

Supplementary file for the paper titled “Adaptive Process Monitoring Using Covariate Information”

To save some space in the main paper, some numerical results are presented in this supplemental file. First, Tables S.1-S.2 present the calculated actual ARL_1 values of the proposed EWMAC chart in cases when $p = 1$ or 5 and other setups are the same as those in Table 2.

In Table S.3, we investigate the impact of model parameters β on the performance of the EWMAC chart in cases when $\beta_0 = 0$, $\beta = (\beta, \beta, \beta)^T$, $\beta = 0.25, 0.5$, or 1 , $ARL_{X,0} = ARL_{Y,0} = 200$, $\lambda = 0.1, 0.3$, or 0.5 , and other setups are the same as those in the example of Table 2. From Table S.3, it can be seen that the ARL_1 values increase with β for detecting shifts of types (I) and (II) when λ, ν and the shift type are given. That is because the shifts are not or minimally related to the covariates in such cases, and the mean shift in Y is about $\delta/\sigma_Y = \delta/\sqrt{\sigma_\varepsilon^2 + 3\beta^2\sigma_x^2}$ times of σ_Y . So, the re-scaled shift size δ/σ_Y would be smaller when β gets larger. Consequently, the ARL_1 values would be larger. For detecting shifts of types (III) and (IV), the opposite is true. Namely, the ARL_1 values would decrease when β increases. That is because the re-scaled shift size in such cases is about $3\beta\delta_x/\sqrt{\sigma_\varepsilon^2 + 3\beta^2\sigma_x^2} = 3\delta_x/\sqrt{(\sigma_\varepsilon/\beta)^2 + 3\sigma_x^2}$, which would increase when β increases.

Table S.4 shows the actual ARL_1 values of the EWMAC chart in cases when $p = 50$, $f(x)$ is linear, $\beta_0 = 0$, the first 10 elements of β are all 0.5, and the remaining elements are 0. Table S.5 presents the actual ARL_1 values of the EWMAC chart in cases when $p = 1$, and $f(x) = 0.5 \exp(x)$. Other setups in the examples of Tables S.4 and S.5 are the same as those in Table 2.

Figure S.1 shows the actual $SDARL_{Y,0}$ values of the EWMAC chart in cases when $p = 1, 3$ or 5 and other setups are the same as those in Figure 2. In Figure S.2, we investigate the effect of IC sample size m on $SDARL_{Y,0}$ of the proposed method in cases when $p = 50$ and $\lambda = 0.1, 0.2$ or 0.3 . Finally, we present the the actual $SDARL_{Y,0}$ values of the EWMAC chart in Figure S.3 in cases when $p = 1$, $f(x) = 0.5 \exp(x)$ and other setups are the same as those in Figure 6.

Table S.1: Calculated ARL_1 values and their standard errors (in parentheses) of the EWMAC chart when the weighting function is chosen to be $\phi_L(x; \lambda, h_{\mathbf{X}})$, $ARL_{Y,0} = 200$, and $p = 1$. In each row, numbers in italic denote cases with the smallest ARL_1 values when comparing different $ARL_{0,\mathbf{X}}$ values with λ fixed, and numbers in bold denote cases with the smallest ARL_1 values when comparing different λ values with $ARL_{\mathbf{X},0}$ fixed.

