

Supplementary Material

S1 Partial Identification

As pointed out by Arellano (2003), identification in a binary choice panel setup is fragile, and it usually hinges on assumptions that are either not satisfied or impossible to verify. Such as when the model is a logit (Chamberlain, 1984, 2010), if a regressor has unbounded support, in which case β_0 is point-identified (Manski, 1987), or if the support of this distribution is finite (Bonhomme, 2012). Chernozhukov et al. (2013) showed that when the covariates have discrete support, the marginal distribution of the individual effects is not identified. The following lemma extends this result to the lack of identification of their copula:

Lemma 1. *Assume that the distribution of X_{igt} is discrete with finite support, and let $\mathbb{P}(Y_g|X_g) = \int_{\mathcal{Y}} P_g(u_g; \mu) dC_X(u_g; \rho)$, where $P_g(u_g; \mu)$ is defined as in the main text and is a measurable function of u_g for each $\mu \in M$, and \mathcal{Y} denotes the support of $\eta(u_g|x_g; \sigma_0)$. Then, for each β , every marginal distribution $F_\eta(\eta_{ig}|x_{ig}; \sigma)$ on the support of η_{ig} , and every copula $C(u_g; \rho)$ on $[0, 1]^N$, there exists a discrete distribution $F_\eta^{k, N, T}$ with no more than 2^{NT} support points such that $\int_{\mathcal{Y}} P_g(z_g, u_g; \mu) dC_X(u_g; \rho) = \int_{\mathcal{Y}} P_g(z_g, u_g; \mu) dF_\eta^{k, N, T}(\eta_g)$.*

Proof. By the definition of the copula, there exists a multivariate distribution \tilde{F}_η such that $C_X(u_g; \rho) = \tilde{F}_\eta(u_g|x_g; \sigma, \rho)$. For each $k = 1, \dots, K$ of the possible values that the vector (X_{11}, \dots, X_{NT}) can take, there are $J = 2^{NT}$ distinct values that the vector (Y_{11}, \dots, Y_{NT}) can take. Apply lemma 7 in Chernozhukov et al. (2013) to $\int_{\mathcal{Y}} P_g(z_g, \eta_g; \beta) d\tilde{F}_\eta(\eta_g; \sigma, \rho)$ to obtain the desired result. \square

S2 Asymptotic Distribution

To derive the asymptotic properties of the estimator, I consider the following assumptions:

Assumption 2. $\theta \neq \theta_0 \Rightarrow \ell_g(z_g; \theta) \neq \ell_g(z_g; \theta_0)$.

Assumption 3. $\theta \in \text{int}\Theta$, where Θ is compact.

Assumption 4. $\ell_{ig}(z_{ig}; \mu)$ is continuous for all $\theta \in \Theta$.

Assumption 5. $\mathbb{E} [\sup_{\theta \in \Theta} |\log(\ell_{ig}(z_{ig}; \mu))|] < \infty$.

Assumption 6. $\ell_{ig}(z_{ig}; \mu)$ is twice continuously differentiable with respect to θ ; $\ell_{ig}(z_{ig}; \mu) > 0$ in a neighborhood \mathcal{N} of θ_0 .

Assumption 7. $\int \sup_{\theta \in \mathcal{N}} \|\nabla_{\mu} \ell_{ig}(z_{ig}; \mu)\| dz_{ig} < \infty$, $\int \sup_{\theta \in \mathcal{N}} \|\nabla_{\mu\mu} \ell_{ig}(z_{ig}; \mu)\| dz_{ig} < \infty$.

Assumption 8. $\Sigma_{\theta}^{CBRE} \equiv \mathbb{E} [\nabla_{\theta} \log(\ell_g(z_g; \theta)) \nabla_{\theta} \log(\ell_g(z_g; \theta))']$ exists and is nonsingular.

Assumption 9. $\mathbb{E} [\sup_{\theta \in \mathcal{N}} \|\nabla_{\theta\theta} \log(\ell_g(z_g; \theta))\|] < \infty$.

Assumption 10. The copula has pdf $c_X(u_g; \rho)$ which is twice continuously differentiable in ρ . Moreover, $\int_{[0,1]^{N_g}} |\nabla_{\rho} c_X(u_g; \rho)| \prod_{i=1}^{N_g} du_{ig} < \infty$ and $\int_{[0,1]^{N_g}} |\nabla_{\rho\rho} c_X(u_g; \rho)| \prod_{i=1}^{N_g} du_{ig} < \infty$.

