Online Supplement to "On asymptotic risk of selecting models for possibly non-stationary time-series"

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1 APPENDIX B: PROOFS OF LEMMAS 1-3

To prove Lemmas 1-3, we need the following five auxiliary lemmas: Lemmas B.1-B.5.

Lemma B.1. Suppose the assumptions in Theorem 1 hold. For $k_0 \le k \le k(n)$, and all $q \ge 2$,

$$\max_{k_0 \le k \le k(n)} P\left(\sigma^2 c_n h - \delta_n \le \chi_n^2(k) \le \sigma^2 c_n h + \delta_n\right) \quad \le \quad C\delta_n^{\alpha}, \tag{B.1}$$

where $h = k - d$, $k(n)$ is defined as in (A.24), and

$$\chi_n^2(k) \quad := \quad \left\| \frac{1}{\sqrt{N}} \sum_{j=K_n}^{n-1} \mathbf{z}_j(h)\varepsilon_{j+1} \right\|^2_{\hat{\Gamma}_n^{-1}(h)}, \ \hat{\Gamma}_n(h) \ := \ \frac{1}{N}\sum_{j=K_n}^{n-1} \mathbf{z}_j(h)\mathbf{z}_j'(h), \tag{B.2}$$

$0 < \delta_n \le C\left(k(n)N^{-1/4}\right)^{q/(\alpha+q)}$ and α is specified as in Assumption NS.

Proof of Lemma B.1. Recall the definition of $\Gamma(h)$ around (A.7) and similar to the proof of Lemma 3 in Ing and Wei (2005),

$$\begin{aligned} \chi_n^2(k) - h\sigma^2 \ &= \ \left\| \frac{1}{\sqrt{N}} \sum_{j=K_n}^{n-1} \mathbf{z}_j(h)\varepsilon_{j+1} \right\|^2_{\hat{\Gamma}_n^{-1}(h) - \Gamma^{-1}(h)} \\ &+ \frac{1}{N}\sum_{j=K_n}^{n-1} \mathbf{z}_j'(h)\Gamma^{-1}(h)\mathbf{z}_j(h)\left(\varepsilon_{j+1}^2 - \sigma^2\right) + \frac{\sigma^2}{N}\sum_{j=K_n}^{n-1}\left(\mathbf{z}_j'(h)\Gamma^{-1}(h)\mathbf{z}_j(h) - h\right) \\ &+ \frac{2}{N}\sum_{l=K_n+1}^{n-1} \mathbf{z}_l'(h)\Gamma^{-1}(h)\left[\sum_{j=K_n}^{l-1} \mathbf{z}_j(h)\varepsilon_{j+1}\right]\varepsilon_{l+1} =: (I) + (IIa) + (IIb) + (III). \end{aligned} \tag{B.3}$$

Consider any $p \ge 2$. By Ing and Wei (2003, Lemma 4 and (2.28)), $E\left|(I)\right|^p < C$. By Ing and Wei (2005, (5.9) and (5.10)), $E\left|(IIa)\right|^p + E\left|(IIb)\right|^p < C$. Therefore, $-\infty < (IV) := (I) + (IIa) + (IIb) < \infty$.

Write $(III) = \sum_{l=K_n+1}^{n-1} \frac{A_l}{\sqrt{N}}\varepsilon_{l+1}$, where

$$A_l^2 \;=\; \frac{4l^2}{N}\left(\mathbf{z}_l'(h)\Gamma^{-1}(h)l^{-1}\sum_{j=K_n}^{l-1}\mathbf{z}_j(h)\varepsilon_{j+1}\right)^2 \;=:\; \frac{4l^2}{N}B_l^2. \tag{B.4}$$

In view of Ing and Wei (2005, Lemma 4), it is suggestive to replace B_l with $\sqrt{\frac{h}{l}}$ and write

$$\sum_{l=K_n+1}^{n-1}\frac{A_l}{\sqrt{N}}\varepsilon_{l+1} \;=\; \sqrt{2h}\sum_{l=K_n+1}^{n-1}\left[\frac{\sqrt{2}}{N}\left(lB_l-\sqrt{l}\right)+\frac{\sqrt{2l}}{N}\right]\varepsilon_{l+1} \;=:\; \sqrt{2h}\left[(V)+(VI)\right].$$

By Ing and Wei (2005, Lemma 4, term (V) in (5.11), and (5.15)-(5.18)), it is straightforward to show that $E\left[(V)\right]^2 \le C$ and thus $-\infty < (V) < \infty$ and $-\infty < (VII) := (IV)+(V) < \infty$. Further, in (VI), $\sum_{l=K_n+1}^{n-1}\frac{2l}{N^2} \to 1$. By these two, (B.3), and the fact that $\sigma^2\left(c_n-1\right)h < \infty$, one may apply Assumption NS and equate the probability, for a general h, in (B.1) as

$$\begin{aligned} &P\left[\frac{\sigma^2\left(c_n-1\right)h-\delta_n}{\sqrt{2h}} \;\le\; \sum_{l=K_n+1}^{n-1}\frac{\sqrt{2l}}{N}\varepsilon_{l+1} \;+\; (VII) \;\le\; \frac{\sigma^2\left(c_n-1\right)h+\delta_n}{\sqrt{2h}}\right] \\ =\;& P\left[\frac{\sigma^2\left(c_n-1\right)h-\delta_n}{\sqrt{2h}} - (VII) \;\le\; \sum_{l=K_n+1}^{n-1}\frac{\sqrt{2l}}{N}\varepsilon_{l+1} \;\le\; \frac{\sigma^2\left(c_n-1\right)h+\delta_n}{\sqrt{2h}} - (VII)\right] \;\le\; C\left[\frac{\delta_n}{\sqrt{h}}\right]^\alpha. \end{aligned}$$

The result follows by taking $h = k_0 - d$. □

Lemma B.2. Suppose the assumptions in Theorem 1 hold. For $k_0 \le k \le k(n)$, and all $q \ge 2$,

$$\begin{aligned} &P\left(\hat{k}_n^S = k, \tilde{k}_m^S \ne k\right) \;+\; P\left(\hat{k}_n^S \ne k, \tilde{k}_m^S = k\right) \\ &\quad + P\left(\hat{k}_n = k, \tilde{k}_m \ne k\right) \;+\; P\left(\hat{k}_n \ne k, \tilde{k}_m = k\right) \;\le\; C\left(k(n)N^{-1/4}\right)^{q\left(1-\frac{q}{\alpha+q}\right)}, \end{aligned} \tag{B.5}$$

where $k(n)$ is defined as in (A.24), α is specified as in Assumption NS, $m = n-\sqrt{n}$, and

$$\tilde{k}_m^S = \arg\min_{k_0\le k\le k(n)}\left(N+c_nk\right)\hat{\sigma}_m^2(k),\;\; \tilde{k}_m = \arg\min_{k_0\le k\le k(n)}\log\hat{\sigma}_m^2(k)+\frac{c_nk}{N}. \tag{B.6}$$

Proof of Lemma B.2. As the proofs for other terms are similar or simpler, we only show that for the third term in (B.5). First consider the case $k+1 \le \tilde{k}_m \le k(n)$. On the $A_n(k,r) = \{\hat{k}_n = k, \tilde{k}_m = r\}$, where $k+1 \le r \le k(n)$,

$$N\left(\hat{\sigma}_n^2(k)-\hat{\sigma}_n^2(r)\right) \;\ge\; c_n\left[r(1+r_n(r))\hat{\sigma}_n^2(r)-k(1+r_n(k))\hat{\sigma}_n^2(k)\right], \tag{B.7}$$

$$N\left(\hat{\sigma}_m^2(k)-\hat{\sigma}_m^2(r)\right) \;\le\; c_n\left[r(1+r_n(r))\hat{\sigma}_m^2(r)-k(1+r_n(k))\hat{\sigma}_m^2(k)\right], \tag{B.8}$$

where referring to (B.52), $r_n(k) := \frac{c_nk}{N}\exp\left(m_n(k)\right)$, $m_n(k)$ lies between 0 and $\frac{c_nk}{N}$. For $0<\delta<1$, define $B_n(k,r) = \left\{\sigma^2-\hat{\sigma}_n^2(k)\le\sigma^2\delta/k, \sigma^2-\hat{\sigma}_n^2(r)\le\sigma^2\delta/r\right\}$. For sufficiently large n, $|\pi-c_n| \le \delta\pi$

and $0 < \max_{k_0 \le k \le k(n)} r_n(k)k \le C\frac{\pi k^2(n)}{N} < \delta$. Combining (B.41) and (B.39) with Ing et al. (2012, Lemma 4.1), for any $q > 0$,

$$\max_{k<r\le k(n)} P\left(B_n^c(k,r)\right) \;\le\; \max_{k<r\le k(n)} Cr^q N^{-q/2} \;\le\; C\left(k(n)\right)^q N^{-q/2}. \tag{B.9}$$

Observing that $\hat{\sigma}_n^2(r) \le \hat{\sigma}_n^2(k) \le \sigma^2$, for sufficiently large n and on the set $(A_n(k,r) \cap B_n(k,r))$,

$$\begin{aligned} D_n &:= N\left(\hat{\sigma}_n^2(k) - \hat{\sigma}_n^2(r)\right)/\sigma^2 + 2\pi\left(1+\delta\right)\delta \;\ge\; c_n\left(r-k\right), \\ D_m &:= N\left(\hat{\sigma}_m^2(k) - \hat{\sigma}_m^2(r)\right)/\sigma^2 \;\le\; c_n\left(r-k\right). \end{aligned}$$

Therefore,

$$\begin{aligned} &\max_{k<r\le k(n)} P\left(A_n(k,r) \cap B_n(k,r)\right) \\ \le\;& \max_{k<r\le k(n)} P\left(D_n - D_m \ge \delta_n\right) + \max_{k<r\le k(n)} P\left(c_n\left(r-k\right) - \delta_n \le D_m \le c_n\left(r-k\right)\right) \;=:\; (I) + (II), \end{aligned} \tag{B.10}$$

where δ_n is specified around (B.1). Consider a modification of $\hat{\Omega}_{d,n}(k)$ in (A.7):

$$\hat{\Omega}_{c,n}(k) \;=\; \begin{pmatrix} N^{-1}\sum_{j=K_n}^{n-1} \mathbf{z}_j(k-d)\mathbf{z}_j'(k-d) & \mathbf{0}_{(k-d)\times d} \\ \mathbf{0}_{d\times(k-d)} & \hat{\Omega}_n(d) \end{pmatrix}, \tag{B.11}$$

and define

$$\begin{aligned} N\left(\hat{\sigma}_n^2(k) - \hat{\sigma}_n^2(r)\right)/\sigma^2 \;&=\; \chi_n^2(r) - \chi_n^2(k) \\ &+ \left[\left\|\frac{1}{\sqrt{N}}\sum_{j=K_n}^{n-1} \mathbf{s}_{j,n}(r)\varepsilon_{j+1}/\sigma\right\|^2_{\hat{\Omega}_n^{-1}(r)-\hat{\Omega}_{c,n}^{-1}(r)} - \left\|\frac{1}{\sqrt{N}}\sum_{j=K_n}^{n-1} \mathbf{s}_{j,n}(k)\varepsilon_{j+1}/\sigma\right\|^2_{\hat{\Omega}_n^{-1}(k)-\hat{\Omega}_{c,n}^{-1}(k)}\right] \\ =:\;& \chi_n^2(r) - \chi_n^2(k) + u_n(k,r), \end{aligned} \tag{B.12}$$

where $\chi_n^2(k)$ is defined as in (B.2). Similarly, $u_m(k,r)$ is defined. First consider (I). By Ing et al. (2010, (B.13) and an analog of (18)),

$$\max_{k<r\le k(n)} E\left|u_n(k,r)\right|^q + E\left|u_m(k,r)\right|^q \;\le\; \max_{k<r\le k(n)} Ck^{2q}N^{-q/2} \;\le\; C\left(k(n)\right)^{2q} N^{-q/2}. \tag{B.13}$$

On the other hand, write

$$\begin{aligned} &\chi_n^2(k) - \chi_m^2(k) \\ =\;& \left(N^{-1/2}\sum_{j=m}^{n-1} \varepsilon_{j+1}\mathbf{z}_j'(k-d)\right)\hat{\Gamma}_n^{-1}(k-d)\left(N^{-1/2}\sum_{j=K_n}^{n-1} \mathbf{z}_j(k-d)\varepsilon_{j+1}\right) \\ &+ \left(N^{-1/2}\sum_{j=K_n}^{m-1} \varepsilon_{j+1}\mathbf{z}_j'(k-d)\right)\left(\hat{\Gamma}_n^{-1}(k-d) - \hat{\Gamma}_m^{-1}(k-d)\right)\left(N^{-1/2}\sum_{j=K_n}^{n-1} \mathbf{z}_j(k-d)\varepsilon_{j+1}\right) \\ &+ \left(N^{-1/2}\sum_{j=K_n}^{m-1} \varepsilon_{j+1}\mathbf{z}_j'(k-d)\right)\hat{\Gamma}_m^{-1}(k-d)\left(N^{-1/2}\sum_{j=m}^{n-1} \mathbf{z}_j(k-d)\varepsilon_{j+1}\right), \end{aligned} \tag{B.14}$$

where $\hat{\Gamma}_n(k-d)$ and $\hat{\Gamma}_m(k-d)$ are defined as in (B.2). By Ing and Wei (2003, (3.17)-(3.20)), since $k(n)=o(n^{1/4})$

$$\begin{aligned} &\max_{k<r\leq k(n)} \left(E\left|\chi_n^2(k)-\chi_m^2(k)\right|^q + E\left|\chi_n^2(r)-\chi_m^2(r)\right|^q \right) \\ \leq\ & \max_{k<r\leq k(n)} C\Big(r^{q/2}\left(N^{-q/4}r^{q/2}\right) + \left(r^q N^{-3q/4}+N^{-q/2}\right) r^q \Big) \ \leq\ C\left(k(n)N^{-1/4}\right)^q. \quad \text{(B.15)} \end{aligned}$$

Therefore, by (B.13)-(B.15), the specification of δ_n, and Lemma B.1 (with $\chi_m^2(r)<\infty$),

$$(I)+(II) \ \leq\ C\left(k(n)N^{-1/4}\right)^{q\left(1-\frac{q}{\alpha+q}\right)}. \tag{B.16}$$

By (B.9), (B.10) and (B.16), $\max_{k+1\leq r\leq k(n)} P\Big(\hat{k}_n=k,\tilde{k}_m=r\Big) \leq C\left(k(n)N^{-1/4}\right)^{q\left(1-\frac{q}{\alpha+q}\right)}$. Similarly, $\max_{k_0\leq r\leq k-1} P\Big(\hat{k}_n=k,\tilde{k}_m=r\Big) \leq C\left(k(n)N^{-1/4}\right)^{q\left(1-\frac{q}{\alpha+q}\right)}$. This completes the proof. □

Lemma B.3. Suppose the assumptions in Theorem 1 hold.

$$\begin{aligned} \sum_{k=k_0+1}^{k(n)} NE\left[\left(\mathbf{f}_n^2(k)-\mathbf{f}_n^2(k_0)\right) I_{\{\hat{k}_n^S=k\}}\right] &= \sum_{k=k_0+1}^{k(n)} NE\left[\left(\mathbf{f}_{2,n}^{*2}(k-d)-\mathbf{f}_{2,n}^{*2}(k_0-d)\right) I_{\{\tilde{k}_m^S=k\}}\right]+o(1); \\ \sum_{k=k_0+1}^{k(n)} NE\left[\left(\mathbf{f}_n^2(k)-\mathbf{f}_n^2(k_0)\right) I_{\{\hat{k}_n=k\}}\right] &= \sum_{k=k_0+1}^{k(n)} NE\left[\left(\mathbf{f}_{2,n}^{*2}(k-d)-\mathbf{f}_{2,n}^{*2}(k_0-d)\right) I_{\{\tilde{k}_m=k\}}\right]+o(1), \quad \text{(B.17)} \end{aligned}$$

where $k(n)$ is defined as in (A.24), $\mathbf{f}_{2,n}^*(.)$ is defined around (A.33), and $\tilde{k}_m^S, \tilde{k}_m$ are defined as in (B.6).