Type ν	$ARL_{\mathbf{X},0} = 100$			$ARL_{\mathbf{X},0} = 200$			$ARL_{\mathbf{X},0} = 300$		
	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$
(I)	14.39(0.08) 13.97(0.11) 17.75(0.16)			11.80(0.07) 13.09(0.10) 16.90(0.15)			11.21(0.06) 12.73(0.10) 16.55(0.15)		
	8.27(0.04) 7.04(0.04) 8.16(0.06)			7.04(0.04) 6.74(0.04) 7.86(0.06)			6.76(0.03) 6.61(0.03) 7.76(0.06)		
	5.81(0.02) 4.54(0.02) 4.81(0.02)			5.04(0.02) 4.38(0.02) 4.67(0.03)			4.87(0.02) 4.32(0.02) 4.62(0.03)		
	4.50(0.02) 3.35(0.02) 3.29(0.02)			3.95(0.02) 3.25(0.02) 3.22(0.02)			3.82(0.02) 3.21(0.01) 3.19(0.01)		
(II)	13.72(0.09) 13.16(0.10) 16.69(0.15)			11.32(0.06) 12.42(0.10) 15.92(0.14)			10.79(0.06) 12.10(0.09) 15.61(0.14)		
	7.83(0.04) 6.60(0.04) 7.55(0.06)			6.71(0.03) 6.34(0.03) 7.31(0.06)			6.44(0.03) 6.23(0.03) 7.19(0.05)		
	5.50(0.02) 4.25(0.02) 4.43(0.03)			4.79(0.02) 4.11(0.02) 4.31(0.03)			4.62(0.02) 4.05(0.02) 4.27(0.03)		
	4.26(0.01) 3.14(0.01) 3.04(0.01)			3.75(0.01) 3.05(0.01) 2.97(0.01)			3.63(0.01) 3.01(0.01) 2.94(0.01)		
(III)	35.73(0.28) 62.05(0.59) 83.34(0.79)			41.42(0.34) 68.22(0.65) 85.12(0.84)			44.39(0.37) 70.17(0.67) 86.47(0.85)		
	24.01(0.17) 40.53(0.37) 56.25(0.54)			27.04(0.20) 45.11(0.42) 59.57(0.57)			28.76(0.22) 46.49(0.43) 61.06(0.59)		
	17.68(0.12) 27.67(0.24) 40.19(0.39)			19.61(0.13) 30.86(0.27) 42.89(0.41)			20.67(0.14) 31.91(0.28) 44.20(0.43)		
	13.73(0.08) 19.95(0.17) 29.01(0.27)			15.16(0.09) 22.03(0.19) 31.21(0.29)			15.90(0.10) 22.95(0.20) 32.00(0.30)		
(IV)	38.18(0.31) 71.45(0.68) 92.71(0.91)			46.81(0.39) 80.65(0.77) 99.76(0.98)			51.88(0.44) 84.11(0.81) 102.84(1.01)		
	26.85(0.20) 51.98(0.49) 72.77(0.71)			32.38(0.25) 60.72(0.58) 79.76(0.79)			35.91(0.28) 64.10(0.61) 82.85(0.81)		
	20.18(0.14) 38.25(0.35) 57.77(0.56)			24.12(0.17) 45.74(0.43) 63.92(0.62)			26.53(0.19) 49.28(0.46) 67.05(0.65)		
	16.02(0.10) 28.88(0.25) 45.85(0.44)			18.87(0.12) 34.98(0.32) 52.02(0.50)			20.69(0.14) 37.73(0.34) 54.66(0.53)		

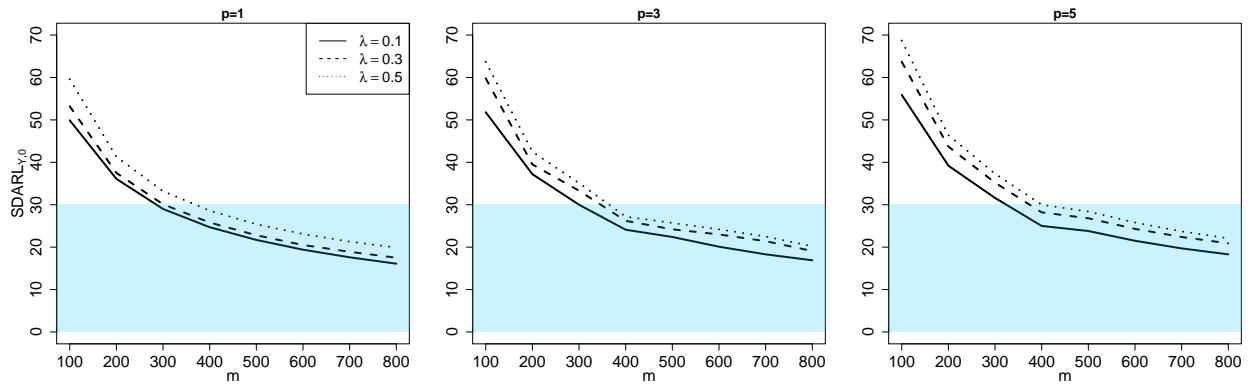


Figure S.1: Actual $SDARLY_0$ values of the EWMAC chart when the IC data size m changes from 100 to 800, $p = 1, 3$ or 5 . Shaded area denotes the $SDARLY_0$ values that are within 15% of the nominal $ARLY_0$ value 200.