Assumption 11. $\int \sup_{\theta \in \mathcal{N}} \left\| \nabla_{\rho} \int_{[0,1]^{N_g}} c_X(u_g; \rho) \prod_{i=1}^{N_g} du_{ig} \right\| dz_g < \infty$ and $\int \sup_{\theta \in \mathcal{N}} \left\| \int_{[0,1]^{N_g}} \nabla_{\rho\rho} c_X(u_g; \rho) \prod_{i=1}^{N_g} du_{ig} \right\| dz_g < \infty$.

Assumption 12. Cluster size is either predetermined, or it is drawn from a distribution with bounded support, independently of all other variables: $N_g \sim F_N(n)$ $n \in \{1, \dots, \bar{N}\}$, for some $\bar{N} \in \mathbb{N}$.

Assumption 2 is the identification condition. It is hard to verify, and a necessary condition is that the number of parameters of the distribution of the random effects (σ, ρ) is not too large relative to the number of time periods and individuals in a cluster. To be more specific, and assuming that the results are conditional on $X = x$, there are $2^{N_g T}$ distinct results of the outcome variable. This implies that there are $2^{N_g T} - 1$ probabilities that vary freely. Consider the matrix in which each column contains the derivatives of these probabilities with respect to each parameter of the distribution of random effects. Assumption 2 is satisfied when the rank of this matrix equals the number of parameters, so it cannot be larger than $2^{N_g T} - 1$. Moreover, if the probabilities satisfy an exchangeability condition, *i.e.* if $\mathbb{P}(Y_1 = y_1, Y_0 = y_0) = \mathbb{P}(Y_1 = y_0, Y_0 = y_1)$ and similarly when the cluster dimension is higher, then the number of probabilities that vary freely is smaller. Specifically,

it would be equal to $\sum_{i_{1,1}=0}^1 \dots \sum_{i_{N_g,T}=0}^1 \mathbf{1} \left(i_{1,1} \leq i_{1,2} \leq \dots \leq i_{1,T} \leq \dots \leq i_{N_g,1} \leq \dots \leq i_{N_g,T} \right)$. Consequently, the maximum number of parameters is at most this number.

Assumptions 1 to 9 mimic the assumptions in theorems 2.5 and 3.3 in Newey and McFadden (1994). With some small modifications, these assumptions work for standard RE estimators. In other words, they allow us to extend any RE estimator to have the cluster dependence described in Section 2. It would be possible to relax some of these assumptions, but it could result in non-standard properties. For example, if Assumption 3 allowed the true value of the parameter lies at the boundary of the parameter space, the asymptotic distribution of $\hat{\theta}$ could be a mixture.

Assumptions 10 and 11 impose smoothness restrictions on the copula, as well as bounds on some functionals of its derivative with respect to the copula parameter. It covers the independence case in which its density equals one everywhere, but not the perfect correlation case in which the copula has no proper density. Assumption 12 limits cluster size to \bar{N} , ruling out the possibility that the size of a group grows to infinity as the sample size grows. This assumption is required to bound the likelihood function, and it should be satisfied in most applications. Regarding its independence with respect to all other variables, it could be relaxed at the cost of complicating the analysis.

The following proposition establishes the asymptotic distribution of the CBRE estimator:

Proposition 3. *Under Assumptions 1 to 12, the CBRE estimator $\hat{\theta}$ is a consistent estimator for θ_0 and its asymptotic distribution is given by $\sqrt{G} \left(\hat{\theta} - \theta_0 \right) \xrightarrow{d} \mathcal{N} \left(0, \Sigma_{\theta}^{CBRE} \right)$.*

Proof. The proposition is shown by checking that Assumptions 1 to 12 satisfy the assumptions in theorems 2.5 and 3.3 in Newey and McFadden (1994). Rather than considering the data *iid* at the individual level, I do it at the cluster level. Begin with the consistency result.