Proof of Lemma B.3. As the proof for $\hat{k}_n^S$ is similar and simpler, we only show that for $\hat{k}_n$. Further, we only prove the case $d\geq 1$, since the proof for the case $d=0$ is similar and simpler. Write

$$\mathbf{f}_{1,n}(d) \ =\ U_{n,n}^{'}(d)\hat{\Omega}_n^{-1}(d)\left(\frac{1}{N}\sum_{j=K_n}^{n-1} U_{j,n}(d)\varepsilon_{j+1}\right), \quad \mathbf{f}_{1,n}^*(d) \ =\ U_{n,n}^{*'}(d)\hat{\Omega}_m^{-1}(d)\left(\frac{1}{N}\sum_{j=K_n}^{m-1} U_{j,n}(d)\varepsilon_{j+1}\right),$$

where $\hat{\Omega}_n(d)$ is defined as in (A.6); $U_{j,n}(d)$ is defined around (B.39) and

$$U_{n,n}^*(d) \ :=\ \left(\frac{1}{N^{d-1/2}}\sum_{j=\sqrt{n}}^{n-1}\kappa_j(d)\varepsilon_{n-j},\ldots,\frac{1}{N^{1/2}}\sum_{j=\sqrt{n}}^{n-1}\kappa_j(1)\varepsilon_{n-j}\right)'.$$

Write

$$\begin{aligned} N\left[\mathbf{f}_n^2(k)-\mathbf{f}_n^2(k_0)\right] \ &=\ N\left[\mathbf{f}_n^2(k)-\left(\mathbf{f}_{1,n}(d)+\mathbf{f}_{2,n}^*(k-d)\right)^2\right] - N\left[\mathbf{f}_n^2(k_0)-\left(\mathbf{f}_{1,n}(d)+\mathbf{f}_{2,n}^*(k_0-d)\right)^2\right] \\ &+2N\left[\mathbf{f}_{1,n}(d)-\mathbf{f}_{1,n}^*(d)\right]\left[\mathbf{f}_{2,n}^*(k-d)-\mathbf{f}_{2,n}^*(k_0-d)\right] + N\left[\mathbf{f}_{2,n}^{*2}(k-d)-\mathbf{f}_{2,n}^{*2}(k_0-d)\right] \\ &+2N\mathbf{f}_{1,n}^*(d)\left[\mathbf{f}_{2,n}^*(k-d)-\mathbf{f}_{2,n}^*(k_0-d)\right] \ =:\ (I)-(II)+2(III)+(IV)+2(V). \quad \text{(B.18)} \end{aligned}$$

For $k_0 \le k \le k(n)$, first consider

$$
\begin{aligned}
& \sqrt{N}\left(\mathbf{f}_n(k) - \mathbf{f}_{1,n}(d) - \mathbf{f}^*_{2,n}(k-d)\right) \\
= \;& \mathbf{s}'_{n,n}(k)\left(\hat{\Omega}_n^{-1}(k) - \hat{\Omega}_{d,n}^{-1}(k)\right) N^{-1/2}\sum_{j=K_n}^{n-1}\mathbf{s}_{j,n}(k)\varepsilon_{j+1} \\
& + \left(\mathbf{z}_n(k-d) - \mathbf{z}^*_n(k-d)\right)'\hat{\Gamma}_n^{-1}(k-d)N^{-1/2}\sum_{j=K_n}^{n-1}\mathbf{z}_j(k-d)\varepsilon_{j+1} \\
& + \mathbf{z}^{*'}_n(k-d)\left(\hat{\Gamma}_n^{-1}(k-d) - \Gamma^{-1}(k-d)\right)N^{-1/2}\sum_{j=K_n}^{n-1}\mathbf{z}_j(k-d)\varepsilon_{j+1} \\
& + \mathbf{z}^{*'}_n(k-d)\Gamma^{-1}(k-d)N^{-1/2}\sum_{j=m}^{n-1}\mathbf{z}_j(k-d)\varepsilon_{j+1} \;=:\; (VIa)+(VIb)+(VIc)+(VId), \quad \text{(B.19)}
\end{aligned}
$$

where $\Gamma(k-d)$ and $\hat{\Gamma}_n(k-d)$ are defined around (A.7) and (B.2) respectively. For any $p \ge 2$, by Ing et al. (2010, (18) of Theorem 1, Lemma B.1 and Lemma B.3),

$$
\begin{aligned}
E|(VIa)|^{2p} \;&\le\; \left|E\left\|\mathbf{s}'_{n,n}(k)\right\|^{12p}\right|^{\frac{1}{6}}\left|E\left\|\hat{\Omega}_n^{-1}(k) - \hat{\Omega}_{d,n}^{-1}(k)\right\|^{3p}\right|^{\frac{2}{3}}\left|E\left\|\frac{1}{\sqrt{N}}\sum_{j=K_n}^{n-1}\mathbf{s}_{j,n}(k)\varepsilon_{j+1}\right\|^{12p}\right|^{\frac{1}{6}} \\
&\le\; Ck^{4p}N^{-p}. \qquad \text{(B.20)}
\end{aligned}
$$

By Ing and Wei (2003, (3.15), (2.27) and Lemma 4),

$$
\begin{aligned}
E|(VIb)|^{2p} \;&\le\; \left|E\left\|\left(\mathbf{z}_n(k-d) - \mathbf{z}^*_n(k-d)\right)'\right\|^{6p}\right|^{\frac{1}{3}}\left|E\left\|\hat{\Gamma}_n^{-1}(k-d)\right\|^{6p}\right|^{\frac{1}{3}}\left|E\left\|\frac{1}{\sqrt{N}}\sum_{j=K_n}^{n-1}\mathbf{z}_j(k-d)\varepsilon_{j+1}\right\|^{6p}\right|^{\frac{1}{3}} \\
&\le\; Ck^{2p}\left(\sum_{j=\sqrt{n}/2-K_n+1}^{\infty} b_j^2\right)^p \;\le\; Ck^{2p}\left(\frac{1}{\sqrt{n}}\sum_{j=\sqrt{n}/2-K_n+1}^{\infty}|j^{1/2}b_j|\right)^p \;\le\; Ck^{2p}N^{-p/2}, \quad \text{(B.21)}
\end{aligned}
$$

since given (1) and $k_0 < \infty$, it is not difficult to show $jb_j^2 \le |j^{1/2}b_j| \le 1$. Further, by Ing and Wei (2003, (3.16), (2.28) and Lemma 4),

$$
\begin{aligned}
& E|(VIc)|^{2p} \\
\le\; & \left|E\left\|\mathbf{z}^*_n(k-d)\right\|^{12p}\right|^{\frac{1}{6}}\left|E\left\|\hat{\Gamma}_n^{-1}(k-d) - \Gamma^{-1}(k-d)\right\|^{3p}\right|^{\frac{2}{3}}\left|E\left\|\frac{1}{\sqrt{N}}\sum_{j=K_n}^{n-1}\mathbf{z}_j(k-d)\varepsilon_{j+1}\right\|^{12p}\right|^{\frac{1}{6}} \\
\le\; & C\left(k^{4p}N^{-p}\right). \qquad \text{(B.22)}
\end{aligned}
$$

By Ing and Wei (2003, (3.16), Remark 1 and Lemma 4),

$$
\begin{aligned}
E|(VId)|^{2p} \;&\le\; \left|E\left\|\mathbf{z}^*_n(k-d)\right\|^{6p}\right|^{\frac{1}{3}}\left|E\left\|\Gamma^{-1}(k-d)\right\|^{6p}\right|^{\frac{1}{3}}\left|E\left\|\frac{1}{\sqrt{N}}\sum_{j=m}^{n-1}\mathbf{z}_j(k-d)\varepsilon_{j+1}\right\|^{6p}\right|^{\frac{1}{3}} \\
&\le\; Ck^{2p}N^{-p/2}. \qquad \text{(B.23)}
\end{aligned}
$$

Next, by Ing (2007, (D.3)) and Ing et al. (2010, (17) of Theorem 1 and Lemma B.1),

$$E\left|\sqrt{N}\mathbf{f}^*_{2,n}(k-d)\right|^{2p} \;\leq\; Ck^p, \;\; E\left|\sqrt{N}\mathbf{f}_{1,n}(d)\right|^{2p} \;\leq\; C. \tag{B.24}$$

Together with (1), (B.20)-(B.24) imply

$$E\left|\sqrt{N}\mathbf{f}_n(k)\right|^{2p} \;\leq\; Ck^p. \tag{B.25}$$

By (B.19)-(B.25), since $k \leq k(n) \leq n^{1/4}$,

$$E|(I)|^p + E|(II)|^p \;\leq\; Ck^{3p/2}N^{-p/4}. \tag{B.26}$$

On the other hand,

$$\begin{aligned}\sqrt{N}\left(\mathbf{f}_{1,n}(d) - \mathbf{f}^*_{1,n}(d)\right) \;&=\; \left(U_{n,n}(d) - U^*_{n,n}(d)\right)'\hat{\Omega}_n^{-1}(d)N^{-1/2}\sum_{j=K_n}^{n-1}U_{j,n}(d)\varepsilon_{j+1} \\ &+U^{*'}_{n,n}(d)\left(\hat{\Omega}_n^{-1}(d) - \hat{\Omega}_m^{-1}(d)\right)N^{-1/2}\sum_{j=K_n}^{n-1}U_{j,n}(d)\varepsilon_{j+1} \\ &+U^{*'}_{n,n}(d)\hat{\Omega}_m^{-1}(d)N^{-1/2}\sum_{j=m}^{n-1}U_{j,n}(d)\varepsilon_{j+1} \;=:\; (VIIa) + (VIIb) + (VIIc).\end{aligned} \tag{B.27}$$

By Ing et al. (2010, (B.38), (17) and (18) of Theorem 1, and Lemma B.1),

$$E\left|(VIIa)\right|^{2p} \;=\; o(N^{-p/2}),\; E\left|(VIIb)\right|^{2p} \;\leq\; CN^{-p},\; E\left|(VIIc)\right|^{2p} \;\leq\; CN^{-p/2}. \tag{B.28}$$

By (B.27)-(B.28) and the first part of (B.24),

$$E|(III)|^p \;\leq\; Ck^{p/2}N^{-p/4}. \tag{B.29}$$

By (B.26), (B.29) and Lemma 1(ii) (particularly the part related to (A.17)), since $k(n) = o(n^{1/4})$,

$$\sum_{k=k_0+1}^{k(n)} E\left[\left((I) - (II) + 2(III)\right)I_{\{\hat{k}_n = k\}}\right] \;\leq\; CN^{-1/4}\sum_{k=k_0+1}^{k(n)} k^{3/2}k^{-q(p-1)/2p} \;=\; o(1), \tag{B.30}$$

if $q \geq 3p/(p-1)$. By the first part of (B.24), $E\left|N\mathbf{f}^{*2}_{2,n}(k-d)\right|^p \leq Ck^p$. By Lemma B.2,

$$\begin{aligned}&\sum_{k=k_0+1}^{k(n)} E\left[|(IV)||I_{\{\hat{k}_n=k\}} - I_{\{\tilde{k}_m\neq k\}}|\right] \\ \leq\;& C\sum_{k=k_0+1}^{k(n)} k\left[P^{\frac{p-1}{p}}\left(\hat{k}_n \neq k, \tilde{k}_m = k\right) + P^{\frac{p-1}{p}}\left(\hat{k}_n = k, \tilde{k}_m \neq k\right)\right] \\ \leq\;& Ck^2(n)\left(k(n)N^{-1/4}\right)^{\frac{(p-1)q}{p}\left(1-\frac{q}{\alpha+q}\right)} \;=\; o(1),\end{aligned} \tag{B.31}$$

if we set q, p such that $\eta < A_{p,q}/4(2 + A_{p,q})$, $A_{p,q} := \frac{(p-1)q}{p}\left(1 - \frac{q}{\alpha+q}\right)$, where we recall from (A.24) that $k(n) \leq n^{\eta}$. Finally consider (V). By construction, it is not difficult to see that

$$\sum_{k=k_0+1}^{k(n)} NE\left[\mathbf{f}_{1,n}^*(d)\left[\mathbf{f}_{2,n}^*(k-d) - \mathbf{f}_{2,n}^*(k_0-d)\right] I_{\{\tilde{k}_m=k\}}\right] \;=\; 0.$$

By (B.24), (B.27) and (B.28), $E\left|(V)\right|^p \leq Ck^{p/2}$. Similarly, set q, p such that $\eta < A_{p,q}/4(3/2 + A_{p,q})$,

$$\sum_{k=k_0+1}^{k(n)} E\left[2|(V)|I_{\{\hat{k}_n=k\}}\right] \;\leq\; C\sum_{k=k_0+1}^{k(n)} k^{1/2}\left[P^{\frac{p-1}{p}}\left(\hat{k}_n \neq k, \tilde{k}_m = k\right) + P^{\frac{p-1}{p}}\left(\hat{k}_n = k, \tilde{k}_m \neq k\right)\right] \;=\; o(1).$$

Together with (B.30) and (B.31), the result follows. This completes the proof. □

Lemma B.4. (Lemma 1 and Lemma 4 in Shibata (1976).) Let $Z_1, Z_2, \cdots$ be independently and identically random variables. Define $S_j := Z_1 + Z_2 + \cdots + Z_j$, $j = 1, 2, \cdots$. Then

$$p_j \;:=\; P\left(S_1 > 0, \cdots, S_j > 0\right) \;=\; \Sigma_j^*\left[\prod_{i=1}^{j}\frac{1}{r_i!}\left(\frac{\alpha_i}{i}\right)^{r_i}\right], \tag{B.32}$$

$$q_j \;:=\; P\left(S_1 \leq 0, \cdots, S_j \leq 0\right) \;=\; \Sigma_j^*\left[\prod_{i=1}^{j}\frac{1}{r_i!}\left(\frac{1-\alpha_i}{i}\right)^{r_i}\right], \tag{B.33}$$

$$e_j \;:=\; E\left(S_j I_{\{S_1>0,\cdots,S_j>0\}}\right) \;=\; \Sigma_j^*\left[\left(\prod_{i=1}^{j}\frac{1}{r_i!}\left(\frac{\alpha_i}{i}\right)^{r_i}\right)\left(\sum_{i=1}^{j}\frac{r_i}{\alpha_i}E\left(S_i I_{S_i>0}\right)\right)\right], \tag{B.34}$$

where $\alpha_i := P\left(S_i > 0\right)$, and the summation Σ_j^* extends over all j-tuples $(r_1, \cdots, r_j)$ of non-negative integers such that $r_1 + 2r_2 + \cdots, +jr_j = j$.

Lemma B.5. (Simplified Lemma A.2 in Sin and Yu (2019).) Consider Model (1). Suppose Assumptions MO, NS, FM and the followings hold:

$$\frac{E\left[\left[\mathcal{S}_n(\hat{k}_n - d) - \mathcal{S}_n\left(k_0 - d\right)\right]^2 I_{\{\hat{k}_n \geq \max\{1,d\}\}}\right]}{L_n\left(k_0\right)} \;=\; o(1), \tag{B.35}$$

$$\frac{E\left[\left[\mathbf{f}_n(\hat{k}_n) - \mathbf{f}_n\left(k_0\right)\right]^2 I_{\{\hat{k}_n \geq \max\{1,d\}\}}\right]}{L_n\left(k_0\right)} \;=\; o(1), \tag{B.36}$$

$$P\left(\hat{k}_n < d\right) \;=\; o(1), \quad \frac{E\left[\left[\mathbf{f}_n(\hat{k}_n) + \mathcal{S}_n\left(\hat{k}_n - d\right)\right]^2 I_{\{\hat{k}_n < d\}}\right]}{L_n\left(k_0\right)} \;=\; o(1). \tag{B.37}$$

Then

$$\lim_{n\to\infty}\frac{E\left(y_{n+1} - \hat{y}_{n+1}(\hat{k}_n)\right)^2 - \sigma^2}{L_n\left(k_0\right)} \;=\; 1. \tag{B.38}$$

The theorem also holds if we replace $\hat{k}_n$ by $\hat{k}_n^S$.

Proof of Lemma 1. When $k \leq k_0 - 1$, obviously $V_n^{-1}(k) \leq C$. For $k_0 + 1 \leq k \leq K_n$,

$$V_n^{-1}(k) \;=\; \frac{L_{n,c_n}(k)}{L_{n,c_n}(k) - L_{n,c_n}(k_0)} \;=\; 1 + \frac{k_0}{k - k_0} \;\leq\; 1 + k_0 \;\leq\; C.$$

As the proof for $d = 0$ is similar to those in Shibata (1980), here we focus on the case $d \geq 1$. For (A.20), since $k \geq d$,

$$\hat{\sigma}_n^2(k) - \sigma^2 \;=\; \left[\hat{\Sigma}_n(k-d) - \sigma^2(k-d)\right] - \left\| N^{-1} \sum_{j=K_n}^{n-1} \mathbf{s}_{j,n}(k)\varepsilon_{j+1,k-d} \right\|^2_{\hat{\Omega}_n^{-1}(k)} + \|\mathbf{a} - \mathbf{a}(k-d)\|_z^2 . \tag{B.39}$$

For positive integer $1 \leq v \leq d$, define $U_{j,n}(v) = (y_j(d)/N^{d-(1/2)}, \cdots, y_j(d-v+1)/N^{d-v+(1/2)})'$. Then, $\mathbf{s}_{j,n}(k) = (\mathbf{z}'(k-d), U'_{j,n}(d))'$ for $d < k \leq K_n$, and $\mathbf{s}_{j,n}(k) = U_{j,n}(k)$ for $k = d$. It follows that

$$\begin{aligned}
&\left\| N^{-1} \sum_{j=K_n}^{n-1} \mathbf{s}_{j,n}(k)\varepsilon_{j+1,k-d} \right\|^2_{\hat{\Omega}_n^{-1}(k)} \\
\leq\;& 2N^{-2} \left\|\hat{\Omega}_n^{-1}(k)\right\| \left[\left\| \sum_{j=K_n}^{n-1} U_{j,n}(d)(\varepsilon_{j+1,k-d} - \varepsilon_{j+1}) \right\|^2 + \left\| \sum_{j=K_n}^{n-1} U_{j,n}(d)\varepsilon_{j+1} \right\|^2 \right. \\
&\left. + \left\| \sum_{j=K_n}^{n-1} \mathbf{z}_j(k-d)(\varepsilon_{j+1,k-d} - \varepsilon_{j+1}) \right\|^2 I_{\{k>d\}} + \left\| \sum_{j=K_n}^{n-1} \mathbf{z}_j(k-d)\varepsilon_{j+1} \right\|^2 I_{\{k>d\}} \right].
\end{aligned} \tag{B.40}$$

By (B.40), (1) and Ing et al. (2010, Theorem 1, Lemmas B.1, B.3, B.4 and B.6), one has for any $q > 0$, all $d \leq k \leq K_n$ and all sufficiently large n,

$$E \left\| N^{-1} \sum_{j=K_n}^{n-1} \mathbf{s}_{j,n}(k)\varepsilon_{j+1,k-d} \right\|^{2q}_{\hat{\Omega}_n^{-1}(k)} \;\leq\; C\frac{k^q}{N^q}. \tag{B.41}$$

Combining (B.41) and (B.39) with Ing et al. (2012, Lemma 4.1) yields for $d \leq k \leq k_0 - 1$,

$$EV_{1n}^q(k) \;\leq\; C\left[\frac{c_n k/N^{1/2} + c_n k^2/N + c_n k \|\mathbf{a} - \mathbf{a}(k-d)\|_z^2}{(c_n - 1)(k + d^2)\sigma^2 + N\|\mathbf{a} - \mathbf{a}(k-d)\|_z^2}\right]^q \;\leq\; Cc_n^q N^{-q}; \tag{B.42}$$

and for $k_0 + 1 \leq k \leq K_n$, since $k \leq K_n = o(N^{1/2})$,

$$EV_{1n}^q(k) \;\leq\; C\left[\frac{c_n k/N^{1/2} + c_n k^2/N}{(c_n - 1)(k + d^2)\sigma^2}\right]^q \;\leq\; C\left[N^{-q/2} + k^q N^{-q}\right] \;\leq\; CN^{-q/2}. \tag{B.43}$$

