Online Supplement for "Industrial Symbiosis: Impact of Competition on Firms' Willingness to Implement" by "Yunxia Zhu, Milind Dawande, Nagesh Gavirneni, and Vaidyanathan Jayaraman"

A Proof of Theorem 1

We consider four scenarios.

Scenario M1: $0 < p_g \leq p_r$

Since Firm 1 charges a lower price for the green variant, all consumers choose to buy either a green variant or nothing. The aggregate market demand is as follows: $Q_r = 0, Q_g = (1-p_g)\theta_1 + (1-\frac{p_g}{a})(1-\theta_1)$. Thus, Firm 1 faces the following profit-maximization problem:

$$\max_{p_r,p_g} \quad \pi_1 = (p_g - c_1^{'})Q_g = (p_g - c_1^{'})[(1-p_g)\theta_1 + (1-\frac{p_g}{a})(1-\theta_1)].$$

The optimal price and optimal profit are as follows:

$$p_r^* \geq p_g^* = \frac{a+(a-1)\theta_1 c_1^{'} + c_1^{'}}{2[(a-1)\theta_1+1]}, \pi_1^{M1} = \frac{[a-(a-1)\theta_1 c_1^{'} - c_1^{'}]^2}{4a[(a-1)\theta_1+1]}.$$

Scenario M2: $p_r \leq p_g \leq ap_r$

Under this scenario, regular consumers purchase either the regular variant or nothing. Meanwhile, flexible and dedicated green consumers purchase either the green variant or nothing. The aggregate market demand is as follows: $Q_r = (1-p_r)\theta_1, Q_g = (1-\frac{p_g}{a})(1-\theta_1)$. Thus, Firm 1 faces the following profit-maximization problem.

$$\begin{aligned}\max_{p_r,p_g} \quad \pi_1 &= (p_r - c_1^{'})Q_r + (p_g - c_1^{'})Q_g \\ &= (p_r - c_1^{'})(1-p_r)\theta_1 + (p_g - c_1^{'})(1-\frac{p_g}{a})(1-\theta_1).\end{aligned}$$

The optimal solution is $p_r^* = \frac{1+c_1^{'}}{2}$, $p_g^* = \frac{a+c_1^{'}}{2}$. Since $a \geq 1$, we have $p_r^* \leq p_g^* \leq ap_r^*$. Thus, Firm 1's maximum profit under Scenario M2 is

$$\pi_1^{M2} = \frac{\theta_1(1-c_1^{'})^2}{4} + \frac{(1-\theta_1)(a-c_1^{'})^2}{4a}.$$

We compare the profits under scenarios M1 and M2.

$$\pi_1^{M2} - \pi_1^{M1} = \frac{\theta_1(1-c_1^{'})^2}{4} + \frac{(1-\theta_1)(a-c_1^{'})^2}{4a} - \frac{[a-(a-1)\theta_1 c_1^{'} - c_1^{'}]^2}{4a[(a-1)\theta_1+1]} = \frac{\theta_1(1-\theta_1)(a-1)^2}{4[(a-1)\theta_1+1]} \geq 0.$$

Scenario M3: $(a-1) + p_r \leq p_g \leq a$

Under this scenario, regular and flexible green consumers purchase either the regular variant or nothing. Dedicated green consumers purchase either the green variant or nothing. The aggregate market demand is as follows: $Q_r = (1-p_r)(\theta_1+\theta_2), Q_g = (1-\frac{p_g}{a})\theta_3$. Thus, Firm 1 faces the following profit-maximization problem:

$$\begin{aligned}\max_{p_r,p_g} \quad \pi_1 &= (p_r - c_1^{'})Q_r + (p_g - c_1^{'})Q_g = (p_r - c_1^{'})(1-p_r)(\theta_1+\theta_2) + (p_g - c_1^{'})(1-\frac{p_g}{a})\theta_3 \\ s.t. \quad & \qquad (a-1) + p_r \leq p_g \leq a.\end{aligned}$$

We first solve the unconstrained problem; the corresponding optimal solution is as follows:

$$p_r^* = \frac{1+c_1^{'}}{2}, p_g^* = \frac{a+c_1^{'}}{2}, \pi_{1,r}^* = \frac{(\theta_1+\theta_2)(1-c_1^{'})^2}{4}, \pi_{1,g}^* = \frac{\theta_3(a-c_1^{'})^2}{4a}.$$

Clearly, Firm 1's maximum profit is bounded from above by that of the unconstrained problem. Thus, $\pi_1^{M3} \leq \frac{(\theta_1+\theta_2)(1-c_1^{'})^2}{4} + \frac{\theta_3(a-c_1^{'})^2}{4a}$. We can then compare the profits under Scenarios M2 and M3.

$$\begin{aligned}\pi_1^{M2} - \pi_1^{M3} &\geq \frac{\theta_1(1-c_1^{'})^2}{4} + \frac{(1-\theta_1)(a-c_1^{'})^2}{4a} - \frac{(\theta_1+\theta_2)(1-c_1^{'})^2}{4} - \frac{\theta_3(a-c_1^{'})^2}{4a} \\ &= \theta_2[\frac{(a-c_1^{'})^2}{4a} - \frac{(1-c_1^{'})^2}{4}] = \frac{\theta_2(a-1)[a^2-(c_1^{'})^2]}{4a} \geq 0.\end{aligned}$$

Scenario M4: $ap_r \leq p_g \leq (a-1) + p_r$

Under this scenario, regular consumers purchase the regular variant or nothing and dedicated green consumers purchase the green variant or nothing. Among flexible green consumers, some buy the regular variant, some others buy the green variant, and the remaining buy nothing. The aggregate market demand is as follows: $Q_r = (1-p_r)\theta_1 + \frac{p_g - ap_r}{a-1}\theta_2, Q_g = (1-\frac{p_g}{a})\theta_3 + (1-\frac{p_g-p_r}{a-1})\theta_2$.