Table S.2: Calculated ARL_1 values and their standard errors (in parentheses) of the EWMAC chart when the weighting function is chosen to be $\phi_L(x; \lambda, h_{\mathbf{X}})$, $ARL_{Y,0} = 200$, and $p = 5$. In each row, numbers in italic denote cases with the smallest ARL_1 values when comparing different $ARL_{0,\mathbf{X}}$ values with λ fixed, and numbers in bold denote cases with the smallest ARL_1 values when comparing different λ values with $ARL_{\mathbf{X},0}$ fixed.

Type ν	$ARL_{\mathbf{X},0} = 100$			$ARL_{\mathbf{X},0} = 200$			$ARL_{\mathbf{X},0} = 300$		
	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$
(I)	27.62(0.20)	26.62(0.21)	33.57(0.32)	18.13(0.12)	22.29(0.19)	29.06(0.27)	16.89(0.11)	21.28(0.18)	27.92(0.26)
	14.44(0.08)	12.96(0.08)	16.05(0.14)	10.40(0.06)	11.36(0.09)	14.28(0.12)	9.87(0.05)	10.98(0.08)	13.82(0.12)
	9.53(0.04)	7.73(0.04)	9.15(0.07)	7.20(0.04)	6.97(0.04)	8.25(0.07)	6.88(0.04)	6.78(0.04)	7.99(0.06)
	7.08(0.03)	5.34(0.03)	5.83(0.04)	5.52(0.02)	4.91(0.02)	5.36(0.03)	5.29(0.02)	4.81(0.02)	5.25(0.03)
(II)	21.36(0.15)	20.25(0.16)	25.81(0.24)	14.87(0.09)	17.50(0.15)	22.76(0.21)	13.95(0.09)	16.86(0.14)	21.98(0.20)
	10.80(0.06)	9.09(0.06)	10.99(0.09)	8.15(0.04)	8.16(0.05)	9.94(0.08)	7.76(0.04)	7.95(0.05)	9.63(0.08)
	7.14(0.02)	5.42(0.02)	5.94(0.03)	5.60(0.02)	5.00(0.02)	5.48(0.03)	5.37(0.02)	4.90(0.02)	5.36(0.03)
	5.40(0.02)	3.82(0.02)	3.85(0.02)	4.30(0.02)	3.57(0.02)	3.63(0.02)	4.14(0.02)	3.51(0.02)	3.56(0.02)
(III)	12.02(0.07)	15.94(0.13)	22.24(0.21)	12.96(0.08)	17.16(0.14)	23.70(0.22)	13.63(0.08)	17.94(0.15)	24.40(0.23)
	8.30(0.04)	9.68(0.07)	12.99(0.11)	8.88(0.05)	10.37(0.08)	13.57(0.12)	9.26(0.05)	10.70(0.08)	14.01(0.12)
	6.31(0.03)	6.63(0.04)	8.28(0.07)	6.71(0.03)	7.01(0.05)	8.75(0.07)	6.97(0.03)	7.23(0.05)	8.97(0.07)
	5.05(0.02)	4.99(0.03)	5.84(0.04)	5.38(0.02)	5.23(0.03)	6.10(0.04)	5.60(0.02)	5.38(0.03)	6.25(0.04)
(IV)	12.26(0.07)	16.75(0.14)	23.59(0.24)	13.37(0.08)	18.37(0.15)	25.43(0.24)	14.21(0.09)	19.29(0.16)	26.39(0.25)
	8.57(0.05)	10.43(0.08)	14.34(0.12)	9.29(0.05)	11.31(0.09)	15.35(0.13)	9.82(0.05)	11.92(0.09)	15.93(0.14)
	6.55(0.03)	7.22(0.05)	9.47(0.08)	7.06(0.03)	7.80(0.05)	10.10(0.09)	7.43(0.03)	8.13(0.06)	10.48(0.09)
	5.27(0.02)	5.44(0.03)	6.65(0.05)	5.69(0.02)	5.83(0.04)	7.11(0.05)	6.00(0.02)	6.07(0.04)	7.39(0.06)