By Assumptions 10 and 12,

$$\ell_g(z_g; \theta) = \int_{[0,1]^{N_g}} \prod_{i=1}^{N_g} P_{ig}(z_{ig}, u_{ig}; \mu) dC_X(u_g; \rho) < \int_{[0,1]^{N_g}} dC_X(u_g; \rho) = 1$$

So $\ell_g(z_{ig}; \theta)$ is well defined and finite. By Assumptions 4 and 10, for any sequence

$\theta_n : \theta_n \rightarrow \theta$, $\prod_{i=1}^{N_g} P_{ig}(z_{ig}, u_{ig}; \mu_n) c_X(u_g; \rho_n) \rightarrow \prod_{i=1}^{N_g} P_{ig}(z_{ig}, u_{ig}; \mu) c_X(u_g; \rho)$ for almost every u_g . Thus, by the dominated convergence theorem, $\ell_g(z_g; \theta_n) \rightarrow \ell_g(z_g; \theta)$, so $\ell_g(z_g; \theta)$ is continuous with respect to θ . By a similar argument, $-\infty < \log(\ell_g(z_g; \theta)) < 0$. To get the lower bound, notice that $\ell_g(z_g; \theta) > 0 \Leftrightarrow \exists u_g : \prod_{i=1}^{N_g} P_{ig}(z_{ig}, u_{ig}; \mu) c_X(u_g; \rho) > 0$. By Assumption 5, $\exists u_{ig}$ be such that $P_{ig}(z_{ig}, u_{ig}^*) > 0 \forall i$. By Assumption 10, the marginals must integrate to 1, and the copula is continuous, so it has a proper pdf. Hence, $\exists u_{1c}^* : c_X(u_{1g}^*, u_{2g}, \dots, u_{N_gg}) > 0 \forall u_{2g}, \dots, u_{N_gg}$. Therefore, $\mathbb{E}[\sup_{\theta \in \Theta} |\log(\ell_g(z_g; \theta))|] < \infty$. These two results, together with Assumptions 1 to 3, verify the conditions in theorem 2.5 in Newey and McFadden (1994) and hence $\hat{\theta} \xrightarrow{P} \theta_0$.

By Assumptions 10, and 12,

$$\begin{aligned} \nabla_\mu \ell_g(z_g; \theta) &= \sum_{i=1}^{N_g} \int_{[0,1]^{N_g}} \nabla_\mu P_{ig}(z_{ig}, u_{ig}; \mu) \prod_{j \neq i} P_{jg}(z_{jg}, u_{jg}; \mu) dC_X(u_g; \rho) \\ &= \sum_{i=1}^{N_g} \int_0^1 \nabla_\mu P_{ig}(z_{ig}, u_{ig}; \mu) \left[\int_{[0,1]^{N_g-1}} \prod_{j \neq i} P_{jg}(z_{jg}, u_{jg}; \mu) dC_X(u_{-ig}|u_{ig}; \rho) \right] du_{ig} \end{aligned}$$

for all $\theta \in \mathcal{N}$, where u_{-ig} denotes the set $\{u_{jg}\}_{j \neq i}$. Note that the term in square brackets is a probability, and hence it takes values on the unit interval. Hence, by Assumption 7,

$$\int \sup_{\theta \in \mathcal{N}} \|\nabla_\mu \ell_g(z_g; \theta)\| dz_g < \overline{N} \max_{i=1, \dots, N_g} \int \sup_{\theta \in \mathcal{N}} \|\nabla_\mu \ell_{ig}(z_{ig}; \mu)\| dz_{ig} < \infty$$

By Assumptions 10 and 12

$$\begin{aligned} \nabla_\rho \ell_g(z_g; \theta) &= \int_{[0,1]^{N_g}} \prod_{i=1}^{N_g} P_{ig}(z_{ig}, u_{ig}; \mu) \nabla_\rho c_X(u_g; \rho) du_{ig} \\ &\leq \int_{[0,1]^{N_g}} \prod_{i=1}^{N_g} P_{ig}(z_{ig}, u_{ig}; \mu) |\nabla_\rho c_X(u_g; \rho)| du_{ig} \\ &\leq \int_{[0,1]^{N_g}} |\nabla_\rho c_X(u_g; \rho)| \prod_{i=1}^{N_g} du_{ig} < \infty \end{aligned}$$

This, together with Assumption 11 implies $\int \sup_{\theta \in \mathcal{N}} \|\nabla_\rho \ell_g(z_g; \theta)\| dz_g < \infty$.