The firm's total profit can be written as follows:

$$\begin{array}{rcl}\max\limits_{p_r,p_g} \quad \pi_1 & = & Q_r p_r + Q_g p_g - c_1^{'}(Q_r+Q_g) \\ & = & -(\theta_1+\frac{a\theta_2}{a-1})p_r^2 + \frac{2\theta_2}{a-1}p_g p_r + [\theta_1 + c_1^{'}(\theta_1+\theta_2)]p_r - (\frac{\theta_3}{a}+\frac{\theta_2}{a-1})p_g^2 + [(\theta_2+\theta_3)+c_1^{'}\frac{\theta_3}{a}]p_g - c_1^{'} \\ s.t. & & p_g - ap_r \geq 0, \\ & & (a-1)+p_r-p_g \geq 0.\end{array}$$

The Lagrangean and the Karush-Kuhn-Tucker optimality conditions are:

$$\begin{array}{rcl} L(p_r,p_g) & = & \pi_1 + \lambda_1(p_g - ap_r) + \lambda_2[(a-1)+p_r-p_g] \\ \frac{\partial L}{\partial p_r} & = & -2(\theta_1+\frac{a\theta_2}{a-1})p_r + \frac{2\theta_2}{a-1}p_g + [\theta_1 + c_1^{'}(\theta_1+\theta_2)] - a\lambda_1 + \lambda_2 = 0 \\ \frac{\partial L}{\partial p_g} & = & -2(\frac{\theta_3}{a}+\frac{\theta_2}{a-1})p_g + \frac{2\theta_2}{a-1}p_r + [(\theta_2+\theta_3)+c_1^{'}\frac{\theta_3}{a}] + \lambda_1 - \lambda_2 = 0 \\ \lambda_1(p_g - ap_r) & = & 0, \\ \lambda_2[(a-1)+p_r-p_g] & = & 0. \end{array}$$

• **Scenario M4.1:** $\lambda_1 = 0$ and $\lambda_2 = 0$

Solving the above system, we have $p_g^* = \frac{a+c_1^{'}}{2}, p_r^* = \frac{1+c_1^{'}}{2}$. However, since $p_g^* - ap_r^* = \frac{(1-a)c_1^{'}}{2} < 0$, the solution is invalid.

• **Scenario M4.2:** $p_g = ap_r$

We can rewrite the demand functions as follows: $Q_r = (1-p_r)\theta_1, Q_g = (1-\frac{p_g}{a})(1-\theta_1)$. Thus, the objective function has the same form as in Scenario M2. However, the optimization problem here has an additional constraint $p_g = ap_r$. Thus, the optimal value obtained in Scenario M2 is at least as good as that under Scenario M4.2.

• **Scenario M4.3:** $p_g = (a-1)+p_r$

We can rewrite the demand functions as follows: $Q_r = (1-p_r)(\theta_1+\theta_2), Q_g = (1-\frac{p_g}{a})\theta_3$. Thus, the objective function has the same form as in Scenario M3. However, the optimization problem here has an additional constraint $p_g = (a-1)+p_r$. Thus, the optimal value obtained in Scenario M3 is at least as good as that under Scenario M4.3.

Combining the analysis above, we can conclude that the decisions obtained under Scenario M2 are optimal. This completes the proof. ■

B Proof of Theorem 2

In equilibrium, Firm 1 provides both regular and green variants. Firm 2 only provides the regular variant. Recall that among the multiple price settings described in Section 3.2, only $p_r < p_g \leq ap_r$ and $ap_r \leq p_g < (a-1)+p_r$ guarantee the existence of both regular and green variants in equilibrium. Thus, following our assumptions in Section 4.2, we consider only these two price settings in our analysis.

1. If $c_1^{'} < \frac{\theta_1}{2(1+\theta_1)}$ and $c_1^{'} < c_2 < \frac{1}{2}[1+\frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$

We first consider the values of the lower and upper bounds of c_2. Since $a > 1$, $a(1-\theta_1)+\theta_1$ decreases with an increase in θ_1. Thus, for $0 \leq \theta_1 \leq 1$, we have $1 \leq a(1-\theta_1)+\theta_1 \leq a$. Therefore, we have $\frac{1}{2}+\frac{c_1^{'}}{2a} \leq \frac{1}{2}[1+\frac{c_1^{'}}{a(1-\theta_1)+\theta_1}] \leq \frac{1}{2}+\frac{c_1^{'}}{2}$. Thus, $c_2 < \frac{1}{2}[1+\frac{c_1^{'}}{a(1-\theta_1)+\theta_1}] \leq (\frac{1}{2}+\frac{c_1^{'}}{2})$.

Since $a > 1$, $0 \leq \theta_1 \leq 1$, and $c_1^{'} < \frac{\theta_1}{2(1+\theta_1)}$, we have

$$\begin{array}{l} (\frac{1}{2}+\frac{c_1^{'}}{2a}) - [\frac{1}{2} - (1-\frac{3}{2a})c_1^{'}] = (1-\frac{1}{a})c_1^{'} > 0, \\ [\frac{1}{2} - (1-\frac{3}{2a})c_1^{'}] - (\frac{1}{2} - \frac{1}{\theta_1}c_1^{'}) = (\frac{3}{2a} + \frac{1-\theta_1}{\theta_1})c_1^{'} > 0, \\ (\frac{1}{2} - \frac{1}{\theta_1}c_1^{'}) - c_1^{'} = \frac{1}{2} - (1+\frac{1}{\theta_1})c_1^{'} > \frac{1}{2} - (1+\frac{1}{\theta_1})\frac{\theta_1}{2(1+\theta_1)} = 0. \end{array}$$

Thus, $c_1^{'} < (\frac{1}{2} - \frac{1}{\theta_1}c_1^{'}) < [\frac{1}{2} - (1-\frac{3}{2a})c_1^{'}] < (\frac{1}{2}+\frac{c_1^{'}}{2a}) \leq \frac{1}{2}[1+\frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$. Therefore, all the three regions, which represent the corresponding three types of equilibria, have positive lengths.