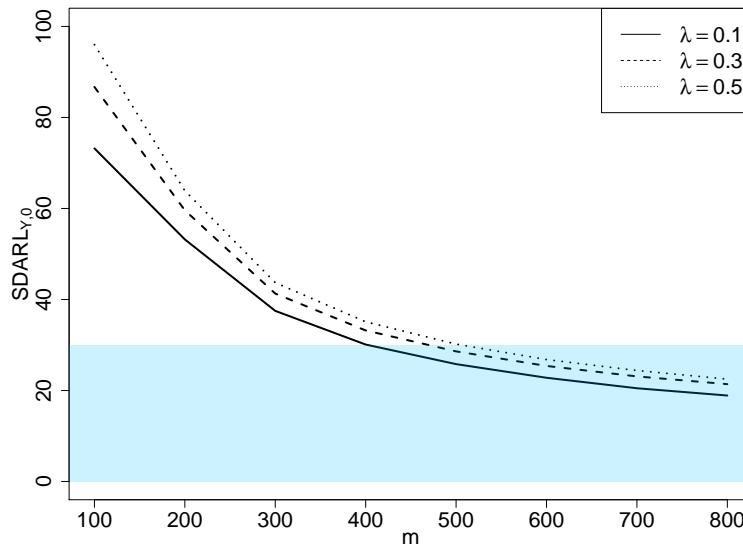


Figure S.2: Actual $SDARLY_0$ values of the EWMAC chart when the IC data size m changes from 100 to 800 and $p = 50$. Shaded area denotes the $SDARLY_0$ values that are within 15% of the nominal $ARL_{Y,0}$ value 200.

Table S.3: Calculated ARL_1 values and their standard errors (in parentheses) of the EWMAC chart with weighting function $\phi_L(x; \lambda, h_{\mathbf{X}})$ for detecting the mean shifts in cases when $p = 3$, $ARL_{\mathbf{X},0} = ARL_{Y,0} = 200$ and β is chosen to be 0.25, 0.50 or 1.00. In each row, numbers in bold denote cases with the smallest ARL_1 values when comparing cases with different λ values with the β value fixed.