By a parallel argument, the second derivatives can be bounded. Consequently, it follows

that $\int \sup_{\theta \in \mathcal{N}} \|\nabla_{\theta} \ell_g(z_g; \theta)\| dz_g < \infty$ and $\int \sup_{\theta \in \mathcal{N}} \|\nabla_{\theta\theta} \ell_g(z_g; \theta)\| dz_g < \infty$. Taking this result, together with Assumptions 1, 3, 6, 8, and 9, the conditions in Theorem 3.3 in Newey and McFadden (1994) are verified and $\sqrt{G}(\hat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, \Sigma_{\theta}^{C BRE})$.

□

S3 Estimation of Average Partial Effects

Frequently, the researcher is interested in the estimation of the APE rather than the slope coefficients. The APE is defined as the marginal effect that increasing the j th regressor $x_{igt,j}$ would have on the probability of the dependent variable being equal to one, averaged over the whole population. Mathematically,

$$APE(x_{igt,j}) \equiv \int_{\mathbb{R}} \frac{\partial}{\partial x_{igt,j}} \mathbb{P}(y_{igt} = 1 | x_{igt}, \eta_{ig}) dF_{\eta}(\eta_{ig} | x_{ig}; \sigma_0) \quad (13)$$

Since it just depends on the marginal distribution of η_{ig} , there is no need to know the copula to identify them, and it can be computed using the sample analogue of Equation 13. It is worth highlighting that the APE depend on the parametric assumptions. See, for instance, Graham and Powell (2012), Chernozhukov et al. (2013), Fernández-Val and Lee (2013), or Escanciano (2016) for discussions on the identification and estimation of APE in this and other related frameworks.

S4 Score and Hessian

Let F_{igt} and f_{igt} be shorthand for $F_{\varepsilon}(-(\eta(u_{ig}|x_{ig}; \sigma) + x'_{igt}\beta))$ and $f_{\varepsilon}(-(\eta(u_{ig}|x_{ig}; \sigma) + x'_{igt}\beta))$, denote the quantile function of $\eta(u_{ig}|x_{ig}; \sigma)$ by $Q_{\eta}(u|x; \sigma) \equiv F_{\eta}^{-1}(u|x; \sigma)$ and by $q_{\eta}(u|x; \sigma)$

its derivative with respect to σ . Then, the score is given by

$$\frac{\partial \mathcal{L}(\theta)}{\partial \beta} = \frac{\sum_{g=1}^G \int_{[0,1]^{N_g}} P_g(z_g, u_g; \mu) \sum_{i=1}^{N_g} \sum_{t=1}^T \frac{f_{igt}}{F_{igt}(1-F_{igt})} (y_{igt} - (1 - F_{igt})) x_{igt} dC_X(u_g; \rho)}{\int_{[0,1]^{N_g}} P_g(z_g, u_g; \mu) dC_X(u_g; \rho)} \quad (14)$$

$$\frac{\partial \mathcal{L}(\theta)}{\partial \sigma} = \frac{\sum_{g=1}^G \int_{[0,1]^{N_g}} P_g(z_g, u_g; \mu) \sum_{i=1}^{N_g} \sum_{t=1}^T \frac{f_{igt}}{F_{igt}(1-F_{igt})} (y_{igt} - (1 - F_{igt})) q_\eta(u_{ig}|x_{ig}; \sigma) dC_X(u_g; \rho)}{\int_{[0,1]^{N_g}} P_g(z_g, u_g; \mu) dC_X(u_g; \rho)} \quad (15)$$

$$\frac{\partial \mathcal{L}(\theta)}{\partial \rho} = \frac{\sum_{g=1}^G \int_{[0,1]^{N_g}} P_g(z_g, u_g; \mu) \frac{\partial c_X(u_g; \rho)}{\partial \rho} \prod_{i=1}^{N_g} du_{ig}}{\int_{[0,1]^{N_g}} P_g(z_g, u_g; \mu) dC_X(u_g; \rho)} \quad (16)$$