We now consider three scenarios:

Scenario D1: $p_r < p_g < ap_r$

Under this scenario, regular consumers only purchase regular products and flexible green consumers only purchase green products. The aggregate market demand is as follows: $Q_r = (1-p_r)\theta_1, Q_g = (1-\frac{p_g}{a})(1-\theta_1)$. Thus, we have $p_r = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1}$. Firm 1's total profit can be written as follows:

$$\max_{q_{1,r},p_g} \quad \pi_1 \quad = \quad q_{1,r}(p_r - c_1^{'}) + Q_g(p_g - c_1^{'}) = q_{1,r}(1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'}) + (1-\frac{p_g}{a})(1-\theta_1)(p_g - c_1^{'}).$$

The first-order conditions are

$$\frac{\partial \pi_1}{\partial q_{1,r}} = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'} - \frac{q_{1,r}}{\theta_1} = 0, \tag{1}$$

$$\frac{\partial \pi_1}{\partial p_g} = (1 - \frac{p_g}{a})(1-\theta_1) - \frac{(1-\theta_1)(p_g - c_1^{'})}{a} = 0. \tag{2}$$

Next we find the Hessian for $\pi_1(q_{1,r}, p_g)$

$$H(q_{1,r}, p_g) = \begin{bmatrix} -\frac{2}{\theta_1} & 0 \\ 0 & -\frac{2(1-\theta_1)}{a} \end{bmatrix}$$

Since $H_1(q_{1,r}, p_g) = -\frac{2}{\theta_1} < 0$, $H_2(q_{1,r}, p_g) = (-\frac{2}{\theta_1})(-\frac{2(1-\theta_1)}{a}) > 0$, the first-order conditions are both necessary and sufficient.
Firm 2's profit is $\max_{q_{2,r}} \pi_2 = (p_r - c_2)q_{2,r} = (1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2)q_{2,r}$. The first-order condition is

$$\frac{\partial \pi_2}{\partial q_{2,r}} = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2 - \frac{q_{2,r}}{\theta_1} = 0 \tag{3}$$

Solving (1), (2), and (3), we obtain the following solution:

	Regular Market	Green Market
q_1^*	$\frac{\theta_1(1-2c_1^{'}+c_2)}{3}$	$\frac{(1-\theta_1)(a-c_1^{'})}{2a}$
q_2^*	$\frac{\theta_1(1+c_1^{'}-2c_2)}{3}$	-
p^*	$\frac{1+c_1^{'}+c_2}{3}$	$\frac{a+c_1^{'}}{2}$

Verifying Validity: We now verify that the quantities derived above are all positive. Since $c_1^{'} < c_2 < \frac{1+c_1^{'}}{2}$, we have $1 - 2c_1^{'} + c_2 = (1 - c_1^{'}) + (c_2 - c_1^{'}) > 0$, and $1 + c_1^{'} - 2c_2 > 0$. Thus, $q_{1,r}^* > 0$ and $q_{2,r}^* > 0$. Also, since $c_1^{'} < \frac{1+c_1^{'}}{2}$, we have $c_1^{'} < 1 < a$. Thus, $q_{1,g}^* > 0$. We also need to verify that the optimal prices satisfy the constraint $p_r < p_g < ap_r$. We have $p_g - p_r = \frac{(3a-2)+c_1^{'}-2c_2}{6} \geq \frac{1+c_1^{'}-2c_2}{6} > 0$. We also have $ap_r - p_g = \frac{-a+(2a-3)c_1^{'}+2ac_2}{6}$. Therefore, if $c_2 > \frac{1}{2} - (1 - \frac{3}{2a})c_1^{'}$, then we have $ap_r^* > p_g^*$. Thus, the above solution is valid.

Scenario D2: $p_g = ap_r$
The aggregate market demand is the same as that in scenario D1. We have $p_r = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1}$ and $p_g = ap_r$. Firm 1's total profit can be written as follows:

$$\begin{aligned} \max_{q_{1,r}} \quad \pi_1 &= q_{1,r}(p_r - c_1^{'}) + Q_g(ap_g - c_1^{'}) = q_{1,r}(p_r - c_1^{'}) + (1 - p_r)(1-\theta_1)(ap_r - c_1^{'}) \\ &= q_{1,r}(1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'}) + \frac{(q_{1,r}+q_{2,r})}{\theta_1}(1-\theta_1)[a(1 - \frac{q_{1,r}+q_{2,r}}{\theta_1}) - c_1^{'}]. \end{aligned}$$

The first-order condition is

$$\frac{\partial \pi_1}{\partial q_{1,r}} = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'} - \frac{q_{1,r}}{\theta_1} + \frac{(1-\theta_1)}{\theta_1}[a(1 - \frac{q_{1,r}+q_{2,r}}{\theta_1}) - c_1^{'}] - \frac{a(q_{1,r}+q_{2,r})}{\theta_1^2}(1-\theta_1) = 0. \tag{4}$$

Firm 2's profit is $\max_{q_{2,r}} \pi_2 = (p_r - c_2)q_{2,r} = (1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2)q_{2,r}$. The first-order condition is

$$\frac{\partial \pi_2}{\partial q_{2,r}} = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2 - \frac{q_{2,r}}{\theta_1} = 0 \tag{5}$$

Solving (4) and (5), we obtain the following solution:

	Regular Market	Green Market
q_1^*	$\frac{\theta_1[\theta_1-2c_1^{'}+c_2(2a-2a\theta_1+\theta_1)]}{2a(1-\theta_1)+3\theta_1}$	$\frac{(1-\theta_1)[(a-a\theta_1+2\theta_1)-c_1^{'}-c_2\theta_1]}{2a(1-\theta_1)+3\theta_1}$
q_2^*	$\frac{\theta_1[(a-a\theta_1+\theta_1)+c_1^{'}-2c_2(a-a\theta_1+\theta_1)]}{2a(1-\theta_1)+3\theta_1}$	-
p^*	$\frac{(a+\theta_1-a\theta_1)+c_1^{'}+c_2\theta_1}{2a(1-\theta_1)+3\theta_1}$	$\frac{a[(a+\theta_1-a\theta_1)+c_1^{'}+c_2\theta_1]}{2a(1-\theta_1)+3\theta_1}$

Verifying Validity: We now verify that the quantities derived above are all positive. Since $c_1^{'} < c_2$, we have $\theta_1 - 2c_1^{'} + c_2(2a - 2a\theta_1 + \theta_1) = \theta_1(1 - c_1^{'}) + (2 - \theta_1)(c_2 - c_1^{'}) + 2(a-1)(1-\theta_1)c_2 > 0$. Thus, $q_{1,r}^* > 0$. Since $c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$, we have $(a - a\theta_1 + \theta_1) + c_1^{'} - 2c_2(a - a\theta_1 + \theta_1) > 0$. Thus, $q_{2,r}^* > 0$. Also, since $c_1^{'} < 1$ and $c_2 < 1$, we have $(a - a\theta_1 + 2\theta_1) - c_1^{'} - c_2\theta_1 > (a - a\theta_1 + 2\theta_1) - 1 - \theta_1 = (a-1)(1-\theta_1) \geq 0$. Thus, $q_{1,g}^* > 0$.

