 and II.2 don’t coexist. As a result, we only have to compare cases I and II.2 when $t<\frac{θ\_{A}+θ\_{B}}{2θ\_{A}}$ , and compare cases II.1 and III when $t>\frac{θ\_{A}+θ\_{B}}{2θ\_{A}}$. We set $j=\left(θ\_{A}-v\_{A}-v\_{B}\right)$, $k=\left(θ\_{A}-θ\_{B}+d\right)$, $r=\left(θ\_{A}+θ\_{B}+d\right)$,

$m=\left(θ\_{A}+θ\_{B}-v\_{A}-v\_{B}\right)$, $ n=\left(θ\_{A}\left(θ\_{A}+d\right)-θ\_{B}^{2}\right)$, and $q=\left(θ\_{A}\left(θ\_{A}-v\_{A}-v\_{B}\right)-θ\_{B}^{2}\right)$. Comparing $π\_{I}$ and $π\_{II.2}$, we find that $π\_{I}>π\_{II.2}$ when $t<\frac{m\left(θ\_{A}r-\sqrt{θ\_{A}rn}\right)}{θ\_{A}θ\_{B}r}$ ($\frac{m\left(θ\_{A}r+\sqrt{θ\_{A}rn}\right)}{θ\_{A}θ\_{B}r}>1>t$, and thus, is excluded). Comparing $π\_{III}$ and $π\_{II.1}$, we find that $π\_{III}>π\_{II.1}$ when $t>\frac{-kq+j\sqrt{θ\_{A}kn}}{θ\_{A}θ\_{B}k}$ ($\frac{-kq-j\sqrt{θ\_{A}kn}}{θ\_{A}θ\_{B}k}<0$, and thus, is excluded). $∎$