Note that if η belongs to a scale family of distributions, *i.e.* if $\eta = \sigma \tilde{\eta}_{ig}$, where $\tilde{\eta}_{ig} \sim F_\eta(1)$, then $Q_\eta(u_{ig}) = \sigma \tilde{\eta}_{ig}$, and thus $q_\eta(u_{ig}; \sigma) = \tilde{\eta}_{ig}$. It is immediate to approximate Equations 14 and 15 using the proposed algorithm presented in this paper. Regarding Equation 16, it is more convenient to numerically evaluate the derivative, *i.e.* $\frac{\partial \mathcal{L}(\theta)}{\partial \rho} \approx \frac{\mathcal{L}(\mu, \rho + \varepsilon) - \mathcal{L}(\mu, \rho)}{\varepsilon}$. Finally, the Hessian is estimated by

$$\hat{H}(\hat{\theta}) = \frac{1}{G} \sum_{g=1}^G \frac{\partial \log(\hat{\ell}_g(z_g; \hat{\theta}))}{\partial \theta} \frac{\partial \log(\hat{\ell}_g(z_g; \hat{\theta}))}{\partial \theta'}$$

S5 Schennach and Wilhelm Test for Copulas

Consider two different parametric copulas, $C_{X,1}(u_g; \rho_1)$ and $C_{X,2}(u_g; \rho_2)$, where both ρ_1 and ρ_2 belong to the interior of their respective parameter spaces. Denote their respective likelihoods by $\ell_{1,g}(z_g; \theta_1)$ and $\ell_{2,g}(z_g; \theta_2)$, where $\theta_1 \equiv (\mu', \rho_1')'$ and $\theta_2 \equiv (\mu', \rho_2')'$ and let

$\omega_g(\hat{\epsilon}_g) = 1 + \hat{\epsilon}_G \mathbf{1}(g \text{ is even})$. $\hat{\epsilon}_G$ is chosen as suggested in Schennach and Wilhelm (2017):

$$\hat{\epsilon}_G = \left(\frac{\hat{C}_{SD}}{\hat{C}_{PL}^*} \right)^{1/3} N^{-1/6} (\ln \ln N)^{1/3}$$

where the constants \hat{C}_{SD} and \hat{C}_{PL}^* are defined in Schennach and Wilhelm (2017). Note that the choice of $\hat{\epsilon}_G$ faces a trade-off between the power and the size of the test, depending on the true model. Then, the test statistic is given by:

$$\tilde{t}_G = \frac{1}{\sqrt{G\hat{\sigma}}} \sum_{g=1}^G \omega_g(\hat{\epsilon}_G) \log(\ell_{1,g}(z_g; \hat{\theta}_1)) - \omega_g(\hat{\epsilon}_G) \log(\ell_{2,g}(z_g; \hat{\theta}_2))$$

where

$$\begin{aligned} \hat{\sigma}^2 \equiv & \frac{1}{G} \sum_{g=1}^G \left[\omega_g(\hat{\epsilon}_G) \log(\ell_{1,g}(z_g; \hat{\theta}_1)) - \omega_g(\hat{\epsilon}_G) \log(\ell_{2,g}(z_g; \hat{\theta}_2)) \right]^2 \\ & - \left[\frac{1}{G} \sum_{g=1}^G \omega_g(\hat{\epsilon}_G) \log(\ell_{1,g}(z_g; \hat{\theta}_1)) - \omega_g(\hat{\epsilon}_G) \log(\ell_{2,g}(z_g; \hat{\theta}_2)) \right]^2 \end{aligned}$$

This test has a limiting normal distribution, uniformly over the subset of distributions that satisfy the null hypothesis of equal fit (Theorem 2 in Schennach and Wilhelm (2017)). Given a size α for the test, the null hypothesis is not rejected if the test is between the $\alpha/2$ and $1 - \alpha/2$ quantiles of the normal distribution, whereas the alternative hypothesis that C_1 is better than C_2 and the other way around are not rejected if the test is below the $\alpha/2$ or above the $1 - \alpha/2$ quantiles, respectively

S6 Elliptical Copulas

Elliptical copulas (Cambanis et al., 1981) constitute one of the major parametric families of copulas, including two of the most widely used copulas, the Gaussian and the t . The algorithm proposed in Section 4 cannot be adapted to these copulas if some restrictions on the correlation structure are imposed.