Scenario D3: $ap_r < p_g < (a-1)+p_r$

Under this scenario, regular consumers purchase the regular variant or nothing. Among flexible green consumers, some buy the regular variant, some others buy the green variant, and the remaining buy nothing. The aggregate market demand is as follows: $Q_r = (1-p_r)\theta_1 + \frac{p_g - ap_r}{a-1}(1-\theta_1), Q_g = (1-\frac{p_g-p_r}{a-1})(1-\theta_1)$.

Recall our assumption (from Section 4.2) that the market price of the regular variant is determined by the Cournot inverse demand function of the regular consumers. Thus, we have $p_r = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1}$. Firm 1's total profit can be written as follows:

$$\max_{q_{1,r},p_g} \quad \pi_1 = [q_{1,r} + \frac{p_g - ap_r}{a-1}(1-\theta_1)](p_r - c_1^{'}) + (1-\frac{p_g-p_r}{a-1})(1-\theta_1)(p_g - c_1^{'})$$

$$= [q_{1,r} + \frac{p_g - a(1-\frac{q_{1,r}+q_{2,r}}{\theta_1})}{a-1}(1-\theta_1)](1-\frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'}) + (1-\frac{p_g-(1-\frac{q_{1,r}+q_{2,r}}{\theta_1})}{a-1})(1-\theta_1)(p_g - c_1^{'}).$$

The first-order conditions are

$$\frac{\partial \pi_1}{\partial q_{1,r}} = [1+\frac{a(1-\theta_1)}{(a-1)\theta_1}](1-\frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'}) - \frac{1}{\theta_1}[q_{1,r} + \frac{p_g - a(1-\frac{q_{1,r}+q_{2,r}}{\theta_1})}{a-1}(1-\theta_1)] - \frac{(1-\theta_1)(p_g-c_1^{'})}{\theta_1(a-1)} = 0, \tag{6}$$

$$\frac{\partial \pi_1}{\partial p_g} = \frac{1-\theta_1}{a-1}(1-\frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'}) + (1-\frac{p_g-(1-\frac{q_{1,r}+q_{2,r}}{\theta_1})}{a-1})(1-\theta_1) - \frac{1-\theta_1}{a-1}(p_g - c_1^{'}) = 0. \tag{7}$$

Next, we consider the Hessian for $\pi_1(q_{1,r}, p_g)$

$$H(q_{1,r},p_g) = \begin{bmatrix} -\frac{2(a-\theta_1)}{\theta_1^2(a-1)} & -\frac{2(1-\theta_1)}{\theta_1(a-1)} \\ -\frac{2(1-\theta_1)}{\theta_1(a-1)} & -\frac{2(1-\theta_1)}{a-1} \end{bmatrix}$$

Since $H_1(q_{1,r},p_g) = -\frac{2(a-\theta_1)}{\theta_1^2(a-1)} < 0$, $H_2(q_{1,r},p_g) = \frac{4(1-\theta_1)}{\theta_1^2(a-1)} > 0$, the first-order conditions are both necessary and sufficient.

Firm 2's profit is $\max_{q_{2,r}} \pi_2 = (p_r - c_2)q_{2,r} = (1-\frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2)q_{2,r}$.

The first-order condition is

$$\frac{\partial \pi_2}{\partial q_{2,r}} = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2 - \frac{q_{2,r}}{\theta_1} = 0 \tag{8}$$

Solving (6), (7), (8), we obtain the following solution:

	Regular Market	Green Market
q_1^*	$\frac{\theta_1[\theta_1 - 2c_1^{'} + c_2(2-\theta_1)]}{2+\theta_1}$	$\frac{(1-\theta_1)}{2}$
q_2^*	$\frac{\theta_1(1+c_1^{'}-2c_2)}{2+\theta_1}$	-
p^*	$\frac{1+c_1^{'}+c_2\theta_1}{2+\theta_1}$	$\frac{(2a+a\theta_1-\theta_1)+2c_1^{'}+2c_2\theta_1}{2(2+\theta_1)}$

Verifying Validity: Since $c_1^{'} < c_2$ and $c_1^{'} < 1$, we have $\theta_1 - 2c_1^{'} + c_2(2-\theta_1) = \theta_1(1-c_1^{'}) + (2-\theta_1)(c_2 - c_1^{'}) > 0$. Thus, $q_{1,r}^* > 0$. Since $c_2 < (\frac{1}{2} + \frac{c_1^{'}}{2})$, we have $q_{2,r}^* > 0$. Also, $q_{1,g}^* > 0$.

We also need to verify that the optimal prices satisfy the constraint $p_g \geq ap_r$. We have $p_g - ap_r = \frac{(a-1)(\theta_1 - 2c_1^{'} - 2\theta_1 c_2)}{2(2+\theta_1)}$. Thus, if $c_2 < (\frac{1}{2} - \frac{1}{\theta_1}c_1^{'})$, then we have $p_g - ap_r > 0$.

We consider the two interior solutions in scenario D1 and scenario D3 (given that each satisfies the corresponding constraint) and one boundary solution in scenario D2. The equilibrium prices in these three scenarios are categorized and validated as follows:

- Type I: If $p_r < p_g < ap_r$, we have $p_r^* = \frac{1+c_1^{'}+c_2}{3}, p_g^* = \frac{a+c_1^{'}}{2}$. Under the imposed condition, $[\frac{1}{2} - (1-\frac{3}{2a}c_1^{'})] < c_2 < \frac{1}{2}[1+\frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$, we indeed have have $p_r^* < p_g^* < ap_r^*$.
- Type II: If $ap_r = p_g$, we have $p_r^* = \frac{(a+\theta_1-a\theta_1)+c_1^{'}+\theta_1 c_2}{2a(1-\theta_1)+3\theta_1}, p_g^* = \frac{a(a+\theta_1-a\theta_1)+ac_1^{'}+a\theta_1 c_2}{2a(1-\theta_1)+3\theta_1}$.
- Type III: If $ap_r < p_g < (a-1)+p_r$, we have $p_r^* = \frac{1+c_1^{'}+\theta_1 c_2}{2+\theta_1}, p_g^* = \frac{(2a+a\theta_1-\theta_1)+2c_1^{'}+2\theta_1 c_2}{2(2+\theta_1)}$. Under the imposed condition, $c_1^{'} < c_2 < (\frac{1}{2} - \frac{1}{\theta_1}c_1^{'})$, we have $ap_r^* < p_g^* < (a-1)+p_r^*$.