Thus (A.20) holds. For (A.21), similarly, for $d \leq k \leq k_0 - 1$,

$$\begin{aligned}
EV_{2n}^q(k) \;\leq\;& C\left[\frac{c_n k_0/N^{1/2} + c_n k_0^2/N + c_n k_0 \|\mathbf{a} - \mathbf{a}(k_0-d)\|_z^2}{(c_n - 1)(k + d^2)\sigma^2 + N\|\mathbf{a} - \mathbf{a}(k-d)\|_z^2}\right]^q \\
\leq\;& C\left[\frac{c_n k_0/N^{1/2} + c_n k_0^2/N + c_n k_0 \|\mathbf{a} - \mathbf{a}(k-d)\|_z^2}{(c_n - 1)(k + d^2)\sigma^2 + N\|\mathbf{a} - \mathbf{a}(k-d)\|_z^2}\right]^q \;\leq\; Cc_n^q N^{-q};
\end{aligned} \tag{B.44}$$

and for $k_0 \le k \le K_n$,

$$EV_{2n}^q(k) \;\le\; C\left[\frac{c_n k_0/N^{1/2} + c_n k_0^2/N}{(c_n-1)(k+d^2)\sigma^2}\right]^q \;\le\; C\left[k^{-q}N^{-q/2} + k^{-q}N^{-q}\right] \;\le\; Ck^{-q}N^{-q/2}. \quad \text{(B.45)}$$

Thus (A.21) holds. For (A.22), since we confine our attention to $d \ge 1$, by (A.7),

$$\hat{\Omega}_{d,n}(k) \;=\; \begin{cases} \hat{\Omega}_n(k), & 1 \le k \le d, \\ \begin{pmatrix} \Gamma(k-d) & \mathbf{0}_{(k-d)\times d} \\ \mathbf{0}_{d\times(k-d)} & \hat{\Omega}_n(d) \end{pmatrix}, & k > d. \end{cases}$$

Then one has

$$NL_{n,c_n}(k)V_{3n}(k) \;\le\; (I) + (II) + (III) + (IV) + (V) + (VI), \quad \text{(B.46)}$$

$$\begin{aligned}
(I) &= \left| (k-d)\sigma^2 - \left\| N^{-1/2}\sum_{j=K_n}^{n-1} \mathbf{z}_j(k-d)\varepsilon_{j+1,k-d} \right\|^2_{\Gamma^{-1}(k-d)} \right| I_{\{k>d\}}, \\
(II) &= \left| (k_0-d)\sigma^2 - \left\| N^{-1/2}\sum_{j=K_n}^{n-1} \mathbf{z}_j(k_0-d)\varepsilon_{j+1,k_0-d} \right\|^2_{\Gamma^{-1}(k_0-d)} \right| I_{\{k_0>d\}}, \\
(III) &= \left\| N^{-1/2}\sum_{j=K_n}^{n-1} U_{j,n}(d)\varepsilon_{j+1,k-d} \right\|^2 \left\| \hat{\Omega}_n^{-1}(d) \right\|, \\
(IV) &= \left\| N^{-1/2}\sum_{j=K_n}^{n-1} U_{j,n}(d)\varepsilon_{j+1,k_0-d} \right\|^2 \left\| \hat{\Omega}_n^{-1}(d) \right\|, \\
(V) &= \left\| N^{-1/2}\sum_{j=K_n}^{n-1} \mathbf{s}_{j,n}(k)\varepsilon_{j+1,k-d} \right\|^2 \left\| \hat{\Omega}_n^{-1}(k) - \hat{\Omega}_{d,n}^{-1}(k) \right\|, \\
(VI) &= \left\| N^{-1/2}\sum_{j=K_n}^{n-1} \mathbf{s}_{j,n}(k_0)\varepsilon_{j+1,k_0-d} \right\|^2 \left\| \hat{\Omega}_n^{-1}(k_0) - \hat{\Omega}_{d,n}^{-1}(k_0) \right\|.
\end{aligned}$$

By (B.46), (1), Ing et al. (2012, Lemma 4.2), and Ing et al. (2010, Theorem 1, Lemmas B.1, B.3, B.4 and B.6), one has

$$EV_{3n}^q(k) \;\le\; C\left[NL_{n,c_n}(k)\right]^{-q}\left[(k-d)^{q/2} + (k_0-d)^{q/2} + 1 + k^{2q}N^{-q/2} + k_0^{2q}N^{-q/2}\right]. \quad \text{(B.47)}$$

In particular, for $d \le k \le k_0 - 1$,

$$EV_{3n}^q(k) \;\le\; C\frac{(k-d)^{q/2} + (k_0-d)^{q/2} + 1 + k^{2q}N^{-q/2} + k_0^{2q}N^{-q/2}}{\left[(c_n-1)(k+d^2)\sigma^2 + N\left\|\mathbf{a} - \mathbf{a}(k-d)\right\|_z^2\right]^q} \;\le\; CN^{-q}; \quad \text{(B.48)}$$

and for $k_0 + 1 \le k \le K_n$,

$$EV_{3n}^q(k) \;\le\; C\frac{(k-d)^{q/2} + (k_0-d)^{q/2} + 1 + k^{2q}N^{-q/2} + k_0^{2q}N^{-q/2}}{\left[(c_n-1)(k+d^2)\sigma^2\right]^q} \;\le\; Ck^{-q/2}c_n^{-q}. \quad \text{(B.49)}$$

Therefore, (A.22) holds. For (A.23), since $d \leq k \leq k_0 - 1$, Ing et al. (2012, Lemma 4.1) yields

$$\begin{aligned} EV_{4n}^q(k) &\leq C[L_{n,c_n}(k)]^{-q}\left[N^{-q/2}\left\|\mathbf{a}(k-d)-\mathbf{a}(k_0-d)\right\|_z^q + N^{-q}\right] \\ &= C\frac{\left\|\mathbf{a}(k-d)-\mathbf{a}(k_0-d)\right\|_z^q N^{-q/2} + N^{-q}}{\left[(c_n-1)(k+d^2)\sigma^2 N^{-1} + \left\|\mathbf{a}-\mathbf{a}(k-d)\right\|_z^2 N^{-1}\right]^q} \leq CN^{-q}; \end{aligned} \tag{B.50}$$

and for $k_0 + 1 \leq k \leq K_n$, $EV_{4n}^q(k) = 0$ as we recall around (A.9) that $\hat{\Sigma}_n(k-d) = \hat{\Sigma}_n(k_0-d)$ and $\sigma^2(k-d) = \sigma^2(k_0-d)$. Thus (A.24) holds. For (A.25), since

$$\begin{aligned} H_n(k) - H_n(k_0) &= \frac{c_n^2 k^2}{N}\exp(m_n(k))\,\sigma^2 + \frac{c_n^2 k^2}{N}\exp(m_n(k))\left(\hat{\sigma}_n^2(k)-\sigma^2\right) \\ &\quad -\frac{c_n^2 k_0^2}{N}\exp(m_n(k_0))\,\sigma^2 - \frac{c_n^2 k_0^2}{N}\exp(m_n(k_0))\left(\hat{\sigma}_n^2(k_0)-\sigma^2\right), \end{aligned} \tag{B.51}$$

where $m_n(k)$ lies between 0 and $\frac{c_n k}{N}$, and $m_n(k_0)$ lies between 0 and $\frac{c_n k_0}{N}$. Since we confine our attention to $k \leq \lfloor (n/c_n)^{\theta_1} \rfloor$, $\frac{c_n k}{N} \leq C$ and thus $m_n(k) < \infty$ and $m_n(k_0) < \infty$ for sufficiently large n. Therefore, (A.25) holds. The proof is complete. □

Proof of Lemma 2. We first show (A.21) for $\hat{k}_n$. Refer to (A.12). By a Taylor's expansion,

$$\begin{aligned} H_n(k) &= N\hat{\sigma}_n^2(k)\left(\exp\left(\frac{c_n k}{N}\right) - 1\right) - c_n k \hat{\sigma}_n^2(k) \\ &= \frac{c_n k}{N}\exp(m_n(k))\, c_n k \hat{\sigma}_n^2(k) =: r_n(k) c_n k \hat{\sigma}_n^2(k), \end{aligned} \tag{B.52}$$

$m_n(k)$ lies between 0 and $\frac{c_n k}{N}$. For sufficiently large n, $\max_{1\leq k\leq K_n} r_n(k) < \infty$. Our proof follows that of Theorem 4.5 in Ing et al. (2012). Write $N\exp[IC_{c_n}(k)] = (N + (1+r_n(k))c_n k)\,\hat{\sigma}_n^2(k)$. Then,

$$\begin{aligned} P\left[\hat{k}_n = k\right] &\leq P\left[(N + (1+r_n(k))c_n k)\hat{\sigma}_n^2(k) \leq (N + (1+r_n(d))c_n d)\hat{\sigma}_n^2(d)\right] \\ \leq\;& P\left[N(\hat{\sigma}_n^2(d-1) - \hat{\sigma}_n^2(d)) \leq (1+r_n(d))c_n d(\sigma^2(0)+M_0)\right] + P\left[\left|\hat{\sigma}_n^2(d) - \sigma^2(0)\right| \geq M_0\right] \\ =:\;& (I) + (II), \end{aligned} \tag{B.53}$$

where M_0 is any positive number and we recall around (A.9) that $\sigma^2(0) = \sigma^2 + \|\mathbf{a} - \mathbf{a}(0)\|_z^2$. Careful inspection shows, with the penalty 2 replaced by $(1+r_n(d))c_n$, the arguments in Ing et al. (2012) go through except their (4.32) and (4.34), which can be modified as: for any $s > 0$,

$$(III) := P\left[\lambda^{-1}(\min) \geq (1-\varepsilon)((1+r_n(d))c_n d)^{-1}(\sigma^2(0)+M_0)^{-1}\right] = O\left(\frac{(1+r_n(d))^s c_n^s}{N^s}\right) = o(1).$$

Thus, by the arguments in Ing et al. (2012, Proof of Theorem 4.5), our conclusion follows. The proof for $\hat{k}_n^S$ is exactly the same, with $(1+r_n(d))$ replaced by 1.

For (A.22), as the proof is exactly the same, we only show that for $\hat{k}_n$. Note by Minkowski's inequality, Hölder's inequality, Wei (1987, Lemma 2) and Ing et al. (2010, Lemma 1 and (B.1)), for any $1 \le k < d$ and all sufficiently large n,

$$E|\mathbf{f}_n(k)|^4 \;\; \le \;\; CN^{2(2d-2k-1)} \;\; \text{and} \;\; E|\mathcal{S}_n(k-d)|^4 \; \le \; CN^{2(2d-2k-1)}.$$

By this, the Cauchy-Schwarz inequality and (A.21) with $q > 8(d-1)$, we obtain

$$\begin{aligned} \frac{E\left[\left[\mathbf{f}_n(\hat{k}_n) + \mathcal{S}_n(\hat{k}_n - d)\right]^2 I_{\{\hat{k}_n < d\}}\right]}{L_n(k_0)} &\le CN \sum_{k=1}^{d-1} \left[E|\mathbf{f}_n(k)|^4 + E|\mathcal{S}_n(k-d)|^4\right]^{1/2} P^{1/2}\left[\hat{k}_n = k\right] \\ &\le CN^{2(d-1)} N^{-q/4} \; = \; o(1). \end{aligned}$$

Thus (A.22) holds. The proof is complete. □

Proof of Lemma 3. When $\log \frac{\hat{\sigma}_n^2(k)}{\hat{\sigma}_n^2(k_0)} \le \frac{c_n(k_0-k)}{N}$, $\hat{\sigma}_n^2(k_0) - \hat{\sigma}_n^2(k) \ge \hat{\sigma}_n^2(k)\left[\exp\left[kc_n(1-(k_0/k))/N\right] - 1\right]$. Denote $\zeta_n(k) = \exp\left[kc_n(1-(k_0/k))/N\right]$. For any $0 < \delta < 1$,

$$\begin{aligned} P\left(\hat{k}_n = k\right) &\le P\left[\log \frac{\hat{\sigma}_n^2(k)}{\hat{\sigma}_n^2(k_0)} \;\le\; \frac{c_n(k_0-k)}{N}\right] \\ &\le P\left[\hat{\sigma}_n^2(k_0) - \hat{\sigma}_n^2(k) \;\ge\; \sigma^2(1-\delta)(\zeta_n(k)-1)\right] + P\left[\sigma^2 - \hat{\sigma}_n^2(k) \;\ge\; \sigma^2\delta\right] \;=:\; (I) + (II), \end{aligned}$$

where we recall from (A.9) that for $k \ge k_0$, $\sigma^2(k-d) = \sigma^2 + \|\mathbf{a} - \mathbf{a}(k-d)\|_z^2 = \sigma^2$, and $\hat{\Sigma}_n(k-d) = \hat{\Sigma}_n(k_0) = N^{-1}\sum_{j=K_n}^{n-1} \varepsilon_{j+1}^2$. By (B.39),

$$\left|\hat{\sigma}_n^2(k) - \hat{\sigma}_n^2(k_0)\right| \;\le\; \left\|N^{-1}\sum_{j=K_n}^{n-1} \mathbf{s}_{j,n}(k_0)\varepsilon_{j+1}\right\|^2_{\hat{\Omega}_n^{-1}(k_0)} + \left\|N^{-1}\sum_{j=K_n}^{n-1} \mathbf{s}_{j,n}(k)\varepsilon_{j+1}\right\|^2_{\hat{\Omega}_n^{-1}(k)}.$$

Therefore, by Chebyshev's inequality, (B.41), $\lfloor (N/c_n)^{\theta_1} \rfloor + 1 \le k \le K_n = o(N^{1/2})$ and δ being arbitrary,

$$(I) \;\le\; C\left(\frac{kN^{-1}}{(1-\delta)\,(N/c_n)^{\theta_1}\,N^{-1}}\right)^q \;\le\; Cc_n^{-q(1-\theta_1)} N^{-q(\theta_1 - 1/2)}. \tag{B.54}$$

Similarly, we can show $(II) \le Cc_n^{-q(1-\theta_1)} N^{-q(\theta_1-1/2)}$. The proof is thus complete. □

2 TABLES FOR AR(d+1)-Garch(1,1), AR(d+2) and AR(d+4), d = 0, 1, 2

Tables II(a)(b)(c) depict the results for DGP(II): AR(d+1)-Garch(1,1). See Equation (24).

Tables III(a)(b)(c) depict the results for DGP(III): Another AR(d+1)-Garch(1,1). See Equation (25).

Tables IV(a)(b)(c) depict the results for DGP(IV): AR(d+2). See Equation (26).

Tables V(a)(b)(c) depict the results for DGP(V): AR(d+4). See Equation (27).

TABLE II(a): The simulated excess risk for DGP (II), $\sigma^2 = 1$, $d = 0$, $k_0 = 1$, $(n-K)E\left[(y_{n+1} - \hat{y}_{n+1})^2 - \sigma^2\right] - \sigma^2$.

n	$K-1$	$AR(1)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	0.121	0.818	0.610	0.573	0.359	17.800	16.924	643.7	625.0	1.304	2.630
	2	0.121	1.362	0.863	0.795	0.408	18.342	17.003	3.119	3.282	171.6	202.5
	3	0.120	1.750	1.052	0.934	0.449	18.866	17.184	3.595	3.889	727.4	911.2
	4	0.114	2.059	1.137	0.972	0.438	19.253	17.327	3.914	4.331	3.692	3.974
	5	0.103	2.314	1.217	1.004	0.449	19.624	17.505	4.247	4.755	4.170	4.186
	6	0.101	2.465	1.217	1.040	0.437	19.947	17.620	4.422	4.976	4.578	4.470
	7	0.104	2.747	1.281	1.067	0.434	20.342	17.826	4.655	5.277	4.948	4.660
	8	0.104	2.897	1.272	1.094	0.440	20.668	17.962	4.807	5.485	5.227	4.832
	9	0.095	2.954	1.276	1.082	0.424	20.915	18.034	4.924	5.645	5.522	4.987
	10	0.097	3.084	1.295	1.041	0.436	21.168	18.190	5.055	5.841	5.836	5.191
	20	0.115	3.567	1.288	1.068	0.451	23.525	19.551	5.703	6.754	7.836	6.462
	30	0.114	3.648	1.331	1.076	0.426	25.174	20.660	5.785	6.943	9.042	7.076
	40	0.121	3.697	1.330	1.075	0.458	26.874	21.761	5.696	6.882	9.877	7.447
	50	0.121	3.840	1.325	1.035	0.403	28.586	23.180	5.367	6.471	10.661	7.859
	60	0.089	3.921	1.258	0.981	0.393	30.367	24.672	5.001	6.040	11.148	8.035
300	1	0.091	0.840	0.574	0.522	0.266	9.217	8.232	1042.9	1007.8	1.396	1.611
	2	0.091	1.175	0.754	0.663	0.325	9.870	8.248	1.776	1.036	289.1	325.4
	3	0.086	1.601	0.906	0.767	0.328	10.373	8.297	2.094	1.431	1282.3	1535.1
	4	0.084	1.991	1.061	0.843	0.327	10.838	8.473	2.574	1.815	3.733	2.796
	5	0.076	2.293	1.143	0.913	0.324	11.088	8.554	2.840	2.039	4.168	3.082
	6	0.074	2.535	1.156	0.907	0.332	11.394	8.628	3.149	2.381	4.588	3.382
	7	0.067	2.752	1.191	0.905	0.321	11.580	8.693	3.361	2.489	4.994	3.635
	8	0.062	2.942	1.215	0.915	0.312	11.934	8.776	3.626	2.753	5.284	3.656
	9	0.061	3.021	1.226	0.898	0.308	12.120	8.839	3.817	2.847	5.517	3.821
	10	0.060	3.051	1.224	0.883	0.296	12.334	8.886	4.024	3.119	5.832	4.018
	20	0.063	3.521	1.292	0.901	0.281	14.138	9.577	5.524	4.763	7.648	5.364
	30	0.058	3.693	1.307	0.900	0.276	15.506	10.350	6.334	5.842	8.899	6.196
	40	0.043	3.694	1.261	0.886	0.268	16.770	11.081	6.927	6.657	9.931	6.971
	50	0.033	3.788	1.299	0.913	0.255	18.063	11.762	7.349	7.344	10.611	7.478
	60	0.050	3.886	1.361	0.950	0.289	19.130	12.474	7.616	7.757	11.265	8.075
500	1	0.015	0.691	0.441	0.396	0.194	4.080	2.982	1733.0	1685.3	1.342	1.382
	2	0.015	1.259	0.740	0.587	0.224	4.665	2.988	1.679	0.720	486.4	534.9
	3	0.018	1.695	0.950	0.695	0.212	5.103	3.045	2.145	1.040	2237.2	2587.9
	4	0.013	2.052	1.020	0.731	0.212	5.529	3.044	2.470	1.146	3.856	2.262
	5	0.014	2.277	1.057	0.781	0.223	5.856	3.077	2.853	1.286	4.423	2.401
	6	0.010	2.559	1.118	0.819	0.215	6.037	3.102	3.151	1.442	4.794	2.537
	7	0.014	2.642	1.101	0.819	0.222	6.334	3.116	3.339	1.517	5.236	2.765
	8	0.009	2.817	1.156	0.842	0.227	6.391	3.147	3.528	1.592	5.660	2.865
	9	0.002	3.053	1.204	0.835	0.230	6.689	3.141	3.719	1.664	5.848	2.941
	10	0.001	3.181	1.238	0.825	0.224	6.788	3.173	3.940	1.811	6.303	3.166
	20	0.021	3.828	1.260	0.802	0.244	8.068	3.371	4.876	2.200	8.317	4.178
	30	0.004	3.950	1.240	0.833	0.168	9.147	3.555	5.523	2.482	9.495	5.186
	40	-0.003	3.956	1.318	0.825	0.165	9.850	3.668	5.964	2.630	10.446	5.849
	50	0.001	3.911	1.321	0.808	0.166	10.546	3.804	6.334	2.746	11.129	6.465
	60	-0.016	4.018	1.311	0.863	0.154	11.080	4.000	6.626	2.802	11.678	6.738
1000	1	0.050	0.661	0.580	0.547	0.193	1.305	0.404	3434.9	3365.8	1.084	1.039
	2	0.050	0.974	0.634	0.523	0.186	1.910	0.402	1.421	0.305	994.7	1068.7
	3	0.049	1.363	0.841	0.587	0.190	2.458	0.401	2.025	0.516	4689.0	5212.6
	4	0.053	1.708	1.007	0.636	0.202	2.874	0.399	2.368	0.675	3.622	1.540
	5	0.053	2.033	1.036	0.639	0.190	3.128	0.409	2.670	0.884	4.156	1.649
	6	0.050	2.317	1.068	0.652	0.188	3.394	0.406	2.928	0.870	4.585	1.630
	7	0.039	2.488	1.103	0.600	0.180	3.488	0.398	2.995	0.818	4.967	1.661
	8	0.035	2.699	1.117	0.611	0.181	3.567	0.393	3.284	0.852	5.339	1.697
	9	0.035	2.723	1.175	0.598	0.180	3.741	0.401	3.504	0.886	5.813	1.736
	10	0.035	2.726	1.127	0.620	0.187	3.989	0.404	3.638	0.991	6.155	1.817
	20	0.036	3.112	1.061	0.559	0.153	5.139	0.421	4.761	1.425	8.371	2.544
	30	0.036	3.290	1.051	0.534	0.147	6.099	0.436	5.520	1.538	9.514	3.012
	40	0.026	3.220	1.044	0.548	0.147	6.611	0.443	5.948	1.669	10.496	3.522
	50	0.052	3.249	1.141	0.613	0.171	7.184	0.491	6.381	1.695	11.222	3.686
	60	0.048	3.171	1.247	0.652	0.170	7.792	0.512	6.602	1.735	11.815	3.933