Type ν	$\beta = 0.25$			$\beta = 0.50$			$\beta = 1.00$		
	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$
(I)	1 11.27(0.06) 12.38(0.09) 15.93(0.14)			14.99(0.09) 17.91(0.15) 23.19(0.21)			27.11(0.20) 34.92(0.32) 44.61(0.42)		
	2 6.79(0.03) 6.43(0.03) 7.43(0.06)			8.82(0.04) 9.02(0.06) 10.98(0.09)			15.39(0.09) 18.43(0.15) 24.09(0.22)		
	3 4.87(0.02) 4.18(0.02) 4.42(0.03)			6.18(0.03) 5.69(0.03) 6.42(0.05)			10.38(0.06) 11.24(0.08) 14.36(0.12)		
	4 3.82(0.01) 3.08(0.01) 3.03(0.01)			4.79(0.02) 4.08(0.02) 4.27(0.03)			7.82(0.04) 7.74(0.04) 9.31(0.08)		
(II)	1 10.64(0.06) 11.49(0.09) 14.59(0.13)			13.29(0.08) 15.31(0.12) 19.79(0.18)			21.16(0.15) 26.92(0.24) 34.92(0.33)		
	2 6.31(0.03) 5.82(0.03) 6.67(0.05)			7.59(0.04) 7.42(0.04) 8.76(0.07)			11.34(0.06) 12.61(0.10) 16.09(0.14)		
	3 4.51(0.02) 3.79(0.02) 3.93(0.02)			5.31(0.02) 4.65(0.02) 5.01(0.03)			7.60(0.04) 7.42(0.04) 8.87(0.07)		
	4 3.53(0.01) 2.83(0.01) 2.70(0.01)			4.11(0.02) 3.37(0.02) 3.36(0.02)			5.68(0.03) 5.12(0.03) 5.66(0.04)		
(III)	1 22.48(0.16) 40.35(0.37) 58.29(0.56)			19.03(0.13) 28.03(0.25) 38.63(0.37)			17.21(0.12) 22.72(0.20) 30.02(0.28)		
	2 14.70(0.09) 24.12(0.21) 36.27(0.34)			12.63(0.07) 16.92(0.14) 23.54(0.21)			11.59(0.07) 13.94(0.11) 18.21(0.16)		
	3 10.76(0.06) 15.66(0.12) 24.00(0.22)			9.38(0.05) 11.37(0.08) 15.34(0.14)			8.65(0.04) 9.42(0.07) 12.01(0.10)		
	4 8.50(0.04) 11.07(0.08) 16.59(0.15)			7.46(0.03) 8.20(0.06) 10.65(0.09)			6.92(0.03) 6.96(0.05) 8.35(0.07)		
(IV)	1 24.10(0.17) 46.50(0.43) 67.34(0.65)			20.00(0.14) 30.95(0.28) 42.57(0.42)			17.80(0.12) 24.06(0.21) 31.83(0.30)		
	2 16.18(0.10) 29.75(0.27) 47.13(0.45)			13.63(0.08) 19.52(0.17) 27.86(0.25)			12.15(0.07) 15.23(0.12) 20.16(0.18)		
	3 12.03(0.06) 20.35(0.17) 33.42(0.31)			10.19(0.06) 13.48(0.11) 19.22(0.17)			9.11(0.05) 10.52(0.08) 13.54(0.12)		
	4 9.58(0.05) 14.49(0.11) 24.21(0.22)			8.15(0.04) 9.87(0.07) 13.60(0.12)			7.32(0.03) 7.75(0.05) 9.73(0.08)		

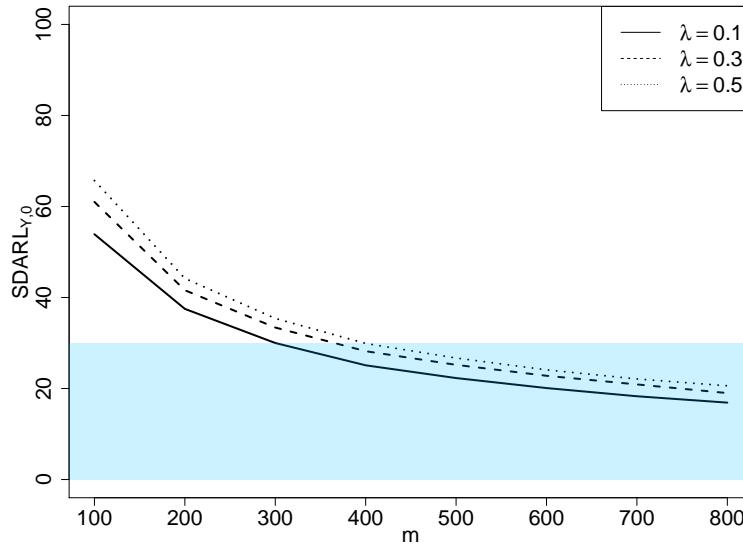


Figure S.3: Actual $SDARLY_0$ values of the EWMAC chart when the IC data size m changes from 100 to 800, $p = 1$, $f(x) = 0.5 \exp(x)$ and $\lambda = 0.1, 0.3$ and 0.5 . Shaded area denotes the $SDARLY_0$ values that are within 15% of the nominal $ARLY_0$ value 200.