Since $[\frac{1}{2} - (1-\frac{3}{2a}c_1^{'})] > [\frac{1}{2} - \frac{1}{\theta_1}c_1^{'}]$, Type I and Type III solutions cannot both be valid. When either of these two solutions is valid, it is straightforward to show that the valid solution is also better than the Type II solution. When neither is valid, the Type II solution is optimal.

2. If $\frac{\theta_1}{2(1+\theta_1)} \leq c_1^{'} < \frac{a}{4a-3}$ and $c_1^{'} < c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$

We have $(\frac{1}{2} - \frac{1}{\theta_1}c_1^{'}) \leq c_1^{'} < [\frac{1}{2} - (1 - \frac{3}{2a}c_1^{'})] < (\frac{1}{2} + \frac{c_1^{'}}{2a})$. Therefore, only the two regions corresponding to Types I and II equilibria have positive lengths.

3. If $\frac{a}{4a-3} \leq c_1^{'} < \frac{1}{2-\frac{1}{a(1-\theta_1)+\theta_1}}$ and $c_1^{'} < c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$

We have $[\frac{1}{2} - (1 - \frac{3}{2a})c_1^{'}] \leq c_1^{'} < (\frac{1}{2} + \frac{c_1^{'}}{2a})$. Only the region corresponding to the Type I equilibrium exists.

This completes the proof. ■

C Proof of Theorem 3

Proof: Since each firm has two strategies, there are four possible combinations. We derive the payoffs of both firms under each combination in the following analysis.

1. Combination (g, g): If both firms only provide the green variant

All consumers choose to buy either a green variant or nothing. The aggregate market demand is as follows:

$$q_{1,g} + q_{2,g} = Q_g = (1 - p_g)\theta_1 + (1 - \frac{p_g}{a})(1 - \theta_1).$$

Thus, $p_g = \frac{1-q_{1,g}-q_{2,g}}{\theta_1+\frac{1-\theta_1}{a}}$. The two firms face the following profit-maximization problems:

$$\begin{aligned} \max_{q_{1,g}} \quad \pi_1 &= (p_g - c_1^{'})q_{1,g}, \\ \max_{q_{2,g}} \quad \pi_2 &= (p_g - c_2)q_{2,g}. \end{aligned}$$

By using the method similar to that used in the proof of Proposition 3, we obtain the equilibrium results as follows:

$$p_g^* = \frac{1}{3}[c_1^{'} + c_2 + \frac{a}{1 + (a-1)\theta_1}],$$

$$q_{1,g}^* = \frac{a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1}{3a}, q_{2,g}^* = \frac{a + c_1^{'} - 2c_2 + (a-1)(c_1^{'} - 2c_2)\theta_1}{3a},$$
$$\pi_1^* = \frac{[a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1]^2}{9a[1 + (a-1)\theta_1]}, \pi_2^* = \frac{[a + c_1^{'} - 2c_2 + (a-1)(c_1^{'} - 2c_2)\theta_1]^2}{9a[1 + (a-1)\theta_1]}.$$

We check whether the production quantities derived are positive.

- If $(2c_1^{'} - c_2) \geq 0$, then $[a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1]$ reaches its minimum when θ_1 reaches its upper bound 1. Thus, $a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1 \geq a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2) = a(1 - 2c_1^{'} + c_2)$. Since $c_2 > 2c_1^{'} - 1$, we have $a(1 - 2c_1^{'} + c_2) > 0$. Thus, $q_{1,g}^* > 0$.
- If $(2c_1^{'} - c_2) < 0$, then $[a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1]$ reaches its minimum when θ_1 reaches its lower bound 0. Thus, $a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1 \geq a - 2c_1^{'} + c_2 \geq (1 - 2c_1^{'} + c_2) > 0$. Thus, $q_{1,g}^* > 0$.

Similarly, we can show $q_{2,g}^* > 0$. Thus, the solution above is valid. These two firms' profits are as follows.

$$\pi_1^{g,g} = \frac{[a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1]^2}{9a[1 + (a-1)\theta_1]}, \pi_2^{g,g} = \frac{[a + c_1^{'} - 2c_2 + (a-1)(c_1^{'} - 2c_2)\theta_1]^2}{9a[1 + (a-1)\theta_1]}.$$

2. Combination (rg, rg): If both firms provide both regular and green variants

Under this scenario, both firms compete in both the regular and the green markets. We obtain the equilibrium results as follows:

$$p_r^* = \frac{1 + c_1^{'} + c_2}{3}, p_g^* = \frac{a + c_1^{'} + c_2}{3}, q_{1,r}^* = \frac{\theta_1(1 - 2c_1^{'} + c_2)}{3},$$
$$q_{2,r}^* = \frac{\theta_1(1 + c_1^{'} - 2c_2)}{3}, q_{1,g}^* = \frac{(1-\theta_1)(a - 2c_1^{'} + c_2)}{3a}, q_{2,g}^* = \frac{(1-\theta_1)(a + c_1^{'} - 2c_2)}{3a}.$$

We have $p_g^* - p_r^* = \frac{(a-1)}{3} > 0$. We can show all four production quantities are positive. These two firms' profits are as follows.

$$\pi_1^{rg,rg} = \frac{\theta_1(1 - 2c_1^{'} + c_2)^2}{9} + \frac{(1-\theta_1)(a - 2c_1^{'} + c_2)^2}{9a}, \quad \pi_2^{rg,rg} = \frac{\theta_1(1 + c_1^{'} - 2c_2)^2}{9} + \frac{(1-\theta_1)(a + c_1^{'} - 2c_2)^2}{9a}.$$

If we compare the profits under this setting with those under (g, g), we have

$$\pi_1^{rg,rg} - \pi_1^{g,g} = \frac{\theta_1(1-\theta_1)(a-1)^2}{9[1 + (a-1)\theta_1]} \geq 0, \quad \pi_2^{rg,rg} - \pi_2^{g,g} = \frac{\theta_1(1-\theta_1)(a-1)^2}{9[1 + (a-1)\theta_1]} \geq 0.$$