TABLE II(b): The simulated excess risk for DGP (II), $\sigma^2 = 1$, $d = 1$, $k_0 = 2$, $(n-K)E\left[(y_{n+1} - \hat{y}_{n+1})^2 - \sigma^2\right] - 3\sigma^2$.

n	$K-2$	$AR(2)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	0.542	1.252	1.040	1.009	0.785	1.3e06	1.3e06	16.602	14.925	2.7e06	2.6e06
	2	0.527	1.798	1.323	1.223	0.829	1.3e06	1.3e06	16.949	15.111	2.4e06	2.2e06
	3	0.512	2.168	1.492	1.362	0.846	1.3e06	1.3e06	17.306	15.331	2.712	2.835
	4	0.490	2.496	1.575	1.432	0.830	1.3e06	1.3e06	17.517	15.427	2.907	3.147
	5	0.477	2.775	1.651	1.469	0.821	1.2e06	1.2e06	17.776	15.562	3.106	3.471
	6	0.467	2.962	1.695	1.477	0.823	1.2e06	1.2e06	18.079	15.769	3.205	3.627
	7	0.461	3.141	1.683	1.486	0.787	1.2e06	1.2e06	18.337	15.882	3.392	3.923
	8	0.445	3.294	1.679	1.484	0.780	1.2e06	1.2e06	18.557	16.017	3.446	4.013
	9	0.435	3.381	1.660	1.464	0.769	1.2e06	1.2e06	18.748	16.136	3.538	4.121
	10	0.426	3.532	1.659	1.425	0.759	1.2e06	1.2e06	18.976	16.305	3.637	4.290
	20	0.376	3.969	1.600	1.411	0.705	1.2e06	1.2e06	21.028	17.546	3.985	4.868
	30	0.314	4.011	1.556	1.330	0.639	1.1e06	1.1e06	22.799	18.709	3.926	4.893
	40	0.279	4.105	1.518	1.298	0.621	1.0e06	1.0e06	24.720	19.776	3.745	4.682
	50	0.240	4.316	1.477	1.187	0.576	9.7e05	9.7e05	26.745	21.217	3.441	4.359
	60	0.161	4.543	1.385	1.132	0.480	9.0e05	9.0e05	28.862	22.665	3.019	3.840
300	1	0.296	1.082	0.830	0.711	0.479	3.6e06	3.6e06	8.485	6.219	7.4e06	7.1e06
	2	0.286	1.444	0.940	0.845	0.517	3.6e06	3.6e06	8.874	6.290	6.4e06	5.7e06
	3	0.275	1.851	1.114	0.975	0.541	3.6e06	3.6e06	9.306	6.449	1.355	-0.031
	4	0.259	2.276	1.254	1.020	0.530	3.6e06	3.6e06	9.599	6.543	1.657	0.353
	5	0.248	2.518	1.299	1.053	0.500	3.6e06	3.6e06	9.763	6.608	1.951	0.638
	6	0.234	2.779	1.297	1.036	0.488	3.6e06	3.6e06	9.987	6.671	2.072	0.765
	7	0.221	2.998	1.320	1.045	0.465	3.6e06	3.6e06	10.226	6.772	2.320	0.996
	8	0.216	3.105	1.310	1.031	0.468	3.6e06	3.6e06	10.330	6.845	2.555	1.195
	9	0.209	3.265	1.304	1.021	0.456	3.6e06	3.6e06	10.511	6.876	2.732	1.458
	10	0.208	3.276	1.303	0.993	0.458	3.6e06	3.6e06	10.635	6.944	2.902	1.716
	20	0.148	3.694	1.333	0.972	0.385	3.5e06	3.5e06	12.321	7.683	4.082	3.169
	30	0.116	3.872	1.403	0.981	0.368	3.3e06	3.3e06	13.584	8.408	4.793	4.129
	40	0.046	3.876	1.277	0.895	0.264	3.2e06	3.2e06	14.760	9.135	5.282	4.842
	50	0.021	3.884	1.294	0.908	0.267	3.1e06	3.1e06	16.051	9.805	5.726	5.608
	60	0.006	3.867	1.327	0.913	0.247	3.0e06	3.0e06	17.206	10.560	5.949	6.060
500	1	0.248	0.995	0.757	0.612	0.401	1.0e07	1.0e07	3.610	0.981	2.1e07	2.0e07
	2	0.248	1.519	1.068	0.814	0.422	1.0e07	1.0e07	4.081	1.015	1.8e07	1.6e07
	3	0.240	1.911	1.239	0.926	0.418	1.0e07	1.0e07	4.469	1.049	1.151	-1.106
	4	0.236	2.263	1.292	0.994	0.418	1.0e07	1.0e07	4.705	1.054	1.471	-0.936
	5	0.228	2.570	1.345	1.024	0.412	1.0e07	1.0e07	4.786	1.092	1.678	-0.832
	6	0.230	2.819	1.359	1.053	0.428	1.0e07	1.0e07	5.004	1.122	1.941	-0.679
	7	0.217	2.970	1.388	1.079	0.392	1.0e07	1.0e07	5.170	1.127	2.048	-0.615
	8	0.207	3.062	1.480	1.041	0.397	1.0e07	1.0e07	5.287	1.138	2.171	-0.600
	9	0.199	3.240	1.467	1.034	0.393	1.0e07	1.0e07	5.519	1.163	2.293	-0.462
	10	0.191	3.353	1.481	1.073	0.389	1.0e07	1.0e07	5.607	1.164	2.449	-0.392
	20	0.163	4.039	1.444	1.002	0.366	1.0e07	1.0e07	6.734	1.366	3.259	0.054
	30	0.118	4.036	1.371	0.988	0.284	9.8e06	9.8e06	7.639	1.547	3.829	0.419
	40	0.073	4.185	1.443	0.942	0.233	9.7e06	9.7e06	8.158	1.684	4.323	0.659
	50	0.024	4.013	1.437	0.870	0.202	9.5e06	9.5e06	8.708	1.790	4.681	0.928
	60	-0.016	3.959	1.397	0.845	0.162	9.3e06	9.3e06	9.306	2.008	4.940	0.907
1000	1	-0.072	0.532	0.423	0.416	0.065	4.2e07	4.2e07	0.857	-1.597	8.3e07	8.0e07
	2	-0.075	0.850	0.506	0.349	0.058	4.2e07	4.2e07	1.372	-1.597	6.9e07	6.4e07
	3	-0.073	1.225	0.686	0.426	0.052	4.2e07	4.2e07	1.801	-1.591	1.064	-1.477
	4	-0.074	1.567	0.823	0.475	0.043	4.2e07	4.2e07	2.188	-1.587	1.210	-1.317
	5	-0.081	1.872	0.895	0.478	0.042	4.2e07	4.2e07	2.365	-1.589	1.519	-1.147
	6	-0.092	2.170	0.985	0.461	0.017	4.2e07	4.2e07	2.509	-1.599	1.710	-1.186
	7	-0.097	2.287	0.965	0.439	-0.003	4.2e07	4.2e07	2.529	-1.601	1.817	-1.252
	8	-0.100	2.463	1.012	0.420	0.002	4.2e07	4.2e07	2.559	-1.598	2.070	-1.207
	9	-0.100	2.581	1.028	0.446	0.011	4.2e07	4.2e07	2.643	-1.597	2.180	-1.186
	10	-0.111	2.631	1.031	0.411	-0.014	4.2e07	4.2e07	2.810	-1.599	2.225	-1.074
	20	-0.110	2.890	0.901	0.360	0.018	4.1e07	4.1e07	4.010	-1.566	3.153	-0.702
	30	-0.130	3.129	0.931	0.361	-0.048	4.1e07	4.1e07	4.692	-1.567	3.799	-0.516
	40	-0.163	3.048	0.876	0.310	-0.085	4.0e07	4.0e07	4.966	-1.545	4.202	-0.382
	50	-0.158	3.012	0.922	0.399	-0.100	4.0e07	4.0e07	5.539	-1.508	4.614	-0.337
	60	-0.175	2.941	1.002	0.469	-0.126	4.0e07	4.0e07	5.954	-1.491	4.805	-0.289

TABLE II(c): The simulated excess risk for DGP (II), $\sigma^2 = 1$, $d = 2$, $k_0 = 3$, $(n-K)E\left[(y_{n+1} - \hat{y}_{n+1})^2 - \sigma^2\right] - 7\sigma^2$.

n	$K-3$	$AR(3)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	1.190	1.963	1.768	1.697	1.420	1.4e10	1.4e10	1.3e06	1.3e06	2.9e10	2.9e10
	2	1.155	2.565	2.021	1.922	1.468	1.4e10	1.4e10	1.3e06	1.3e06	13.866	11.274
	3	1.108	2.962	2.149	2.023	1.455	1.4e10	1.4e10	1.3e06	1.3e06	14.042	11.441
	4	1.068	3.268	2.235	2.085	1.431	1.4e10	1.4e10	1.2e06	1.2e06	14.162	11.557
	5	1.042	3.561	2.347	2.116	1.410	1.3e10	1.3e10	1.2e06	1.2e06	14.555	11.764
	6	1.013	3.731	2.322	2.119	1.371	1.3e10	1.3e10	1.2e06	1.2e06	14.731	11.885
	7	0.978	3.882	2.323	2.065	1.319	1.3e10	1.3e10	1.2e06	1.2e06	14.982	12.020
	8	0.945	4.029	2.268	2.012	1.293	1.3e10	1.3e10	1.2e06	1.2e06	15.154	12.169
	9	0.916	4.059	2.270	1.971	1.265	1.3e10	1.3e10	1.2e06	1.2e06	15.350	12.326
	10	0.893	4.204	2.255	1.975	1.248	1.3e10	1.3e10	1.2e06	1.2e06	15.489	12.479
	20	0.679	4.659	1.962	1.752	1.025	1.2e10	1.2e10	1.1e06	1.1e06	17.611	13.681
	30	0.491	4.605	1.864	1.573	0.846	1.2e10	1.2e10	1.1e06	1.1e06	19.639	14.831
	40	0.359	4.549	1.727	1.461	0.726	1.1e10	1.1e10	1.0e06	1.0e06	21.991	15.937
	50	0.274	4.886	1.695	1.366	0.648	1.0e10	1.0e10	9.6e05	9.6e05	24.681	17.498
	60	0.126	5.409	1.523	1.196	0.494	9.4e09	9.4e09	8.9e05	9.0e05	27.465	19.240
300	1	0.809	1.628	1.323	1.241	0.962	1.1e11	1.1e11	3.6e06	3.6e06	2.4e11	2.4e11
	2	0.789	1.988	1.484	1.401	1.020	1.1e11	1.1e11	3.6e06	3.6e06	6.100	2.406
	3	0.761	2.403	1.625	1.449	1.000	1.1e11	1.1e11	3.6e06	3.6e06	6.372	2.525
	4	0.738	2.744	1.774	1.537	0.992	1.1e11	1.1e11	3.6e06	3.6e06	6.552	2.617
	5	0.711	3.034	1.829	1.542	0.964	1.1e11	1.1e11	3.6e06	3.6e06	6.728	2.655
	6	0.687	3.226	1.867	1.530	0.932	1.1e11	1.1e11	3.6e06	3.6e06	6.841	2.758
	7	0.671	3.404	1.837	1.525	0.916	1.1e11	1.1e11	3.6e06	3.6e06	6.978	2.842
	8	0.654	3.556	1.826	1.481	0.888	1.1e11	1.1e11	3.6e06	3.6e06	7.023	2.878
	9	0.640	3.758	1.792	1.447	0.868	1.1e11	1.1e11	3.6e06	3.6e06	7.226	2.943
	10	0.615	3.766	1.780	1.423	0.849	1.1e11	1.1e11	3.5e06	3.5e06	7.314	3.004
	20	0.452	4.134	1.631	1.240	0.716	1.1e11	1.1e11	3.4e06	3.4e06	8.899	3.736
	30	0.305	4.107	1.599	1.108	0.545	1.0e11	1.0e11	3.3e06	3.3e06	10.156	4.441
	40	0.168	4.078	1.434	1.015	0.391	1.0e11	1.0e11	3.2e06	3.2e06	11.294	5.170
	50	0.059	4.014	1.390	0.927	0.283	9.6e10	9.6e10	3.1e06	3.1e06	12.631	5.902
	60	-0.023	3.908	1.318	0.873	0.211	9.2e10	9.2e10	3.0e06	3.0e06	13.879	6.565
500	1	0.385	1.104	0.873	0.774	0.554	9.0e11	9.0e11	1.0e07	1.0e07	2.0e12	2.0e12
	2	0.366	1.637	1.182	0.985	0.552	9.0e11	9.0e11	1.0e07	1.0e07	1.349	-2.988
	3	0.357	2.062	1.276	1.119	0.550	9.0e11	9.0e11	1.0e07	1.0e07	1.572	-2.926
	4	0.343	2.395	1.387	1.151	0.530	9.0e11	9.0e11	1.0e07	1.0e07	1.661	-2.915
	5	0.339	2.687	1.399	1.152	0.555	9.0e11	9.0e11	1.0e07	1.0e07	1.750	-2.898
	6	0.320	2.975	1.451	1.140	0.531	9.0e11	9.0e11	1.0e07	1.0e07	1.939	-2.864
	7	0.299	3.165	1.439	1.083	0.518	9.0e11	9.0e11	1.0e07	1.0e07	2.047	-2.869
	8	0.288	3.307	1.476	1.100	0.503	9.0e11	9.0e11	1.0e07	1.0e07	2.148	-2.835
	9	0.274	3.412	1.484	1.133	0.464	8.9e11	8.9e11	1.0e07	1.0e07	2.352	-2.832
	10	0.269	3.542	1.465	1.134	0.467	8.9e11	8.9e11	1.0e07	1.0e07	2.366	-2.815
	20	0.166	4.079	1.348	0.987	0.370	8.8e11	8.8e11	1.0e07	1.0e07	3.250	-2.596
	30	0.038	4.021	1.285	0.887	0.208	8.6e11	8.6e11	9.8e06	9.8e06	4.040	-2.443
	40	-0.059	3.999	1.274	0.821	0.098	8.4e11	8.4e11	9.6e06	9.6e06	4.567	-2.270
	50	-0.178	3.731	1.191	0.717	-0.010	8.3e11	8.3e11	9.4e06	9.5e06	5.092	-2.188
	60	-0.270	3.832	1.128	0.614	-0.083	8.1e11	8.1e11	9.3e06	9.3e06	5.640	-2.008
1000	1	0.094	0.691	0.609	0.542	0.243	1.5e13	1.5e13	4.2e07	4.2e07	3.3e13	3.3e13
	2	0.091	1.010	0.724	0.511	0.238	1.5e13	1.5e13	4.2e07	4.2e07	-1.192	-5.591
	3	0.085	1.406	0.857	0.555	0.217	1.5e13	1.5e13	4.2e07	4.2e07	-0.807	-5.586
	4	0.075	1.680	0.954	0.586	0.212	1.5e13	1.5e13	4.2e07	4.2e07	-0.654	-5.590
	5	0.059	2.062	1.078	0.542	0.193	1.5e13	1.5e13	4.2e07	4.2e07	-0.594	-5.600
	6	0.049	2.264	1.118	0.546	0.197	1.5e13	1.5e13	4.2e07	4.2e07	-0.491	-5.602
	7	0.042	2.455	1.121	0.465	0.180	1.5e13	1.5e13	4.2e07	4.2e07	-0.501	-5.598
	8	0.035	2.670	1.090	0.523	0.174	1.5e13	1.5e13	4.2e07	4.2e07	-0.447	-5.596
	9	0.019	2.764	1.151	0.495	0.120	1.5e13	1.5e13	4.1e07	4.2e07	-0.438	-5.599
	10	0.007	2.811	1.091	0.458	0.114	1.5e13	1.5e13	4.1e07	4.1e07	-0.367	-5.608
	20	-0.016	3.101	1.047	0.425	0.065	1.5e13	1.5e13	4.1e07	4.1e07	0.682	-5.561
	30	-0.087	3.125	0.936	0.416	-0.008	1.4e13	1.4e13	4.1e07	4.1e07	1.245	-5.562
	40	-0.143	3.065	0.914	0.322	-0.072	1.4e13	1.4e13	4.0e07	4.0e07	1.590	-5.548
	50	-0.162	3.055	0.983	0.374	-0.097	1.4e13	1.4e13	4.0e07	4.0e07	2.057	-5.509
	60	-0.190	3.093	0.860	0.348	-0.106	1.4e13	1.4e13	4.0e07	4.0e07	2.596	-5.483