Table S.4: Calculated ARL_1 values and their standard errors (in parentheses) of the EWMAC chart when the weighting function is chosen to be $\phi_L(x; \lambda, h_{\mathbf{X}})$, $ARL_{Y,0} = 200$, and $p = 50$. In each row, numbers in italic denote cases with the smallest ARL_1 values when comparing different $ARL_{0,\mathbf{X}}$ values with λ fixed, and numbers in bold denote cases with the smallest ARL_1 values when comparing different λ values with $ARL_{0,\mathbf{X}}$ fixed.

Type ν	$ARL_{\mathbf{X},0} = 100$			$ARL_{\mathbf{X},0} = 200$			$ARL_{\mathbf{X},0} = 300$		
	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$
(I)	1 51.08(0.42) 42.50(0.39) 50.33(0.48)			24.04(0.17) 32.19(0.29) 41.69(0.40)			22.01(0.16) 30.19(0.27) 39.60(0.38)		
	2 24.20(0.16) 20.83(0.18) 26.04(0.24)			13.79(0.08) 16.36(0.13) 21.52(0.20)			12.88(0.08) 15.61(0.13) 20.56(0.19)		
	3 14.89(0.08) 12.09(0.09) 14.71(0.13)			9.48(0.05) 10.02(0.07) 12.55(0.11)			8.91(0.05) 9.63(0.07) 12.03(0.10)		
	4 10.69(0.05) 8.08(0.06) 9.30(0.08)			7.14(0.03) 6.91(0.05) 8.15(0.07)			6.79(0.03) 6.67(0.04) 7.87(0.06)		
(II)	1 28.51(0.21) 25.87(0.22) 31.88(0.30)			16.46(0.11) 20.54(0.18) 26.96(0.25)			15.33(0.10) 19.49(0.17) 25.79(0.24)		
	2 13.08(0.07) 10.68(0.08) 12.77(0.11)			8.77(0.04) 9.04(0.06) 11.08(0.09)			8.28(0.04) 8.72(0.06) 10.66(0.09)		
	3 8.31(0.03) 6.07(0.04) 6.66(0.05)			5.87(0.02) 5.35(0.03) 6.00(0.04)			5.58(0.02) 5.20(0.03) 5.82(0.03)		
	4 6.08(0.02) 4.16(0.02) 4.21(0.03)			4.33(0.02) 3.76(0.02) 3.87(0.02)			4.23(0.01) 3.67(0.02) 3.78(0.02)		
(III)	1 7.74(0.04) 8.60(0.06) 10.90(0.09)			8.09(0.04) 8.91(0.06) 11.25(0.10)			8.37(0.04) 9.13(0.07) 11.50(0.10)		
	2 5.42(0.02) 5.32(0.03) 6.25(0.05)			5.67(0.02) 5.50(0.03) 6.45(0.05)			5.84(0.02) 5.63(0.03) 6.56(0.05)		
	3 4.18(0.02) 3.82(0.02) 4.11(0.03)			4.39(0.02) 3.94(0.02) 4.24(0.03)			4.52(0.02) 4.03(0.02) 4.33(0.03)		
	4 3.40(0.01) 2.99(0.01) 3.02(0.02)			3.59(0.01) 3.09(0.01) 3.11(0.02)			3.70(0.01) 3.15(0.01) 3.17(0.02)		
(IV)	1 7.82(0.04) 8.82(0.06) 11.29(0.10)			8.22(0.04) 9.22(0.07) 11.74(0.10)			8.56(0.04) 9.48(0.07) 12.08(0.10)		
	2 5.51(0.02) 5.52(0.03) 6.62(0.05)			5.81(0.02) 5.75(0.03) 6.86(0.05)			6.04(0.03) 5.92(0.04) 7.02(0.05)		
	3 4.26(0.02) 3.96(0.02) 4.37(0.03)			4.50(0.02) 4.15(0.03) 4.53(0.03)			4.68(0.02) 4.25(0.02) 4.65(0.03)		
	4 3.47(0.01) 3.11(0.01) 3.21(0.02)			3.69(0.01) 3.24(0.01) 3.33(0.01)			3.84(0.01) 3.32(0.01) 3.41(0.02)		

Table S.5: Calculated ARL_1 values and their standard errors (in parentheses) of the EWMAC chart when the weighting function is chosen to be $\phi_L(x; \lambda, h_{\mathbf{X}})$, $ARL_{Y,0} = 200$, $p = 1$, and $f(x) = 0.5 \exp(x)$. In each row, numbers in italic denote cases with the smallest ARL_1 values when comparing different $ARL_{0,\mathbf{X}}$ values with λ fixed, and numbers in bold denote cases with the smallest ARL_1 values when comparing different λ values with $ARL_{0,\mathbf{X}}$ fixed.