3. Combination (g, rg): If Firm 1 only provides the green variant, Firm 2 provides both regular and green variants

   We now compare these two firms' profits under this setting and those under the setting (rg, rg). First, both firms obtain the same amount of profits from the green market under both settings. Second, Firm 1 produces the regular variant under the setting (rg, rg) but not under the setting (g, rg). Thus, in the regular market, Firm 1 obtains positive profit under the setting (rg, rg) but 0 under the setting (g, rg). Third, in the regular market, Firm 2 competes with Firm 1 under the setting (rg, rg) but is the exclusive supplier under the setting (g, rg). Thus, Firm 2 obtains more profit in the regular market under the setting (rg, rg) than under the setting (g, rg). Therefore, we have the following results:

$$\pi_1^{rg,rg} > \pi_1^{g,rg}, \quad \pi_2^{g,rg} > \pi_2^{rg,rg}.$$

4. Combination (rg, g): If Firm 1 provides both regular and green variants, Firm 2 only provides the green variant

   This scenario is similar to Combination (g, rg). If we compare these two firms' profits under this setting and under the setting (rg, rg), we have the following results:

$$\pi_1^{rg,g} > \pi_1^{rg,rg}, \quad \pi_2^{rg,rg} > \pi_2^{rg,g}.$$

Now we derive the Nash equilibrium. Since we have $\pi_1^{rg,g} > \pi_1^{rg,rg} > \pi_1^{g,g}$ and $\pi_1^{rg,rg} > \pi_1^{g,rg}$, Firm 1's dominating strategy is rg no matter which strategy Firm 2 chooses. Similarly, we have $\pi_2^{rg,rg} > \pi_2^{rg,g}$ and $\pi_2^{g,rg} > \pi_2^{rg,rg} > \pi_2^{g,g}$. Thus, Firm 2's dominating strategy is rg as well. Therefore, (rg, rg) is the only Nash equilibrium. Both Firms provide both the regular and the green variants. This completes the proof. ■

D Proof of Theorem 4

Since $c'_{1,p} = c_{1,p}$ and $c'_{1,s} = c_{1,s}$, then we have

$$\Delta_M - \Delta_{CR} = [\frac{(1-c_{1,p})^2}{4} - \frac{(1-2c_{1,p}+c_{2,p})^2}{9}](\theta_1 - 1) + [\frac{(1-c_{1,s})^2}{4} - \frac{(1-2c_{1,s}+c_{2,s})^2}{9}](\theta_1 - 1)$$

Since $\frac{(1-c_{1,p})}{2} - \frac{(1-2c_{1,p}+c_{2,p})}{3} = \frac{(1+c_{1,p}-2c_{2,p})}{6} > 0$, we have $\frac{(1-c_{1,p})^2}{4} - \frac{(1-2c_{1,p}+c_{2,p})^2}{9} > 0$. Similarly, we have $\frac{(1-c_{1,s})^2}{4} - \frac{(1-2c_{1,s}+c_{2,s})^2}{9} > 0$. Since $\theta_1 \le 1$, we have $\Delta_M \le \Delta_{CR}$. ■

E Proof of Theorem 5

If $c'_{1,p} = c_{1,p}$ and $c'_{1,s} = c_{1,s}$, then

$$\begin{aligned}
& \Delta_{CG} - \Delta_M \\
= & \frac{(1-\theta_1)(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} + \frac{(1-\theta_1)(1-c_{1,p})^2}{4} - \frac{(1-\theta_1)(a_p - c_{1,p})^2}{4a_p} \\
& + \frac{(1-\theta_1)(a_s - 2c_{1,s} + c_{2,s})^2}{9a_s} + \frac{(1-\theta_1)(1-c_{1,s})^2}{4} - \frac{(1-\theta_1)(a_s - c_{1,s})^2}{4a_s}
\end{aligned}$$

(a) If $a_p = 1$ and $a_s = 1$, then

$$\Delta_{CG} - \Delta_M = \frac{(1-\theta_1)(1-2c_{1,p}+c_{2,p})^2}{9} + \frac{(1-\theta_1)(1-2c_{1,s}+c_{2,s})^2}{9} \ge 0.$$

(b) If $c_{2,p} \le c_{1,p}$ and $c_{2,s} \le c_{1,s}$, then

$$\begin{aligned}
\Delta_{CG} - \Delta_M \le & \frac{(1-\theta_1)(a_p - c_{1,p})^2}{9a_p} + \frac{(1-\theta_1)(1-c_{1,p})^2}{4} - \frac{(1-\theta_1)(a_p - c_{1,p})^2}{4a_p} \\
& + \frac{(1-\theta_1)(a_s - c_{1,s})^2}{9a_s} + \frac{(1-\theta_1)(1-c_{1,s})^2}{4} - \frac{(1-\theta_1)(a_s - c_{1,s})^2}{4a_s} \\
= & \frac{(1-\theta_1)}{36a_p}[(-5a_p^2 + 9a_p) - 8a_p c_{1,p} + (9a_p - 5)c_{1,p}^2] + \frac{(1-\theta_1)}{36a_s}[(-5a_s^2 + 9a_s) - 8a_s c_{1,s} + (9a_s - 5)c_{1,s}^2]
\end{aligned}$$

If $a_p \ge \frac{9}{5}$, then $(-5a_p^2 + 9a_p) \le 0$. Since $1 > c_{1,p}$, we have

$$\begin{aligned}
& \frac{(1-\theta_1)}{36a_p}[(-5a_p^2 + 9a_p) - 8a_p c_{1,p} + (9a_p - 5)c_{1,p}^2] \\
\le & \frac{(1-\theta_1)}{36a_p}[(-5a_p^2 + 9a_p)c_{1,p}^2 - 8a_p c_{1,p}^2 + (9a_p - 5)c_{1,p}^2] = \frac{-5(1-\theta_1)(a_p - 1)^2 c_{1,p}^2}{36a_p} \le 0.
\end{aligned}$$

Similarly, we have $\frac{(1-\theta_1)}{36a_s}[(-5a_s^2 + 9a_s) - 8a_s c_{1,s} + (9a_s - 5)c_{1,s}^2] \le 0$. Therefore, we have $\Delta_{CG} \le \Delta_M$.