TABLE III(a): The simulated excess risk for DGP (III), $\sigma^2 = 1$, $d = 0$, $k_0 = 1$, $(n-K)E\left[(y_{n+1}-\hat{y}_{n+1})^2-\sigma^2\right]-\sigma^2$.

n	$K-1$	$AR(1)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	0.816	4.035	3.906	3.829	3.355	18.035	16.931	648.8	636.1	4.640	6.111
	2	0.816	5.801	5.391	5.324	4.443	19.243	17.857	6.756	7.459	202.7	234.3
	3	0.813	6.746	6.115	5.892	4.667	19.969	18.449	7.241	8.333	711.8	889.0
	4	0.811	7.514	6.659	6.405	4.943	20.447	18.662	7.207	8.823	8.440	8.924
	5	0.797	7.977	6.893	6.573	4.974	20.974	19.151	7.319	9.026	8.886	8.989
	6	0.793	8.139	6.978	6.649	4.963	21.127	19.270	7.221	9.366	9.044	8.949
	7	0.789	8.531	7.108	6.679	4.882	21.452	19.442	7.198	9.549	9.248	9.234
	8	0.809	8.630	7.101	6.740	4.877	21.769	19.496	7.180	9.772	9.456	9.384
	9	0.791	8.723	7.085	6.635	4.887	22.006	19.553	7.092	9.658	9.484	9.515
	10	0.800	8.822	7.056	6.764	4.880	22.322	19.655	7.112	9.827	9.752	9.689
	20	0.854	9.330	7.058	6.739	4.835	23.607	20.397	7.107	10.566	11.369	10.675
	30	0.912	9.729	7.234	6.873	5.144	25.208	21.211	6.805	10.276	12.173	11.412
	40	0.933	9.892	7.452	6.969	5.020	26.410	22.036	6.402	9.980	13.014	11.950
	50	0.964	10.143	7.510	7.056	5.001	27.969	23.179	6.105	9.737	13.756	12.521
	60	0.960	10.410	7.586	7.206	5.021	29.495	24.474	5.709	9.319	14.368	13.118
300	1	1.028	3.485	3.207	3.178	2.830	10.884	9.263	1045.9	1018.5	3.788	4.238
	2	1.028	6.249	5.861	5.520	4.818	13.325	11.066	6.489	6.379	321.5	359.6
	3	1.016	7.348	6.683	6.347	5.245	13.965	11.495	6.908	7.158	1278.9	1522.1
	4	1.015	8.381	7.345	6.914	5.573	14.302	11.910	7.145	7.754	8.949	8.171
	5	1.010	8.964	7.848	7.345	5.917	14.712	12.450	7.320	8.198	9.621	8.432
	6	1.027	10.048	8.585	8.261	6.808	15.412	13.188	7.584	8.925	10.939	9.862
	7	1.003	10.371	8.847	8.774	7.465	15.790	13.118	8.120	9.661	11.177	10.020
	8	1.002	10.890	9.581	9.040	7.953	16.796	13.676	8.504	10.175	11.567	10.427
	9	0.997	17.743	16.344	15.601	14.688	21.583	20.341	15.280	16.886	18.135	17.690
	10	0.997	17.723	16.007	15.568	14.632	21.462	20.044	16.775	18.481	19.274	18.675
	20	1.032	21.288	16.036	15.417	14.283	20.590	20.295	12.386	17.719	22.733	20.267
	30	1.011	21.583	16.275	15.527	14.109	20.410	20.235	11.856	18.269	12.985	11.363
	40	0.996	21.622	16.104	15.324	13.849	19.179	15.226	12.194	18.790	13.945	12.429
	50	1.010	21.483	15.722	15.241	13.740	20.189	15.178	9.855	16.541	14.473	13.071
	60	1.019	21.651	15.607	14.959	13.465	21.276	15.787	8.948	12.259	14.954	14.259
500	1	0.906	3.638	3.421	3.422	2.785	6.058	4.276	1735.6	1696.1	4.130	4.294
	2	0.906	5.192	4.827	4.824	4.009	7.278	5.261	4.949	4.492	524.3	574.5
	3	0.895	7.095	6.418	6.266	5.234	8.699	6.288	6.020	5.648	2240.9	2577.8
	4	0.890	7.259	6.141	5.874	4.228	8.788	5.723	5.635	5.271	8.403	7.030
	5	0.890	8.055	6.596	6.213	4.583	8.912	6.060	5.974	5.685	9.137	7.767
	6	0.888	8.517	6.868	6.426	4.690	9.398	6.073	5.922	6.055	9.579	7.873
	7	0.904	8.746	6.953	6.489	4.796	9.567	5.956	6.057	5.976	9.734	7.736
	8	0.900	8.991	6.923	6.469	4.786	9.601	6.015	5.953	6.024	10.041	7.946
	9	0.885	9.054	7.004	6.538	4.798	9.822	5.951	5.998	5.994	10.433	8.081
	10	0.891	9.076	6.963	6.532	4.881	9.737	6.041	6.041	6.179	10.638	8.120
	20	0.917	9.620	7.390	6.669	4.800	10.540	5.732	6.674	6.375	11.668	9.043
	30	0.908	9.751	7.196	6.520	4.788	11.294	6.013	7.126	6.947	12.548	9.836
	40	0.939	9.928	7.066	6.619	4.708	11.972	6.288	7.630	7.344	13.367	10.477
	50	0.953	9.955	7.199	6.367	4.715	12.415	6.332	7.906	7.405	14.071	10.728
	60	0.965	9.957	7.321	6.442	4.683	13.022	6.449	8.245	7.835	14.815	11.483
1000	1	1.343	4.602	4.077	4.059	3.622	4.361	2.262	3447.9	3390.7	4.614	4.842
	2	1.343	8.831	8.261	8.061	7.144	8.118	5.626	8.116	7.054	1056.5	1133.7
	3	1.347	10.744	9.987	9.291	8.136	9.278	6.342	8.608	8.334	4702.7	5235.5
	4	1.353	16.002	14.755	14.446	12.868	13.858	10.608	12.700	12.795	15.705	12.753
	5	1.359	15.111	15.373	15.037	13.298	12.592	10.297	12.379	12.444	16.740	13.314
	6	1.352	15.292	15.457	15.116	13.227	12.004	9.729	12.136	12.716	16.144	12.953
	7	1.339	15.606	15.531	15.162	13.332	11.937	9.804	12.204	12.914	16.082	13.334
	8	1.333	15.645	15.479	15.066	13.153	11.622	9.633	12.328	13.479	15.263	13.057
	9	1.343	15.844	15.663	15.119	13.230	11.770	9.636	12.109	12.738	15.423	13.131
	10	1.343	15.805	15.677	15.137	13.207	11.489	8.889	11.738	12.665	15.520	13.113
	20	1.344	16.086	15.549	15.120	13.006	11.782	8.877	11.706	12.702	16.948	12.633
	30	1.331	16.048	15.492	14.851	13.111	12.933	8.892	12.032	12.352	18.025	13.711
	40	1.334	15.986	15.874	15.359	13.514	12.812	8.560	11.501	13.949	18.918	13.925
	50	1.371	16.083	15.938	15.215	13.449	13.275	8.933	12.879	12.521	19.611	15.175
	60	1.356	16.277	16.136	15.324	13.549	13.714	8.640	11.398	13.174	20.095	18.332

TABLE III(b): The simulated excess risk for DGP (III), $\sigma^2 = 1$, $d = 1$, $k_0 = 2$, $(n-K)E\left[(y_{n+1}-\hat{y}_{n+1})^2-\sigma^2\right]-3\sigma^2$.

n	$K-2$	$AR(2)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	1.391	4.525	4.209	4.121	3.700	1.3e06	1.3e06	16.772	14.888	2.7e06	2.6e06
	2	1.374	6.293	5.769	5.598	4.732	1.3e06	1.3e06	17.985	16.090	2.4e06	2.3e06
	3	1.366	7.238	6.423	6.163	4.894	1.3e06	1.3e06	18.407	16.439	5.582	6.972
	4	1.342	8.011	7.003	6.605	5.248	1.3e06	1.3e06	18.923	16.829	5.583	7.361
	5	1.331	8.564	7.311	6.808	5.328	1.2e06	1.2e06	19.260	17.249	5.572	7.528
	6	1.313	8.749	7.391	6.863	5.302	1.2e06	1.2e06	19.310	17.339	5.560	7.744
	7	1.325	8.989	7.484	7.093	5.255	1.2e06	1.2e06	19.519	17.418	5.437	7.910
	8	1.300	9.133	7.442	6.965	5.206	1.2e06	1.2e06	19.615	17.441	5.305	7.844
	9	1.300	9.141	7.353	6.972	5.192	1.2e06	1.2e06	19.881	17.587	5.356	7.774
	10	1.295	9.257	7.375	7.009	5.181	1.2e06	1.2e06	19.977	17.599	5.286	7.935
	20	1.290	9.855	7.418	6.983	5.087	1.2e06	1.2e06	21.283	18.471	5.203	8.485
	30	1.268	10.172	7.528	7.087	5.132	1.1e06	1.1e06	23.022	19.392	4.775	7.921
	40	1.263	10.269	7.683	7.221	5.083	1.0e06	1.0e06	24.281	20.060	4.426	7.590
	50	1.293	10.381	7.707	7.225	5.155	9.7e05	9.7e05	26.133	21.058	4.010	7.211
	60	1.245	10.968	7.623	7.209	5.164	9.0e05	9.0e05	27.971	22.570	3.621	6.835
300	1	1.111	3.534	3.268	3.214	2.925	3.6e06	3.6e06	10.185	7.251	7.4e06	7.1e06
	2	1.093	6.301	5.713	5.552	4.846	3.6e06	3.6e06	12.214	9.109	6.4e06	5.8e06
	3	1.085	7.414	6.465	6.144	5.071	3.6e06	3.6e06	12.682	9.520	5.521	5.768
	4	1.074	8.258	7.066	6.721	5.536	3.6e06	3.6e06	13.045	9.971	5.718	6.504
	5	1.082	9.051	7.749	7.120	5.926	3.6e06	3.6e06	13.467	10.428	5.961	6.548
	6	1.053	9.977	8.503	7.715	6.733	3.6e06	3.6e06	13.979	11.019	6.412	7.208
	7	1.045	10.345	8.821	8.215	7.266	3.6e06	3.6e06	14.612	11.262	6.691	7.847
	8	1.037	10.915	9.239	8.676	7.751	3.6e06	3.6e06	15.203	11.577	7.069	8.359
	9	1.034	17.794	15.902	15.335	14.344	3.6e06	3.6e06	21.310	18.100	13.787	15.287
	10	1.039	17.610	15.569	15.301	14.312	3.6e06	3.6e06	20.058	18.053	11.992	13.433
	20	1.005	21.292	15.831	15.037	14.133	3.5e06	3.5e06	18.177	18.264	11.259	15.982
	30	0.990	21.807	16.372	15.456	14.328	3.3e06	3.3e06	18.411	18.594	10.979	16.623
	40	0.916	21.329	15.833	15.010	13.571	3.2e06	3.2e06	17.203	12.972	9.813	17.159
	50	0.912	21.191	15.485	15.134	13.492	3.1e06	3.1e06	18.005	13.097	6.995	9.837
	60	0.904	21.618	15.271	14.717	13.256	3.0e06	3.0e06	19.158	13.627	7.257	10.269
500	1	1.225	3.971	3.773	3.642	3.135	1.0e07	1.0e07	5.633	2.244	2.1e07	2.0e07
	2	1.212	5.592	5.118	5.123	4.472	1.0e07	1.0e07	6.843	3.238	1.8e07	1.6e07
	3	1.203	7.476	6.575	6.327	5.375	1.0e07	1.0e07	7.826	4.154	4.585	3.793
	4	1.197	7.672	6.333	5.941	4.530	1.0e07	1.0e07	8.006	3.826	4.302	3.558
	5	1.191	8.376	6.796	6.285	4.784	1.0e07	1.0e07	8.250	4.038	4.484	3.881
	6	1.206	8.841	7.043	6.458	5.018	1.0e07	1.0e07	8.526	3.831	4.592	4.089
	7	1.194	9.109	7.071	6.488	4.993	1.0e07	1.0e07	8.574	3.791	4.593	3.804
	8	1.174	9.194	7.079	6.585	4.994	1.0e07	1.0e07	8.488	3.787	4.429	3.914
	9	1.175	9.361	7.168	6.566	5.058	1.0e07	1.0e07	8.394	3.938	4.617	4.000
	10	1.167	9.514	7.133	6.568	5.045	1.0e07	1.0e07	8.546	3.977	4.652	4.026
	20	1.155	9.789	7.261	6.615	5.016	1.0e07	1.0e07	8.949	3.673	5.091	4.368
	30	1.134	9.942	7.287	6.574	4.876	9.9e06	9.9e06	9.714	3.932	5.637	4.765
	40	1.133	10.089	7.306	6.507	5.028	9.7e06	9.7e06	10.331	4.241	5.976	5.273
	50	1.105	9.979	7.341	6.489	4.969	9.5e06	9.5e06	10.790	4.220	6.401	5.686
	60	1.090	9.901	7.353	6.600	4.761	9.3e06	9.3e06	11.276	4.500	6.620	5.974
1000	1	1.267	4.328	4.055	3.949	3.496	4.2e07	4.2e07	3.918	0.282	8.3e07	8.1e07
	2	1.270	8.725	8.189	7.873	7.006	4.2e07	4.2e07	7.313	3.583	7.0e07	6.5e07
	3	1.274	10.743	9.987	9.526	8.051	4.2e07	4.2e07	9.085	4.309	6.739	6.394
	4	1.278	15.936	14.722	14.356	12.842	4.2e07	4.2e07	13.068	8.514	10.998	10.873
	5	1.267	16.836	15.328	14.787	13.160	4.2e07	4.2e07	11.779	8.347	10.690	10.461
	6	1.251	17.045	15.319	14.879	13.026	4.2e07	4.2e07	11.368	7.947	10.489	10.579
	7	1.246	17.264	15.499	14.950	13.182	4.2e07	4.2e07	11.134	8.062	10.437	10.570
	8	1.252	17.343	15.496	14.942	13.101	4.2e07	4.2e07	10.905	7.914	10.602	11.137
	9	1.252	17.465	15.522	14.946	13.124	4.2e07	4.2e07	10.538	7.801	10.468	10.487
	10	1.244	17.409	15.514	14.945	13.091	4.2e07	4.2e07	10.286	6.934	9.936	10.336
	20	1.244	17.633	15.408	14.820	12.816	4.1e07	4.1e07	11.005	7.542	9.770	10.435
	30	1.219	17.758	15.420	14.754	12.908	4.1e07	4.1e07	11.373	7.412	10.257	10.315
	40	1.200	15.983	15.846	15.058	13.150	4.1e07	4.1e07	11.339	6.528	9.544	11.694
	50	1.206	17.897	15.791	15.156	13.246	4.0e07	4.0e07	12.046	6.950	11.126	10.512
	60	1.189	18.048	16.084	15.182	13.216	4.0e07	4.0e07	12.002	6.571	9.620	11.181

TABLE III(c): The simulated excess risk for DGP (III), $\sigma^2 = 1$, $d = 2$, $k_0 = 3$, $(n-K)E\left[(y_{n+1} - \hat{y}_{n+1})^2 - \sigma^2\right] - 7\sigma^2$.