Type ν	$ARL_{\mathbf{X},0} = 100$			$ARL_{\mathbf{X},0} = 200$			$ARL_{\mathbf{X},0} = 300$			
	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.3$	$\lambda = 0.5$	
(I)	1	24.03(0.15)	27.82(0.24)	40.67(0.39)	14.12(0.08)	19.06(0.16)	28.53(0.26)	13.04(0.08)	17.41(0.14)	25.86(0.24)
	2	12.29(0.06)	11.81(0.09)	17.09(0.15)	8.13(0.04)	8.94(0.06)	12.45(0.10)	7.62(0.04)	8.41(0.06)	11.45(0.09)
	3	8.13(0.03)	6.75(0.04)	8.57(0.07)	5.71(0.02)	5.52(0.03)	6.79(0.05)	5.39(0.02)	5.26(0.03)	6.39(0.05)
	4	6.10(0.02)	4.62(0.02)	5.26(0.03)	4.43(0.02)	3.91(0.02)	4.37(0.03)	4.20(0.01)	3.78(0.01)	4.16(0.03)
(II)	1	14.30(0.10)	13.45(0.12)	15.20(0.14)	10.35(0.06)	11.28(0.09)	13.31(0.12)	9.82(0.06)	10.77(0.09)	12.72(0.11)
	2	9.08(0.05)	7.89(0.06)	9.20(0.08)	6.70(0.03)	6.63(0.05)	7.76(0.06)	6.35(0.03)	6.37(0.04)	7.40(0.06)
	3	6.55(0.03)	5.27(0.03)	5.89(0.04)	4.93(0.02)	4.54(0.03)	5.05(0.04)	4.70(0.02)	4.37(0.03)	4.82(0.03)
	4	5.13(0.02)	3.86(0.02)	4.07(0.03)	3.92(0.01)	3.37(0.01)	3.54(0.02)	3.74(0.01)	3.27(0.01)	3.41(0.02)
(III)	1	39.17(0.34)	60.25(0.56)	73.21(0.71)	42.00(0.37)	62.06(0.60)	73.41(0.72)	44.23(0.39)	63.60(0.62)	74.11(0.72)
	2	26.58(0.22)	40.94(0.39)	52.23(0.51)	27.53(0.22)	41.53(0.40)	52.16(0.51)	28.77(0.23)	42.07(0.40)	51.88(0.51)
	3	19.23(0.15)	28.88(0.27)	37.98(0.37)	19.63(0.15)	28.83(0.27)	37.09(0.36)	20.30(0.15)	29.12(0.27)	36.85(0.36)
	4	14.58(0.11)	20.96(0.19)	27.97(0.27)	14.89(0.10)	20.83(0.19)	27.17(0.26)	15.47(0.11)	21.02(0.19)	26.89(0.26)
(IV)	1	40.65(0.36)	63.94(0.62)	77.74(0.76)	45.88(0.41)	68.60(0.67)	80.18(0.79)	49.93(0.44)	71.37(0.69)	81.43(0.80)
	2	28.30(0.24)	46.09(0.44)	58.64(0.58)	31.79(0.27)	50.04(0.48)	61.16(0.60)	34.74(0.29)	52.62(0.51)	62.73(0.61)
	3	20.98(0.17)	33.93(0.32)	44.67(0.44)	23.33(0.18)	37.28(0.36)	46.95(0.46)	25.49(0.20)	39.22(0.38)	48.25(0.48)
	4	16.04(0.12)	25.62(0.24)	34.62(0.34)	17.94(0.13)	28.01(0.26)	36.36(0.35)	19.58(0.15)	29.81(0.28)	37.73(0.36)