■

F Proof of Theorem 6

If $c'_{1,p} = c_{1,p}$, and $c'_{1,s} = c_{1,s}$, then we have

$$\begin{aligned} & \Delta_{CG} - \Delta_{CR} \\ = & (1-\theta_1)[\frac{(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} - \frac{(a_p - c_{1,p})^2}{4a_p} + \frac{(1 - 2c_{1,p} + c_{2,p})^2}{9}] \\ & +(1-\theta_1)[\frac{(a_s - 2c_{1,s} + c_{2,s})^2}{9a_s} - \frac{(a_s - c_{1,s})^2}{4a_s} + \frac{(1 - 2c_{1,s} + c_{2,s})^2}{9}] \end{aligned}$$

Since $\frac{\partial \frac{(a_p-2c_{1,p}+c_{2,p})^2}{9a_p}}{\partial a_p} = \frac{(a_p-2c_{1,p}+c_{2,p})(a_p+2c_{1,p}-c_{2,p})}{9a_p^2} > 0$, then $\frac{(a_p-2c_{1,p}+c_{2,p})^2}{9a_p}$ increases with an increase in a_p. Thus, for $a_p \geq 1$, we have

$$\frac{(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} \geq \frac{(1 - 2c_{1,p} + c_{2,p})^2}{9}.$$

Therefore,

$$\frac{(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} - \frac{(a_p - c_{1,p})^2}{4a_p} + \frac{(1 - 2c_{1,p} + c_{2,p})^2}{9} \leq \frac{2(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} - \frac{(a_p - c_{1,p})^2}{4a_p}.$$

Since $c_{2,p} \leq c_{1,p}$, we have

$$\frac{2(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} - \frac{(a_p - c_{1,p})^2}{4a_p} \leq \frac{2(a_p - c_{1,p})^2}{9a_p} - \frac{(a_p - c_{1,p})^2}{4a_p} = \frac{-(a_p - c_{1,p})^2}{36a_p} < 0.$$

Similarly, we can show $\frac{(a_s-2c_{1,s}+c_{2,s})^2}{9a_s} - \frac{(a_s-c_{1,s})^2}{4a_s} + \frac{(1-2c_{1,s}+c_{2,s})^2}{9} < 0$. Thus, we have $\Delta_{CG} < \Delta_{CR}$. ■

G Proof of Theorem 7

Since $\frac{\partial \frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p}}{\partial a_p} = \frac{(1-\theta_1)(a_p-c'_{1,p})(a_p+c'_{1,p})}{8a_p^2} > 0$, we have $\frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p}$ increases with an increase in a_p. Thus, for $a_p \geq 2$, we have $\frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p} \geq \frac{(1-\theta_1)(2-c'_{1,p})^2}{16} > \frac{(1-\theta_1)(2-c'_{1,p}-c_{2,p})^2}{18}$. Therefore, $\frac{\theta_1(2-c'_{1,p}-c_{2,p})^2}{18} + \frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p} > \frac{\theta_1(2-c'_{1,p}-c_{2,p})^2}{18} + \frac{(1-\theta_1)(2-c'_{1,p}-c_{2,p})^2}{18} = \frac{(2-c'_{1,p}-c_{2,p})^2}{18}$.

Since $c'_{1,p} \leq c_{1,p}$, we have $\frac{(2-c_{1,p}-2c_{2,p})^2}{18} \leq \frac{(2-c'_{1,p}-2c_{2,p})^2}{18}$. Thus, we have $\frac{\theta_1(2-c'_{1,p}-c_{2,p})^2}{18} + \frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p} > \frac{(2-c_{1,p}-c_{2,p})^2}{18}$. Similarly, we have $\frac{\theta_1(2-c'_{1,s}-c_{2,s})^2}{18} + \frac{(1-\theta_1)(a_s-c'_{1,s})^2}{8a_s} > \frac{(2-c_{1,s}-c_{2,s})^2}{18}$. Thus, $W^a_{CR} - W^b_{CR} = \frac{\theta_1(2-c'_{1,p}-c_{2,p})^2}{18} + \frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p} + \frac{\theta_1(2-c'_{1,s}-c_{2,s})^2}{18} + \frac{(1-\theta_1)(a_s-c'_{1,s})^2}{8a_s} - \frac{(2-c_{1,p}-c_{2,p})^2}{18} - \frac{(2-c_{1,s}-c_{2,s})^2}{18} > 0$. This completes the proof. ■

H Proof of Theorem 8

- Since $a_p = 2$, $a_s = 2$, $c'_{1,p} = c_{1,p}$, and $c'_{1,s} = c_{1,s}$, we have $W^a_{CR} > W^b_{CR}$ from Theorem 7.
- If $a_p = 2$, $c'_{1,p} = c_{1,p}$, we have

$$\begin{aligned} & \frac{\theta_1(1 - 2c'_{1,p} + c_{2,p})^2}{9} + \frac{(1-\theta_1)(a_p - c'_{1,p})^2}{4a_p} - \frac{(1 - 2c_{1,p} + c_{2,p})^2}{9} \\ = & \frac{\theta_1(1 - 2c_{1,p} + c_{2,p})^2}{9} + \frac{(1-\theta_1)(2 - c_{1,p})^2}{8} - \frac{(1 - 2c_{1,p} + c_{2,p})^2}{9} \\ = & \frac{(1-\theta_1)(2 - c_{1,p})^2}{8} - \frac{(1-\theta_1)(1 - 2c_{1,p} + c_{2,p})^2}{9} \geq \frac{(1-\theta_1)(2 - c_{1,p})^2}{8} - \frac{(1-\theta_1)(1 - 2c_{1,p} + c_{2,p})^2}{8} \\ = & \frac{(1-\theta_1)}{8}[(2 - c_{1,p})^2 - (1 - 2c_{1,p} + c_{2,p})^2] = \frac{(1-\theta_1)}{8}[(1 + c_{1,p} - c_{2,p})(3 - 3c_{1,p} + c_{2,p})] > 0. \end{aligned}$$

Similarly, we have $\frac{\theta_1(1-2c'_{1,s}+c_{2,s})^2}{9} + \frac{(1-\theta_1)(a_s-c'_{1,s})^2}{4a_s} - \frac{(1-2c_{1,s}+c_{2,s})^2}{9} \geq \frac{(1-\theta_1)}{8}[(2 - c_{1,s})^2 - (1 - 2c_{1,s} + c_{2,s})^2]$.

Since $K = \frac{(1-\theta_1)}{8}[(2 - c_{1,p})^2 - (1 - 2c_{1,p} + c_{2,p})^2] + \frac{(1-\theta_1)}{8}[(2 - c_{1,s})^2 - (1 - 2c_{1,s} + c_{2,s})^2]$, we have $\Delta_{CR} \geq 0$.