n	$K-3$	$AR(3)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	2.167	5.373	5.182	5.107	4.644	1.4e10	1.4e10	1.3e06	1.3e06	2.9e10	3.0e10
	2	2.134	7.130	6.692	6.537	5.569	1.4e10	1.4e10	1.3e06	1.3e06	15.223	12.036
	3	2.082	7.937	7.158	6.929	5.670	1.4e10	1.4e10	1.3e06	1.3e06	15.541	12.599
	4	2.045	8.599	7.693	7.280	5.816	1.4e10	1.4e10	1.2e06	1.2e06	15.760	12.805
	5	2.011	9.070	8.087	7.622	5.857	1.3e10	1.3e10	1.2e06	1.2e06	16.151	13.258
	6	2.002	9.365	8.139	7.664	5.850	1.3e10	1.3e10	1.2e06	1.2e06	16.189	13.559
	7	1.959	9.478	8.077	7.593	5.776	1.3e10	1.3e10	1.2e06	1.2e06	16.229	13.539
	8	1.934	9.577	8.036	7.511	5.738	1.3e10	1.3e10	1.2e06	1.2e06	16.408	13.627
	9	1.903	9.690	7.933	7.483	5.712	1.3e10	1.3e10	1.2e06	1.2e06	16.421	13.620
	10	1.877	9.800	8.008	7.520	5.596	1.3e10	1.3e10	1.2e06	1.2e06	16.415	13.702
	20	1.700	10.423	7.963	7.550	5.419	1.2e10	1.2e10	1.1e06	1.1e06	18.044	14.511
	30	1.558	10.671	7.891	7.408	5.119	1.2e10	1.2e10	1.1e06	1.1e06	19.708	15.398
	40	1.414	10.643	7.723	7.311	5.117	1.1e10	1.1e10	1.0e06	1.0e06	21.470	16.245
	50	1.418	11.389	7.887	7.451	5.130	1.0e10	1.0e10	9.6e05	9.6e05	23.875	17.462
	60	1.361	11.989	7.771	7.373	4.961	9.4e09	9.4e09	8.9e05	9.0e05	26.368	19.205
300	1	1.837	4.090	3.901	3.793	3.450	1.1e11	1.1e11	3.6e06	3.6e06	2.4e11	2.4e11
	2	1.819	6.854	6.314	6.125	5.380	1.1e11	1.1e11	3.6e06	3.6e06	9.337	4.998
	3	1.795	7.944	7.303	6.710	5.700	1.1e11	1.1e11	3.6e06	3.6e06	9.752	5.464
	4	1.793	8.829	7.867	7.275	6.150	1.1e11	1.1e11	3.6e06	3.6e06	9.695	5.936
	5	1.745	9.603	8.335	7.830	6.390	1.1e11	1.1e11	3.6e06	3.6e06	10.170	6.327
	6	1.723	10.494	9.069	8.394	7.310	1.1e11	1.1e11	3.6e06	3.6e06	10.598	6.992
	7	1.707	10.856	9.313	8.750	7.800	1.1e11	1.1e11	3.6e06	3.6e06	11.230	7.142
	8	1.689	11.419	10.003	9.474	8.240	1.1e11	1.1e11	3.6e06	3.6e06	11.485	7.746
	9	1.689	17.940	16.296	15.757	14.600	1.1e11	1.1e11	3.6e06	3.6e06	17.308	14.122
	10	1.660	17.845	16.245	15.674	14.500	1.1e11	1.1e11	3.5e06	3.5e06	16.152	13.902
	20	1.511	21.338	15.880	15.221	14.100	1.1e11	1.1e11	3.4e06	3.4e06	14.596	14.217
	30	1.406	22.005	15.994	15.395	14.300	1.0e11	1.0e11	3.3e06	3.3e06	15.580	13.941
	40	1.246	21.495	15.561	15.017	13.692	1.0e11	1.0e11	3.2e06	3.2e06	13.784	9.284
	50	1.166	21.183	15.294	14.793	13.386	9.6e10	9.6e10	3.1e06	3.1e06	14.721	9.548
	60	1.114	21.480	15.210	14.591	13.119	9.3e10	9.3e10	3.0e06	3.0e06	15.843	10.046
500	1	1.533	4.051	3.954	3.871	3.300	9.1e11	9.1e11	1.0e07	1.0e07	2.0e12	2.0e12
	2	1.510	5.781	5.309	5.211	4.570	9.0e11	9.0e11	1.0e07	1.0e07	4.075	-0.739
	3	1.499	7.686	6.814	6.413	5.560	9.0e11	9.0e11	1.0e07	1.0e07	4.667	0.437
	4	1.486	7.623	6.623	5.998	4.570	9.0e11	9.0e11	1.0e07	1.0e07	4.404	-0.207
	5	1.492	8.304	7.144	6.336	4.970	9.0e11	9.0e11	1.0e07	1.0e07	4.914	0.104
	6	1.475	8.736	7.361	6.498	5.180	9.0e11	9.0e11	1.0e07	1.0e07	4.919	0.064
	7	1.450	8.915	7.482	6.598	5.210	9.0e11	9.0e11	1.0e07	1.0e07	5.188	0.027
	8	1.444	9.111	7.446	6.680	5.220	9.0e11	9.0e11	1.0e07	1.0e07	5.146	0.051
	9	1.434	9.192	7.488	6.624	5.190	8.9e11	8.9e11	1.0e07	1.0e07	5.183	-0.099
	10	1.437	9.287	7.491	6.653	5.290	8.9e11	8.9e11	1.0e07	1.0e07	5.362	0.060
	20	1.345	9.691	7.510	6.680	4.890	8.8e11	8.8e11	1.0e07	1.0e07	5.314	-0.283
	30	1.241	9.747	7.381	6.487	4.730	8.6e11	8.6e11	9.8e06	9.8e06	6.039	0.136
	40	1.176	9.852	7.463	6.490	4.780	8.4e11	8.4e11	9.6e06	9.7e06	6.624	0.096
	50	1.076	9.905	7.441	6.374	4.640	8.3e11	8.3e11	9.5e06	9.5e06	7.056	0.274
	60	1.044	9.798	7.403	6.318	4.530	8.1e11	8.1e11	9.3e06	9.3e06	7.625	0.454
1000	1	1.765	4.712	4.549	4.396	4.030	1.5e13	1.5e13	4.2e07	4.2e07	3.3e13	3.3e13
	2	1.763	9.110	8.798	8.365	7.540	1.5e13	1.5e13	4.2e07	4.2e07	4.460	-0.513
	3	1.762	11.068	10.483	9.719	8.610	1.5e13	1.5e13	4.2e07	4.2e07	6.472	0.199
	4	1.750	16.435	15.206	14.810	13.300	1.5e13	1.5e13	4.2e07	4.2e07	9.679	4.585
	5	1.731	15.442	15.800	15.349	13.600	1.5e13	1.5e13	4.2e07	4.2e07	8.035	4.032
	6	1.720	15.637	15.895	15.399	13.700	1.5e13	1.5e13	4.2e07	4.2e07	8.339	3.743
	7	1.723	15.980	16.093	15.551	13.800	1.5e13	1.5e13	4.2e07	4.2e07	8.281	3.793
	8	1.717	15.994	16.084	15.467	13.600	1.5e13	1.5e13	4.2e07	4.2e07	7.798	3.659
	9	1.702	16.030	16.012	15.451	13.500	1.5e13	1.5e13	4.2e07	4.2e07	7.541	3.523
	10	1.683	16.070	16.098	15.428	13.400	1.5e13	1.5e13	4.2e07	4.2e07	7.029	2.947
	20	1.654	16.443	15.826	15.281	13.200	1.5e13	1.5e13	4.1e07	4.1e07	7.594	3.411
	30	1.579	16.434	15.853	15.174	13.300	1.5e13	1.5e13	4.1e07	4.1e07	8.102	3.451
	40	1.526	16.529	16.209	15.332	13.600	1.4e13	1.4e13	4.1e07	4.1e07	7.665	2.683
	50	1.509	16.528	16.127	15.301	13.600	1.4e13	1.4e13	4.0e07	4.0e07	8.148	2.827
	60	1.453	16.595	16.267	15.387	13.500	1.4e13	1.4e13	4.0e07	4.0e07	8.588	2.525

TABLE IV(a): The simulated excess risk for DGP(IV), $\sigma^2 = 1$, $d = 0$, $k_0 = 2$, $(n-K)E\left[(y_{n+1} - \hat{y}_{n+1})^2 - \sigma^2\right] - 2\sigma^2$.

n	$K-2$	$AR(2)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	-0.010	0.890	1.242	1.340	2.229	1.554	2.189	1.241	2.121	504.424	563.901
	2	-0.014	1.224	1.386	1.469	2.242	2.294	2.387	1.869	2.293	415.820	468.157
	3	-0.016	1.549	1.488	1.575	2.264	2.873	2.509	2.341	2.464	1.947	0.305
	4	-0.011	1.769	1.557	1.614	2.294	3.291	2.656	2.709	2.554	2.402	0.420
	5	-0.013	1.974	1.622	1.675	2.305	3.673	2.776	3.019	2.701	2.770	0.516
	6	-0.012	2.123	1.643	1.664	2.319	3.995	2.881	3.360	2.728	3.015	0.606
	7	-0.006	2.245	1.657	1.678	2.342	4.242	2.962	3.640	2.879	3.340	0.683
	8	-0.006	2.369	1.609	1.670	2.362	4.513	3.017	3.890	2.952	3.581	0.767
	9	-0.007	2.468	1.606	1.655	2.340	4.752	3.042	4.047	2.988	3.802	0.843
	10	-0.017	2.587	1.616	1.635	2.341	5.002	3.106	4.229	3.048	3.958	0.888
	20	-0.027	2.903	1.667	1.663	2.333	6.681	3.859	5.735	3.694	5.570	1.567
	30	-0.009	2.964	1.747	1.711	2.366	7.758	4.482	6.627	4.159	6.651	2.265
	40	-0.005	3.094	1.764	1.807	2.372	8.643	5.020	7.220	4.568	7.498	2.695
	50	-0.006	3.201	1.733	1.740	2.317	9.224	5.725	7.812	4.974	8.107	3.174
	60	-0.055	3.236	1.692	1.671	2.237	9.838	6.341	8.068	5.050	8.665	3.561
300	1	0.023	0.728	0.753	0.819	1.731	1.484	1.803	1.091	1.717	874.076	954.732
	2	0.022	1.119	0.945	0.939	1.733	2.250	2.013	1.690	1.898	726.095	798.953
	3	0.014	1.460	1.075	1.012	1.737	2.839	2.087	2.287	1.994	1.984	0.298
	4	0.012	1.679	1.129	1.065	1.745	3.231	2.220	2.714	2.128	2.516	0.417
	5	0.004	1.871	1.172	1.079	1.716	3.628	2.186	3.104	2.225	2.839	0.465
	6	0.009	1.976	1.223	1.109	1.769	3.958	2.296	3.399	2.317	3.086	0.504
	7	0.004	2.138	1.238	1.109	1.758	4.224	2.378	3.661	2.340	3.375	0.574
	8	0.009	2.252	1.249	1.119	1.745	4.511	2.415	3.876	2.410	3.677	0.626
	9	0.007	2.335	1.249	1.137	1.760	4.791	2.509	4.100	2.550	3.952	0.710
	10	0.012	2.344	1.259	1.152	1.784	5.046	2.581	4.338	2.615	4.200	0.744
	20	0.007	2.771	1.346	1.185	1.894	6.819	3.154	5.932	3.194	5.764	1.212
	30	0.012	2.799	1.351	1.167	2.075	8.004	3.746	6.922	3.656	6.760	1.618
	40	0.037	2.827	1.468	1.273	2.152	8.801	4.063	7.665	4.039	7.488	1.951
	50	0.032	2.828	1.496	1.412	2.242	9.447	4.535	8.261	4.426	8.137	2.223
	60	0.032	2.873	1.475	1.461	2.289	10.148	4.889	8.703	4.611	8.743	2.509
500	1	-0.074	0.484	0.356	0.294	0.398	1.151	0.645	1.041	0.573	1470.506	1579.785
	2	-0.071	0.855	0.555	0.438	0.428	1.849	0.733	1.759	0.705	1240.460	1338.988
	3	-0.065	1.135	0.653	0.520	0.448	2.377	0.822	2.220	0.748	1.957	0.112
	4	-0.068	1.383	0.708	0.534	0.443	2.752	0.892	2.522	0.869	2.432	0.165
	5	-0.063	1.531	0.774	0.551	0.465	3.202	0.947	2.951	0.899	2.813	0.202
	6	-0.058	1.735	0.779	0.536	0.460	3.679	1.009	3.272	0.934	2.997	0.198
	7	-0.064	1.887	0.790	0.531	0.451	4.014	1.053	3.540	0.969	3.311	0.213
	8	-0.065	2.009	0.805	0.529	0.448	4.194	1.155	3.817	1.038	3.692	0.286
	9	-0.068	2.085	0.848	0.555	0.428	4.510	1.220	3.998	1.173	3.913	0.368
	10	-0.067	2.155	0.863	0.543	0.450	4.642	1.337	4.299	1.263	4.062	0.363
	20	-0.107	2.359	0.787	0.496	0.463	6.326	1.664	5.776	1.861	5.631	0.478
	30	-0.095	2.407	0.811	0.573	0.529	7.561	2.135	6.866	2.383	6.657	0.655
	40	-0.092	2.375	0.874	0.577	0.559	8.377	2.663	7.565	2.556	7.405	0.772
	50	-0.067	2.483	0.899	0.610	0.620	9.148	3.023	8.191	2.928	8.123	0.939
	60	-0.112	2.487	0.884	0.609	0.603	9.587	3.162	8.522	3.138	8.549	0.986
1000	1	-0.063	0.515	0.301	0.154	0.017	1.288	0.205	0.992	0.126	3023.258	3192.131
	2	-0.066	0.874	0.467	0.259	0.038	2.086	0.312	1.749	0.204	2555.396	2701.100
	3	-0.059	1.155	0.664	0.344	0.042	2.666	0.345	2.377	0.304	2.245	0.134
	4	-0.066	1.356	0.703	0.352	0.036	2.981	0.368	2.765	0.383	2.636	0.099
	5	-0.070	1.470	0.731	0.394	0.033	3.311	0.365	3.125	0.431	2.958	0.110
	6	-0.071	1.541	0.770	0.387	0.030	3.578	0.351	3.575	0.513	3.279	0.131
	7	-0.071	1.601	0.786	0.400	0.040	3.865	0.308	3.881	0.531	3.632	0.104
	8	-0.068	1.710	0.796	0.381	0.039	4.045	0.421	4.082	0.554	3.877	0.117
	9	-0.069	1.728	0.744	0.388	0.048	4.358	0.352	4.309	0.581	4.225	0.117
	10	-0.077	1.751	0.724	0.396	0.042	4.535	0.392	4.599	0.642	4.408	0.090
	20	-0.069	1.970	0.729	0.428	0.038	6.127	0.718	6.116	1.117	6.007	0.143
	30	-0.056	2.115	0.781	0.417	0.009	7.470	1.134	7.119	1.625	6.854	0.110
	40	-0.046	2.180	0.815	0.444	0.044	8.220	1.578	8.090	2.119	7.776	0.104
	50	-0.039	2.171	0.802	0.405	0.061	8.898	1.747	8.628	2.337	8.473	0.126
	60	-0.050	2.159	0.734	0.358	0.048	9.334	1.891	8.999	2.609	8.838	0.181

TABLE IV(b): The simulated excess risk for DGP(IV), $\sigma^2 = 1$, $d = 1$, $k_0 = 3$, $(n-K)E\left[(y_{n+1}-\hat{y}_{n+1})^2-\sigma^2\right]-4\sigma^2$.

n	$K-3$	$AR(3)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	0.067	0.984	1.311	1.444	2.357	8.3e+04	8.3e+04	1.359	0.387	2.0e+4	1.4e+4
	2	0.053	1.363	1.449	1.603	2.380	8.3e+04	8.3e+04	1.846	0.553	1.201	0.460
	3	0.048	1.684	1.561	1.670	2.407	8.2e+04	8.2e+04	2.260	0.734	1.474	0.559
	4	0.035	1.971	1.646	1.711	2.390	8.2e+04	8.2e+04	2.493	0.821	1.690	0.628
	5	0.023	2.154	1.689	1.733	2.375	8.2e+04	8.2e+04	2.721	0.913	1.953	0.761
	6	0.020	2.280	1.721	1.765	2.401	8.1e+04	8.1e+04	2.941	0.980	2.214	0.848
	7	0.014	2.366	1.679	1.748	2.411	8.1e+04	8.1e+04	3.189	1.054	2.381	0.938
	8	0.000	2.502	1.664	1.704	2.438	8e+04	8e+04	3.371	1.070	2.573	0.974
	9	-0.016	2.538	1.662	1.693	2.401	8e+04	8e+04	3.573	1.114	2.747	1.013
	10	-0.034	2.621	1.633	1.660	2.397	8e+04	8e+04	3.754	1.189	2.896	1.103
	20	-0.093	2.985	1.666	1.703	2.329	7.6e+04	7.6e+04	5.287	2.030	4.189	1.675
	30	-0.132	3.001	1.735	1.683	2.328	7.2e+04	7.2e+04	6.299	2.639	5.015	2.053
	40	-0.174	3.118	1.708	1.685	2.353	6.8e+04	6.8e+04	7.148	3.243	5.675	2.497
	50	-0.205	3.180	1.651	1.633	2.291	6.4e+04	6.4e+04	7.872	4.010	6.173	2.827
	60	-0.302	3.355	1.554	1.543	2.158	5.9e+04	5.9e+04	8.460	4.660	6.465	2.857
300	1	0.076	0.771	0.813	0.878	1.785	2.3e+05	2.3e+05	1.375	-0.031	4.6e+4	3.2e+4
	2	0.061	1.175	0.985	1.008	1.792	2.3e+05	2.3e+05	1.846	0.105	1.054	-0.070
	3	0.053	1.539	1.136	1.064	1.781	2.3e+05	2.3e+05	2.158	0.201	1.442	0.074
	4	0.039	1.809	1.177	1.089	1.762	2.3e+05	2.3e+05	2.437	0.211	1.669	0.152
	5	0.038	1.954	1.176	1.103	1.816	2.3e+05	2.3e+05	2.718	0.279	2.001	0.232
	6	0.030	2.115	1.219	1.106	1.802	2.3e+05	2.3e+05	2.982	0.365	2.189	0.316
	7	0.030	2.226	1.253	1.107	1.789	2.3e+05	2.3e+05	3.187	0.386	2.385	0.351
	8	0.021	2.247	1.238	1.131	1.776	2.3e+05	2.3e+05	3.430	0.455	2.629	0.427
	9	0.020	2.390	1.234	1.105	1.789	2.3e+05	2.3e+05	3.709	0.521	2.817	0.539
	10	0.011	2.350	1.272	1.128	1.799	2.3e+05	2.3e+05	3.846	0.551	2.934	0.598
	20	-0.051	2.693	1.236	1.148	1.849	2.2e+05	2.2e+05	5.412	1.181	4.345	1.184
	30	-0.084	2.758	1.236	1.113	1.922	2.1e+05	2.1e+05	6.394	1.669	5.287	1.575
	40	-0.137	2.817	1.279	1.161	1.973	2.1e+05	2.1e+05	7.173	2.057	5.995	1.888
	50	-0.190	2.785	1.240	1.194	2.029	2e+05	2e+05	7.808	2.501	6.590	2.230
	60	-0.226	2.745	1.280	1.188	2.083	1.9e+05	1.9e+05	8.473	2.805	7.019	2.521
500	1	-0.045	0.480	0.373	0.324	0.430	6.5e+05	6.5e+05	1.178	-1.392	1.1e+5	7.9e+4
	2	-0.044	0.878	0.547	0.458	0.471	6.5e+05	6.5e+05	1.493	-1.278	1.184	-1.356
	3	-0.049	1.145	0.667	0.494	0.469	6.5e+05	6.5e+05	1.802	-1.200	1.378	-1.348
	4	-0.047	1.364	0.723	0.480	0.471	6.5e+05	6.5e+05	2.065	-1.188	1.653	-1.153
	5	-0.047	1.506	0.795	0.496	0.475	6.5e+05	6.5e+05	2.412	-1.128	1.947	-1.105
	6	-0.056	1.691	0.777	0.502	0.453	6.5e+05	6.5e+05	2.759	-1.112	2.120	-1.133
	7	-0.060	1.869	0.797	0.509	0.470	6.4e+05	6.4e+05	3.017	-1.007	2.300	-1.033
	8	-0.068	1.934	0.796	0.512	0.453	6.4e+05	6.4e+05	3.229	-0.930	2.503	-0.996
	9	-0.074	2.013	0.811	0.494	0.457	6.4e+05	6.4e+05	3.377	-0.874	2.708	-0.912
	10	-0.079	2.024	0.780	0.504	0.466	6.4e+05	6.4e+05	3.588	-0.757	2.946	-0.797
	20	-0.147	2.245	0.665	0.473	0.433	6.3e+05	6.3e+05	4.905	-0.408	4.212	-0.219
	30	-0.166	2.325	0.695	0.472	0.480	6.2e+05	6.2e+05	6.026	-0.002	5.208	0.155
	40	-0.176	2.236	0.760	0.475	0.488	6.1e+05	6.1e+05	6.792	0.472	5.835	0.447
	50	-0.213	2.322	0.739	0.497	0.480	6e+05	6e+05	7.402	0.847	6.480	0.885
	60	-0.282	2.186	0.707	0.439	0.431	5.9e+05	5.9e+05	7.863	1.105	6.818	0.914
1000	1	0.130	0.668	0.483	0.352	0.232	2.6e+06	2.6e+06	1.210	-1.824	3.7e+5	2.8e+5
	2	0.135	1.026	0.684	0.456	0.247	2.6e+06	2.6e+06	1.736	-1.754	1.274	-1.847
	3	0.127	1.251	0.813	0.495	0.253	2.6e+06	2.6e+06	1.996	-1.704	1.581	-1.720
	4	0.121	1.465	0.850	0.528	0.248	2.6e+06	2.6e+06	2.224	-1.644	1.940	-1.604
	5	0.118	1.475	0.925	0.542	0.249	2.6e+06	2.6e+06	2.559	-1.654	2.249	-1.584
	6	0.115	1.629	0.892	0.561	0.229	2.6e+06	2.6e+06	2.625	-1.668	2.581	-1.491
	7	0.117	1.710	0.944	0.559	0.232	2.6e+06	2.6e+06	2.799	-1.672	2.785	-1.505
	8	0.114	1.810	0.947	0.590	0.246	2.6e+06	2.6e+06	3.004	-1.607	2.929	-1.437
	9	0.103	1.879	0.965	0.591	0.227	2.6e+06	2.6e+06	3.220	-1.677	3.163	-1.431
	10	0.110	1.898	1.003	0.577	0.233	2.6e+06	2.6e+06	3.323	-1.598	3.329	-1.375
	20	0.092	2.147	0.939	0.557	0.201	2.6e+06	2.6e+06	4.600	-1.255	4.619	-0.951
	30	0.076	2.190	0.905	0.550	0.172	2.6e+06	2.6e+06	5.855	-0.938	5.558	-0.469
	40	0.064	2.351	0.920	0.533	0.180	2.5e+06	2.5e+06	6.634	-0.646	6.366	0.014
	50	0.059	2.232	0.899	0.464	0.184	2.5e+06	2.5e+06	7.182	-0.365	6.909	0.300
	60	0.037	2.273	0.883	0.409	0.131	2.5e+06	2.5e+06	7.663	-0.207	7.256	0.491

TABLE IV(c): The simulated excess risk for DGP(IV), $\sigma^2 = 1$, $d = 2$, $k_0 = 4$, $(n-K)E\left[(y_{n+1}-\hat{y}_{n+1})^2-\sigma^2\right]-8\sigma^2$.

n	$K-4$	$AR(4)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	0.113	1.134	1.435	1.581	2.574	9.7e+08	9.7e+08	8.3e+04	8.3e+04	-1.392	-3.448
	2	0.094	1.509	1.602	1.725	2.636	9.7e+08	9.7e+08	8.2e+04	8.2e+04	-1.155	-3.300
	3	0.060	1.819	1.685	1.763	2.614	9.6e+08	9.6e+08	8.2e+04	8.2e+04	-0.919	-3.197
	4	0.030	2.021	1.744	1.811	2.558	9.6e+08	9.6e+08	8.2e+04	8.2e+04	-0.845	-3.152
	5	0.002	2.189	1.789	1.825	2.524	9.5e+08	9.5e+08	8.1e+04	8.1e+04	-0.644	-3.017
	6	-0.024	2.308	1.762	1.810	2.535	9.5e+08	9.5e+08	8.1e+04	8.1e+04	-0.448	-2.971
	7	-0.053	2.414	1.735	1.757	2.513	9.4e+08	9.4e+08	8e+04	8e+04	-0.243	-2.939
	8	-0.093	2.489	1.743	1.732	2.451	9.4e+08	9.4e+08	8e+04	8e+04	-0.116	-2.894
	9	-0.116	2.599	1.716	1.709	2.439	9.3e+08	9.3e+08	8e+04	8e+04	0.057	-2.808
	10	-0.126	2.693	1.737	1.721	2.413	9.3e+08	9.3e+08	7.9e+04	7.9e+04	0.238	-2.731
	20	-0.338	2.890	1.527	1.567	2.221	8.8e+08	8.8e+08	7.5e+04	7.5e+04	1.662	-1.965
	30	-0.525	2.917	1.465	1.449	2.124	8.3e+08	8.3e+08	7.1e+04	7.1e+04	2.690	-1.298
	40	-0.645	2.936	1.356	1.335	2.022	7.7e+08	7.7e+08	6.7e+04	6.7e+04	3.561	-0.616
	50	-0.752	3.030	1.244	1.219	1.842	7.2e+08	7.2e+08	6.3e+04	6.3e+04	4.428	0.358
	60	-0.947	3.495	1.080	1.048	1.623	6.6e+08	6.6e+08	5.9e+04	5.9e+04	5.057	1.130
300	1	0.012	0.724	0.744	0.880	1.868	7.5e+09	7.5e+09	2.3e+05	2.3e+05	-1.440	-4.032
	2	-0.011	1.130	0.932	1.010	1.889	7.5e+09	7.5e+09	2.3e+05	2.3e+05	-1.231	-3.920
	3	-0.038	1.465	1.008	1.058	1.872	7.5e+09	7.5e+09	2.3e+05	2.3e+05	-0.998	-3.854
	4	-0.050	1.689	1.100	1.061	1.892	7.5e+09	7.5e+09	2.3e+05	2.3e+05	-0.752	-3.737
	5	-0.074	1.831	1.106	1.054	1.835	7.5e+09	7.5e+09	2.3e+05	2.3e+05	-0.580	-3.728
	6	-0.086	1.986	1.131	1.089	1.848	7.4e+09	7.4e+09	2.3e+05	2.3e+05	-0.403	-3.666
	7	-0.106	2.097	1.160	1.086	1.849	7.4e+09	7.4e+09	2.3e+05	2.3e+05	-0.244	-3.618
	8	-0.116	2.152	1.156	1.081	1.832	7.4e+09	7.4e+09	2.3e+05	2.3e+05	-0.002	-3.505
	9	-0.137	2.252	1.132	1.049	1.835	7.4e+09	7.4e+09	2.3e+05	2.3e+05	0.204	-3.513
	10	-0.152	2.271	1.151	1.033	1.857	7.3e+09	7.3e+09	2.3e+05	2.3e+05	0.378	-3.447
	20	-0.305	2.525	1.096	1.003	1.831	7.1e+09	7.1e+09	2.2e+05	2.2e+05	1.723	-2.942
	30	-0.412	2.540	0.975	0.897	1.846	6.9e+09	6.9e+09	2.1e+05	2.1e+05	2.786	-2.391
	40	-0.543	2.402	0.941	0.875	1.788	6.6e+09	6.6e+09	2.1e+05	2.1e+05	3.536	-1.935
	50	-0.655	2.438	0.889	0.836	1.809	6.4e+09	6.4e+09	2e+05	2e+05	4.176	-1.516
	60	-0.753	2.246	0.874	0.786	1.731	6.2e+09	6.2e+09	1.9e+05	1.9e+05	4.817	-1.210
500	1	-0.322	0.158	0.046	0.049	0.199	5.9e+10	5.9e+10	6.5e+05	6.5e+05	-1.473	-5.318
	2	-0.334	0.571	0.275	0.161	0.215	5.9e+10	5.9e+10	6.5e+05	6.5e+05	-1.497	-5.252
	3	-0.340	0.792	0.405	0.195	0.205	5.9e+10	5.9e+10	6.5e+05	6.5e+05	-1.312	-5.169
	4	-0.349	1.044	0.443	0.161	0.183	5.8e+10	5.8e+10	6.5e+05	6.5e+05	-1.087	-5.189
	5	-0.365	1.217	0.473	0.181	0.179	5.8e+10	5.8e+10	6.4e+05	6.5e+05	-0.737	-5.172
	6	-0.378	1.393	0.494	0.200	0.186	5.8e+10	5.8e+10	6.4e+05	6.4e+05	-0.407	-5.095
	7	-0.389	1.501	0.508	0.189	0.185	5.8e+10	5.8e+10	6.4e+05	6.4e+05	-0.280	-4.995
	8	-0.403	1.620	0.499	0.180	0.174	5.8e+10	5.8e+10	6.4e+05	6.4e+05	-0.148	-4.904
	9	-0.414	1.666	0.452	0.158	0.170	5.8e+10	5.8e+10	6.4e+05	6.4e+05	0.017	-4.864
	10	-0.430	1.657	0.476	0.158	0.137	5.8e+10	5.8e+10	6.4e+05	6.4e+05	0.219	-4.786
	20	-0.556	1.885	0.326	0.096	0.078	5.7e+10	5.7e+10	6.3e+05	6.3e+05	1.453	-4.349
	30	-0.653	1.823	0.229	0.002	-0.002	5.6e+10	5.6e+10	6.2e+05	6.2e+05	2.441	-3.937
	40	-0.722	1.743	0.226	-0.051	-0.014	5.5e+10	5.5e+10	6.1e+05	6.1e+05	3.236	-3.579
	50	-0.782	1.874	0.187	-0.072	-0.038	5.4e+10	5.4e+10	5.9e+05	6e+05	3.848	-3.150
	60	-0.882	1.762	0.082	-0.150	-0.072	5.2e+10	5.2e+10	5.8e+05	5.9e+05	4.297	-3.010
1000	1	0.356	0.887	0.731	0.572	0.438	9.5e+11	9.5e+11	2.6e+06	2.6e+06	-1.568	-5.826
	2	0.341	1.213	0.959	0.697	0.438	9.4e+11	9.4e+11	2.6e+06	2.6e+06	-1.346	-5.744
	3	0.331	1.442	1.076	0.742	0.428	9.4e+11	9.4e+11	2.6e+06	2.6e+06	-1.068	-5.746
	4	0.324	1.654	1.118	0.772	0.416	9.4e+11	9.4e+11	2.6e+06	2.6e+06	-0.915	-5.658
	5	0.319	1.699	1.185	0.801	0.423	9.4e+11	9.4e+11	2.6e+06	2.6e+06	-0.662	-5.681
	6	0.319	1.801	1.175	0.808	0.401	9.4e+11	9.4e+11	2.6e+06	2.6e+06	-0.569	-5.677
	7	0.310	1.910	1.200	0.826	0.419	9.4e+11	9.4e+11	2.6e+06	2.6e+06	-0.407	-5.696
	8	0.297	2.041	1.196	0.786	0.418	9.4e+11	9.4e+11	2.6e+06	2.6e+06	-0.316	-5.630
	9	0.300	2.058	1.227	0.785	0.411	9.4e+11	9.4e+11	2.6e+06	2.6e+06	-0.146	-5.657
	10	0.300	2.071	1.201	0.784	0.418	9.4e+11	9.4e+11	2.6e+06	2.6e+06	0.022	-5.599
	20	0.229	2.294	1.116	0.741	0.340	9.3e+11	9.3e+11	2.6e+06	2.6e+06	1.238	-5.299
	30	0.180	2.429	1.057	0.707	0.280	9.2e+11	9.2e+11	2.6e+06	2.6e+06	2.310	-4.904
	40	0.120	2.422	1.109	0.656	0.207	9.1e+11	9.1e+11	2.5e+06	2.5e+06	3.031	-4.709
	50	0.084	2.306	0.965	0.545	0.202	9.1e+11	9.1e+11	2.5e+06	2.5e+06	3.619	-4.440
	60	-0.001	2.205	0.878	0.413	0.127	9e+11	9e+11	2.5e+06	2.5e+06	4.061	-4.256

TABLE V(a): The simulated excess risk for DGP(V), $\sigma^2 = 1$, $d = 0$, $k_0 = 4$, $(n-K)E\left[(y_{n+1}-\hat{y}_{n+1})^2-\sigma^2\right]-4\sigma^2$.

n	$K-4$	$AR(4)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	0.108	1.637	2.198	2.400	3.750	4.388	5.757	3.441	3.886	3.289	2.706
	2	0.106	1.897	2.352	2.518	3.742	5.167	6.116	4.156	4.140	3.933	2.933
	3	0.123	2.142	2.426	2.521	3.706	5.858	6.521	4.780	4.403	4.631	3.112
	4	0.119	2.342	2.472	2.539	3.682	6.328	6.734	5.149	4.528	5.054	3.201
	5	0.125	2.480	2.536	2.563	3.715	6.929	7.061	5.479	4.665	5.400	3.268
	6	0.121	2.625	2.517	2.612	3.748	7.222	7.157	5.770	4.790	5.652	3.308
	7	0.145	2.691	2.512	2.633	3.763	7.627	7.445	6.019	4.938	5.835	3.286
	8	0.146	2.873	2.517	2.626	3.766	7.908	7.670	6.211	4.994	6.042	3.311
	9	0.153	2.922	2.522	2.658	3.729	8.225	7.900	6.449	5.095	6.220	3.332
	10	0.154	2.984	2.503	2.632	3.746	8.420	8.036	6.505	5.134	6.337	3.375
	20	0.177	3.268	2.442	2.488	3.782	10.261	9.478	7.626	5.421	7.415	3.588
	30	0.139	3.381	2.359	2.426	3.694	11.094	10.309	8.449	6.021	8.213	3.870
	40	0.168	3.388	2.383	2.493	3.590	11.619	11.110	8.915	6.267	8.755	4.031
	50	0.118	3.271	2.170	2.299	3.426	11.715	11.560	9.284	6.415	9.187	4.307
	60	0.122	3.495	2.071	2.168	3.243	11.684	11.934	9.466	6.583	9.442	4.413
300	1	-0.052	1.395	2.068	2.429	4.372	2.855	4.800	4.521	4.526	4.489	4.586
	2	-0.058	1.686	2.213	2.482	4.331	3.756	5.038	5.705	5.010	5.871	5.306
	3	-0.055	1.956	2.296	2.504	4.356	4.594	5.180	6.503	5.387	6.869	5.596
	4	-0.066	2.159	2.294	2.467	4.376	5.193	5.446	7.024	5.654	7.532	5.815
	5	-0.066	2.351	2.311	2.511	4.355	5.662	5.617	7.600	5.896	8.043	5.968
	6	-0.070	2.515	2.323	2.513	4.377	6.124	5.755	8.111	6.195	8.593	6.135
	7	-0.071	2.601	2.304	2.529	4.320	6.506	5.893	8.359	6.289	8.928	6.181
	8	-0.077	2.712	2.312	2.549	4.271	6.868	5.972	8.616	6.355	9.129	6.206
	9	-0.070	2.714	2.372	2.567	4.290	7.109	6.089	8.836	6.519	9.296	6.221
	10	-0.066	2.721	2.376	2.589	4.369	7.578	6.307	9.103	6.641	9.583	6.165
	20	-0.036	2.952	2.451	2.610	4.199	9.852	7.143	10.730	7.389	10.766	6.136
	30	-0.012	3.068	2.484	2.748	4.285	11.508	8.011	11.578	7.780	11.506	6.130
	40	0.011	3.064	2.512	2.786	4.280	12.616	8.881	12.110	8.011	11.986	6.150
	50	-0.020	3.008	2.463	2.658	4.253	13.520	9.527	12.476	8.185	12.396	6.107
	60	0.014	3.136	2.548	2.728	4.241	14.291	10.537	12.668	8.225	12.623	6.137
500	1	0.050	1.287	1.879	2.351	4.934	2.015	4.484	5.003	5.265	4.834	5.685
	2	0.055	1.581	2.026	2.476	4.911	3.013	4.758	6.769	5.957	7.043	7.143
	3	0.057	1.864	2.111	2.519	4.887	3.912	4.950	7.969	6.616	8.739	8.031
	4	0.063	2.041	2.074	2.534	4.917	4.669	5.047	8.702	7.028	9.923	8.651
	5	0.053	2.264	2.172	2.563	4.948	5.185	5.239	9.583	7.536	10.974	9.237
	6	0.049	2.392	2.170	2.566	4.907	5.622	5.132	10.400	8.040	11.641	9.580
	7	0.072	2.549	2.240	2.549	4.969	6.185	5.163	10.826	8.246	12.350	9.718
	8	0.081	2.693	2.291	2.608	5.007	6.555	5.304	11.310	8.498	12.673	9.776
	9	0.089	2.782	2.300	2.629	5.013	6.887	5.373	11.808	8.869	13.191	10.019
	10	0.095	2.846	2.301	2.575	5.012	7.279	5.436	12.163	9.007	13.566	10.081
	20	0.099	3.111	2.213	2.662	5.017	9.636	5.944	14.623	10.468	15.082	10.114
	30	0.089	3.124	2.268	2.677	4.903	11.236	6.171	15.540	10.924	15.944	9.901
	40	0.106	3.065	2.303	2.712	4.994	12.492	6.520	16.207	11.226	16.440	9.679
	50	0.129	3.018	2.382	2.795	5.010	13.403	6.889	16.627	11.502	16.727	9.436
	60	0.090	2.940	2.307	2.769	5.075	14.269	7.253	16.925	11.491	16.958	9.249
1000	1	-0.240	0.594	0.607	0.799	3.024	1.562	3.145	3.496	3.803	3.399	7.838
	2	-0.243	0.899	0.829	0.922	2.994	2.598	3.563	5.986	5.394	6.567	10.888
	3	-0.263	1.149	1.008	0.969	2.939	3.433	3.724	7.693	6.603	8.344	12.616
	4	-0.251	1.365	1.131	1.007	2.908	4.107	3.840	9.462	7.604	10.872	13.771
	5	-0.260	1.599	1.139	1.023	2.836	4.740	3.880	11.331	8.804	13.284	15.281
	6	-0.269	1.807	1.105	1.070	2.943	5.371	4.068	12.576	9.734	15.024	16.082
	7	-0.257	1.832	1.137	1.088	3.012	5.805	4.146	13.509	10.425	16.833	17.209
	8	-0.258	2.011	1.146	1.097	3.014	6.233	4.350	14.551	10.828	18.516	18.292
	9	-0.253	2.055	1.141	1.102	3.017	6.737	4.405	15.485	11.615	19.897	18.782
	10	-0.233	2.097	1.127	1.078	3.062	7.185	4.319	16.152	12.420	20.980	19.644
	20	-0.235	2.523	1.200	1.065	3.041	9.579	4.729	21.412	16.292	26.371	21.764
	30	-0.233	2.623	1.186	1.116	3.105	11.493	4.825	23.932	18.208	27.645	21.749
	40	-0.212	2.627	1.286	1.212	3.137	12.664	5.261	25.360	19.285	28.312	21.597
	50	-0.197	2.566	1.325	1.343	3.224	13.595	5.598	26.273	19.909	28.647	21.363
	60	-0.173	2.637	1.331	1.366	3.389	14.377	5.648	26.951	20.315	28.885	21.159