- Since $a_p = 2$, $a_s = 2$, $c'_{1,p} = c_{1,p}$, $c'_{1,s} = c_{1,s}$, we have

$$\begin{aligned} \Delta_M = & \frac{\theta_1(1 - c'_{1,p})^2}{4} + \frac{(1-\theta_1)(a_p - c'_{1,p})^2}{4a_p} - \frac{(1 - c_{1,p})^2}{4} + \frac{\theta_1(1 - c'_{1,s})^2}{4} + \frac{(1-\theta_1)(a_s - c'_{1,s})^2}{4a_s} - \frac{(1 - c_{1,s})^2}{4} - K \\ = & \frac{\theta_1(1 - c_{1,p})^2}{4} + \frac{(1-\theta_1)(2 - c_{1,p})^2}{8} - \frac{(1 - c_{1,p})^2}{4} + \frac{\theta_1(1 - c_{1,s})^2}{4} + \frac{(1-\theta_1)(2 - c_{1,s})^2}{8} - \frac{(1 - c_{1,s})^2}{4} - K \\ = & \frac{(1-\theta_1)}{8}[(2 - c_{1,p})^2 - 2(1 - c_{1,p})^2] + \frac{(1-\theta_1)}{8}[(2 - c_{1,s})^2 - 2(1 - 2c_{1,s})^2] - K. \end{aligned}$$

Since $K = \frac{(1-\theta_1)}{8}[(2-c_{1,p})^2 - (1-2c_{1,p}+c_{2,p})^2] + \frac{(1-\theta_1)}{8}[(2-c_{1,s})^2 - (1-2c_{1,s}+c_{2,s})^2]$, we have

$\Delta_M = \frac{(1-\theta_1)}{8}[(1-2c_{1,p}+c_{2,p})^2 - 2(1-c_{1,p})^2] + \frac{(1-\theta_1)}{8}[(1-2c_{1,s}+c_{2,s})^2 - 2(1-2c_{1,s})^2]$. Since $c_{2,p} \leq c_{1,p}$, $c_{2,s} \leq c_{1,s}$, we have

$$\begin{aligned}
\Delta_M &= \frac{(1-\theta_1)}{8}[(1-2c_{1,p}+c_{2,p})^2 - 2(1-c_{1,p})^2] + \frac{(1-\theta_1)}{8}[(1-2c_{1,s}+c_{2,s})^2 - 2(1-2c_{1,s})^2] \\
&< \frac{(1-\theta_1)}{8}[(1-2c_{1,p}+c_{2,p})^2 - (1-c_{1,p})^2] + \frac{(1-\theta_1)}{8}[(1-2c_{1,s}+c_{2,s})^2 - (1-2c_{1,s})^2] \\
&= \frac{(1-\theta_1)(c_{2,p}-c_{1,p})}{8}[(1-2c_{1,p}+c_{2,p}) + (1-c_{1,p})] + \frac{(1-\theta_1)(c_{2,s}-c_{1,s})}{8}[(1-2c_{1,s}+c_{2,s}) + (1-2c_{1,s})] \leq 0.
\end{aligned}$$

The result follows. ■

I Proof of Theorem 9

We first compare the prices of the regular variants of Product P before and after the implementation.

$$p^b_{r,p} - p^a_{r,p} = \frac{1+c_{1,p}+c_{2,p}}{3} - \frac{1+c'_{1,p}+c_{2,p}}{3} = \frac{c_{1,p}-c'_{1,p}}{3}.$$

Since $c'_{1,p} \leq c_{1,p}$, we have $p^b_{r,p} - p^a_{r,p} \geq 0$. Next, we compare the prices of the green variants of Product P before and after the implementation.

$$p^b_{g,p} - p^a_{g,p} = \frac{a_p + c_{2,p}}{2} - \frac{a_p + c'_{1,p} + c_{2,p}}{3} = \frac{a_p - 2c'_{1,p} + c_{2,p}}{6} \geq 0.$$

Similarly, for Product S, we have $p^b_{r,s} \geq p^a_{r,s}$ and $p^b_{g,s} \geq p^a_{g,s}$. Since $p^b_{r,p} \geq p^a_{r,p}$, $p^b_{g,p} \geq p^a_{g,p}$, $p^b_{r,s} \geq p^a_{r,s}$, and $p^b_{g,s} \geq p^a_{g,s}$, then we have $W^a_{CG} > W^b_{CG}$. ■

J Proof of Theorem 10

- Since $c'_{1,p} = c_{1,p}$, $c'_{1,s} = c_{1,s}$, then from Theorem 9, we have $W^a_{CG} \geq W^b_{CG}$.
- If $a_p = 1$, $a_s = 1$, $c'_{1,p} = c_{1,p}$, $c'_{1,s} = c_{1,s}$, we have

$$\Delta_M = \frac{\theta_1(1-c_{1,p})^2}{4} + \frac{(1-\theta_1)(1-c_{1,p})^2}{4} - \frac{(1-c_{1,p})^2}{4} + \frac{\theta_1(1-c_{1,s})^2}{4} + \frac{(1-\theta_1)(1-c_{1,s})^2}{4} - \frac{(1-c_{1,s})^2}{4} - K = -K < 0.$$

- We also have

$$\begin{aligned}
\Delta_{CG} &= \frac{\theta_1(1-2c'_{1,p}+c_{2,p})^2}{9} + \frac{(1-\theta_1)(a_p-2c'_{1,p}+c_{2,p})^2}{9a_p} - \frac{\theta_1(1-2c_{1,p}+c_{2,p})^2}{9} \\
&\quad + \frac{\theta_1(1-2c'_{1,s}+c_{2,s})^2}{9} + \frac{(1-\theta_1)(a_s-2c'_{1,s}+c_{2,s})^2}{9a_s} - \frac{\theta_1(1-2c_{1,s}+c_{2,s})^2}{9} - K \\
&= \frac{(1-\theta_1)(1-2c_{1,p}+c_{2,p})^2}{9} + \frac{(1-\theta_1)(1-2c_{1,s}+c_{2,s})^2}{9} - K.
\end{aligned}$$

Since $K \leq \frac{(1-\theta_1)(1-2c_{1,p}+c_{2,p})^2}{9} + \frac{(1-\theta_1)(1-2c_{1,s}+c_{2,s})^2}{9}$, we have $\Delta_{CG} \geq 0$.
This completes the proof. ■