TABLE V(b): The simulated excess risk for DGP(V), $\sigma^2 = 1$, $d = 1$, $k_0 = 5$, $(n-K)E\left[(y_{n+1} - \hat{y}_{n+1})^2 - \sigma^2\right] - 6\sigma^2$.

n	$K-5$	$AR(5)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	0.174	1.702	2.377	2.591	4.490	8.2e+04	8.2e+04	4.085	4.338	2.873	2.172
	2	0.181	1.994	2.520	2.719	4.504	8.1e+04	8.1e+04	4.658	4.792	3.407	2.501
	3	0.171	2.255	2.589	2.746	4.502	8.1e+04	8.1e+04	5.225	5.073	3.810	2.657
	4	0.165	2.449	2.661	2.813	4.518	8.1e+04	8.1e+04	5.620	5.386	4.086	2.777
	5	0.152	2.645	2.683	2.839	4.544	8e+04	8e+04	6.011	5.642	4.349	2.900
	6	0.167	2.798	2.715	2.891	4.556	8e+04	8e+04	6.295	5.848	4.552	3.029
	7	0.160	2.902	2.704	2.905	4.582	7.9e+04	7.9e+04	6.506	6.108	4.732	3.097
	8	0.157	2.991	2.696	2.891	4.564	7.9e+04	7.9e+04	6.723	6.194	4.924	3.159
	9	0.152	3.034	2.719	2.868	4.569	7.9e+04	7.9e+04	7.005	6.519	5.063	3.249
	10	0.154	3.062	2.694	2.853	4.527	7.8e+04	7.8e+04	7.142	6.585	5.094	3.237
	20	0.089	3.440	2.628	2.745	4.564	7.4e+04	7.4e+04	8.731	8.026	6.049	3.468
	30	0.028	3.505	2.528	2.744	4.671	7e+04	7e+04	9.534	8.971	6.799	3.901
	40	0.029	3.531	2.589	2.734	4.680	6.6e+04	6.6e+04	10.046	9.845	7.278	4.189
	50	-0.071	3.485	2.408	2.659	4.717	6.2e+04	6.2e+04	10.011	10.048	7.511	4.225
	60	-0.088	3.958	2.428	2.622	4.685	5.8e+04	5.8e+04	9.875	10.414	7.834	4.488
300	1	-0.003	1.442	2.141	2.487	4.447	2.3e+05	2.3e+05	2.721	3.007	4.555	3.124
	2	-0.007	1.753	2.301	2.565	4.467	2.3e+05	2.3e+05	3.320	3.246	5.252	3.574
	3	-0.022	1.983	2.349	2.601	4.481	2.3e+05	2.3e+05	3.977	3.423	5.893	3.821
	4	-0.030	2.262	2.397	2.614	4.467	2.3e+05	2.3e+05	4.422	3.663	6.286	4.125
	5	-0.042	2.444	2.507	2.579	4.461	2.3e+05	2.3e+05	4.772	3.808	6.757	4.381
	6	-0.049	2.575	2.470	2.602	4.399	2.3e+05	2.3e+05	5.159	3.926	7.011	4.454
	7	-0.063	2.700	2.422	2.604	4.375	2.3e+05	2.3e+05	5.522	4.045	7.204	4.548
	8	-0.061	2.760	2.395	2.564	4.399	2.3e+05	2.3e+05	5.808	4.221	7.414	4.609
	9	-0.068	2.798	2.388	2.616	4.408	2.3e+05	2.3e+05	6.163	4.423	7.549	4.694
	10	-0.078	2.845	2.398	2.601	4.381	2.2e+05	2.2e+05	6.418	4.480	7.746	4.788
	20	-0.067	3.084	2.456	2.687	4.378	2.2e+05	2.2e+05	8.430	5.518	9.180	5.473
	30	-0.104	3.043	2.451	2.768	4.381	2.1e+05	2.1e+05	10.080	6.379	9.880	5.821
	40	-0.151	3.072	2.388	2.671	4.393	2.1e+05	2.1e+05	11.052	7.262	10.425	5.945
	50	-0.240	2.893	2.389	2.544	4.288	2e+05	2e+05	11.943	8.116	10.775	6.132
	60	-0.203	3.080	2.415	2.609	4.369	1.9e+05	1.9e+05	12.766	9.165	10.968	6.121
500	1	0.179	1.404	2.040	2.489	5.125	6.5e+05	6.5e+05	1.998	2.578	5.895	4.108
	2	0.177	1.711	2.135	2.608	5.115	6.5e+05	6.5e+05	2.712	2.807	7.030	4.814
	3	0.179	1.872	2.213	2.607	5.141	6.4e+05	6.4e+05	3.514	3.004	8.081	5.490
	4	0.164	2.088	2.192	2.621	5.035	6.4e+05	6.4e+05	4.038	3.037	8.695	5.817
	5	0.156	2.324	2.284	2.609	5.053	6.4e+05	6.4e+05	4.456	3.159	9.324	6.205
	6	0.176	2.513	2.318	2.635	5.140	6.4e+05	6.4e+05	4.975	3.190	9.882	6.684
	7	0.182	2.633	2.368	2.692	5.160	6.4e+05	6.4e+05	5.437	3.248	10.180	6.765
	8	0.186	2.777	2.352	2.693	5.179	6.4e+05	6.4e+05	5.770	3.388	10.601	6.973
	9	0.190	2.851	2.370	2.639	5.169	6.4e+05	6.4e+05	6.012	3.312	10.911	7.281
	10	0.196	2.931	2.353	2.673	5.212	6.4e+05	6.4e+05	6.366	3.474	11.208	7.378
	20	0.179	3.135	2.284	2.651	5.030	6.3e+05	6.3e+05	8.287	3.919	13.067	8.543
	30	0.129	3.105	2.335	2.674	4.987	6.2e+05	6.2e+05	9.711	4.185	13.906	8.892
	40	0.109	3.079	2.287	2.661	4.963	6e+05	6e+05	10.884	4.499	14.552	9.304
	50	0.070	3.026	2.310	2.679	5.009	5.9e+05	5.9e+05	11.868	4.879	15.016	9.589
	60	0.040	3.058	2.294	2.684	5.030	5.8e+05	5.8e+05	12.681	5.338	15.194	9.526
1000	1	-0.153	0.726	0.715	0.961	2.958	2.6e+06	2.6e+06	1.736	1.248	4.970	3.068
	2	-0.175	0.971	0.915	1.028	2.925	2.6e+06	2.6e+06	2.566	1.556	7.553	4.818
	3	-0.164	1.263	1.139	1.080	2.960	2.6e+06	2.6e+06	3.051	1.748	9.062	5.916
	4	-0.173	1.436	1.228	1.103	2.997	2.6e+06	2.6e+06	3.606	1.876	10.500	7.148
	5	-0.183	1.633	1.273	1.110	2.985	2.6e+06	2.6e+06	3.988	1.961	11.763	8.114
	6	-0.172	1.859	1.198	1.163	2.977	2.6e+06	2.6e+06	4.548	2.143	12.811	8.955
	7	-0.176	1.958	1.238	1.179	2.989	2.6e+06	2.6e+06	4.943	2.191	13.755	9.470
	8	-0.172	2.162	1.237	1.167	3.003	2.6e+06	2.6e+06	5.320	2.275	14.632	10.197
	9	-0.152	2.201	1.220	1.147	2.980	2.6e+06	2.6e+06	5.767	2.299	15.400	10.855
	10	-0.171	2.259	1.189	1.114	3.026	2.6e+06	2.6e+06	6.085	2.270	16.053	11.624
	20	-0.174	2.619	1.248	1.176	3.008	2.6e+06	2.6e+06	8.311	2.642	20.364	14.807
	30	-0.171	2.676	1.265	1.223	3.162	2.6e+06	2.6e+06	9.931	2.902	22.499	16.442
	40	-0.162	2.697	1.252	1.328	3.308	2.5e+06	2.5e+06	11.026	3.352	23.772	17.496
	50	-0.191	2.630	1.223	1.408	3.370	2.5e+06	2.5e+06	11.838	3.453	24.556	18.041
	60	-0.178	2.761	1.304	1.430	3.426	2.5e+06	2.5e+06	12.577	3.645	25.181	18.326

TABLE V(c): The simulated excess risk for DGP(V), $\sigma^2 = 1$, $d = 2$, $k_0 = 6$, $(n-K)E\left[(y_{n+1} - \hat{y}_{n+1})^2 - \sigma^2\right] - 10\sigma^2$.

n	$K-6$	$AR(6)$	AIC	$AIC3$	$HQIC$	BIC	yLa	$yALa$	ΔyLa	$\Delta yALa$	$\Delta^2 yLa$	$\Delta^2 yALa$
180	1	0.463	2.181	3.060	3.328	5.597	9.5e+08	9.5e+08	8.1e+04	8.1e+04	1.470	0.889
	2	0.437	2.435	3.176	3.449	5.649	9.5e+08	9.5e+08	8.1e+04	8.1e+04	1.767	1.217
	3	0.415	2.702	3.193	3.425	5.670	9.4e+08	9.4e+08	8e+04	8.1e+04	2.309	1.683
	4	0.380	2.963	3.183	3.425	5.629	9.4e+08	9.4e+08	8e+04	8e+04	2.525	1.863
	5	0.379	3.110	3.280	3.438	5.633	9.3e+08	9.3e+08	8e+04	8e+04	2.854	2.114
	6	0.351	3.201	3.210	3.484	5.611	9.3e+08	9.3e+08	7.9e+04	7.9e+04	3.037	2.357
	7	0.331	3.270	3.172	3.444	5.581	9.2e+08	9.2e+08	7.9e+04	7.9e+04	3.235	2.531
	8	0.312	3.404	3.226	3.474	5.544	9.2e+08	9.2e+08	7.8e+04	7.9e+04	3.515	2.733
	9	0.295	3.432	3.203	3.429	5.505	9.1e+08	9.1e+08	7.8e+04	7.8e+04	3.738	2.918
	10	0.262	3.477	3.176	3.444	5.533	9.1e+08	9.1e+08	7.8e+04	7.8e+04	3.899	3.037
	20	0.071	3.637	3.014	3.281	5.510	8.5e+08	8.5e+08	7.4e+04	7.4e+04	5.210	4.355
	30	-0.076	3.790	2.884	3.136	5.334	8e+08	8e+08	7e+04	7e+04	6.058	5.570
	40	-0.159	3.845	2.904	3.070	5.336	7.5e+08	7.5e+08	6.6e+04	6.6e+04	6.501	6.283
	50	-0.350	3.771	2.515	2.750	5.162	7e+08	7e+08	6.1e+04	6.2e+04	6.361	6.527
	60	-0.425	4.788	2.444	2.645	5.042	6.4e+08	6.4e+08	5.7e+04	5.7e+04	6.160	6.659
300	1	0.082	1.700	2.492	2.909	4.980	7.4e+09	7.4e+09	2.3e+05	2.3e+05	0.010	-0.748
	2	0.053	2.020	2.602	2.945	4.943	7.4e+09	7.4e+09	2.3e+05	2.3e+05	0.464	-0.601
	3	0.031	2.241	2.683	2.991	4.902	7.4e+09	7.4e+09	2.3e+05	2.3e+05	0.937	-0.359
	4	0.007	2.426	2.697	2.977	4.887	7.4e+09	7.4e+09	2.3e+05	2.3e+05	1.233	-0.099
	5	-0.013	2.615	2.740	2.952	4.921	7.3e+09	7.3e+09	2.3e+05	2.3e+05	1.633	0.010
	6	-0.035	2.741	2.713	2.995	4.853	7.3e+09	7.3e+09	2.3e+05	2.3e+05	1.932	0.091
	7	-0.046	2.860	2.708	2.986	4.839	7.3e+09	7.3e+09	2.3e+05	2.3e+05	2.208	0.340
	8	-0.058	2.907	2.654	2.999	4.854	7.3e+09	7.3e+09	2.3e+05	2.3e+05	2.497	0.575
	9	-0.083	2.885	2.681	2.969	4.870	7.3e+09	7.3e+09	2.2e+05	2.2e+05	2.743	0.584
	10	-0.096	2.940	2.636	2.932	4.891	7.2e+09	7.2e+09	2.2e+05	2.2e+05	2.982	0.782
	20	-0.177	3.195	2.649	2.838	4.791	7e+09	7e+09	2.2e+05	2.2e+05	4.934	2.000
	30	-0.284	3.092	2.487	2.793	4.787	6.8e+09	6.8e+09	2.1e+05	2.1e+05	6.483	2.857
	40	-0.392	2.960	2.435	2.613	4.736	6.5e+09	6.5e+09	2e+05	2e+05	7.535	3.700
	50	-0.527	2.781	2.293	2.492	4.530	6.3e+09	6.3e+09	2e+05	2e+05	8.262	4.512
	60	-0.567	2.950	2.281	2.482	4.523	6.1e+09	6.1e+09	1.9e+05	1.9e+05	9.135	5.634
500	1	0.309	1.514	2.237	2.689	5.533	5.8e+10	5.8e+10	6.5e+05	6.5e+05	-0.511	-1.294
	2	0.302	1.845	2.328	2.752	5.487	5.8e+10	5.8e+10	6.4e+05	6.4e+05	0.112	-1.221
	3	0.279	2.012	2.366	2.707	5.486	5.8e+10	5.8e+10	6.4e+05	6.4e+05	0.642	-1.009
	4	0.262	2.240	2.350	2.694	5.453	5.8e+10	5.8e+10	6.4e+05	6.4e+05	1.062	-1.032
	5	0.272	2.552	2.498	2.708	5.464	5.8e+10	5.8e+10	6.4e+05	6.4e+05	1.494	-0.807
	6	0.270	2.691	2.538	2.743	5.402	5.8e+10	5.8e+10	6.4e+05	6.4e+05	1.811	-0.769
	7	0.265	2.820	2.519	2.713	5.392	5.8e+10	5.8e+10	6.4e+05	6.4e+05	2.173	-0.707
	8	0.262	2.948	2.481	2.661	5.434	5.8e+10	5.8e+10	6.4e+05	6.4e+05	2.546	-0.665
	9	0.261	3.072	2.482	2.694	5.419	5.7e+10	5.7e+10	6.4e+05	6.4e+05	2.762	-0.601
	10	0.249	3.160	2.521	2.704	5.423	5.7e+10	5.7e+10	6.4e+05	6.4e+05	3.050	-0.459
	20	0.149	3.190	2.357	2.711	5.333	5.6e+10	5.6e+10	6.2e+05	6.2e+05	4.842	0.023
	30	0.050	3.107	2.322	2.756	5.197	5.5e+10	5.5e+10	6.1e+05	6.1e+05	6.130	0.205
	40	-0.031	3.153	2.256	2.703	5.106	5.4e+10	5.4e+10	6e+05	6e+05	7.294	0.492
	50	-0.142	2.956	2.201	2.637	5.033	5.3e+10	5.3e+10	5.9e+05	5.9e+05	8.262	1.024
	60	-0.197	2.877	2.229	2.537	5.091	5.2e+10	5.2e+10	5.8e+05	5.8e+05	9.122	1.508
1000	1	-0.542	0.323	0.391	0.592	2.794	9.4e+11	9.4e+11	2.6e+06	2.6e+06	-0.895	-2.786
	2	-0.532	0.616	0.589	0.687	2.898	9.4e+11	9.4e+11	2.6e+06	2.6e+06	-0.267	-2.434
	3	-0.546	0.892	0.758	0.657	2.841	9.4e+11	9.4e+11	2.6e+06	2.6e+06	0.137	-2.240
	4	-0.558	1.061	0.837	0.683	2.821	9.4e+11	9.4e+11	2.6e+06	2.6e+06	0.522	-2.104
	5	-0.551	1.276	0.878	0.718	2.825	9.4e+11	9.4e+11	2.6e+06	2.6e+06	0.932	-2.066
	6	-0.560	1.550	0.874	0.765	2.823	9.4e+11	9.4e+11	2.6e+06	2.6e+06	1.425	-1.939
	7	-0.561	1.630	0.880	0.759	2.820	9.4e+11	9.4e+11	2.6e+06	2.6e+06	1.791	-1.862
	8	-0.544	1.711	0.867	0.750	2.820	9.4e+11	9.4e+11	2.6e+06	2.6e+06	2.148	-1.786
	9	-0.565	1.738	0.875	0.731	2.820	9.4e+11	9.4e+11	2.6e+06	2.6e+06	2.620	-1.698
	10	-0.555	1.824	0.920	0.747	2.821	9.3e+11	9.3e+11	2.6e+06	2.6e+06	2.897	-1.834
	20	-0.613	2.151	0.855	0.739	2.843	9.3e+11	9.3e+11	2.6e+06	2.6e+06	4.915	-1.371
	30	-0.623	2.237	0.901	0.800	2.899	9.2e+11	9.2e+11	2.5e+06	2.6e+06	6.342	-1.123
	40	-0.661	2.332	0.807	0.817	2.874	9.1e+11	9.1e+11	2.5e+06	2.5e+06	7.408	-0.740
	50	-0.705	2.244	0.813	0.824	3.032	9e+11	9e+11	2.5e+06	2.5e+06	8.255	-0.480
	60	-0.720	2.201	0.822	0.908	2.937	8.9e+11	8.9e+11	2.5e+06	2.5e+06	8.869	-0.300