Let R denote the correlation matrix of a d -variate normal distribution and Φ_R its cdf. The Gaussian copula with correlation R is given by $C(u; R) = \Phi_R(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_d))$. If R is positive definite, then it is possible to obtain the Cholesky decomposition, denoted by A . It is possible to express the Gaussian copula in terms of d independent normal distributions and the coefficients of A (Embrechts et al., 2001), where the (i, j) element is denoted by a_{ji} . Hence, it is possible to rewrite the integral that is required to evaluate the likelihood as

$$\mathcal{I} = \int_{[0,1]^d} \prod_{i=1}^d \ell_i(u_i) dC(u; R) = \int_{[0,1]^d} \prod_{i=1}^d \ell_i \left(\Phi \left(\sum_{j=1}^i a_{ji} \Phi^{-1}(v_j) \right) \right) \prod_{j=1}^d dv_j$$

The likelihood can be decomposed into d independent random variables. However, the dimensionality of the integral is not reduced as it was the case with the Archimedean copulas.

A similar reformulation of the integral for the t (or any other elliptical) copula is possible: denote by $t_{\nu, R}$ the cdf of the d -variate t distribution with ν degrees of freedom and correlation matrix R , then the t copula is given by $C(u; \nu, R) = t_{\nu, R}(t_{\nu}^{-1}(u_1), \dots, t_{\nu}^{-1}(u_d))$. Again, if R is positive definite, and following Embrechts et al. (2001), the copula can be written in terms of d independent normal variables and a χ^2 with ν degrees of freedom, and its cdf is denoted by F_{ν} . The integral \mathcal{I} is then given by

$$\mathcal{I} = \int_{[0,1]^d} \prod_{i=1}^d \ell_i(u_i) dC(u; \nu, R) = \int_{[0,1]^{d+1}} \prod_{i=1}^d \ell_i \left(t_{\nu} \left(\frac{\sqrt{\nu}}{\sqrt{F_{\nu}^{-1}(w)}} \sum_{j=1}^i a_{ji} \Phi^{-1}(v_j) \right) \right) \prod_{j=1}^d dv_j dw$$

With respect to the Gaussian copula, the only remarkable difference is the inclusion of the χ^2 , which results in an increase of the dimension of the integral from d to $d + 1$.

If one is willing to adopt a symmetric correlation among the elements of the copula, *i.e.* if all the off-diagonal elements of R were equal to ρ , then it would be possible to obtain a reduction of the dimensionality of the integral similar to the one attained for the Archimedean copulas. To see this, notice that by the properties of the normal distribution, it is possible to obtain a d -variate normal distribution with covariance function $R = (1 - \rho) I_d + \rho \iota_d \iota_d'$, where ι_d is a vector of ones. Each element is the sum of two independent random normals,

one specific to each dimension, and one common to all, with weights $\sqrt{1-\rho}$ and $\sqrt{\rho}$. Hence, when the copula is Gaussian, the integral \mathcal{I} can be rewritten as

$$\mathcal{I} = \int_0^1 \prod_{i=1}^d \left[\int_0^1 \ell_i \left(\Phi \left(\sqrt{\rho} \Phi^{-1}(z) + \sqrt{1-\rho} \Phi^{-1}(v_i) \right) \right) dv_i \right] dz$$

For the t copula a similar decomposition is feasible, but the dimensionality of the resulting integral is 3, because of the χ^2 distribution:

$$\mathcal{I} = \int_{[0,1]^2} \prod_{i=1}^d \left[\int_0^1 \ell_i \left(t_\nu \left(\frac{\sqrt{\nu}}{\sqrt{F_\nu^{-1}(w)}} \left(\sqrt{\rho} \Phi^{-1}(z) + \sqrt{1-\rho} \Phi^{-1}(v_i) \right) \right) \right) dv_i \right] dz dw$$

S7 Bernstein Copulas

Bernstein copulas are a family of multivariate copulas introduced by Sancetta and Satchell (2004). The M -th degree Bernstein polynomial is given by $P_{v,M}(u) = \binom{M}{v} u^v (1-u)^{M-v}$ for $0 \leq v \leq M \in \mathbb{N}$ and $u \in [0, 1]$. Define the map $C_B : [0, 1]^d \rightarrow [0, 1]$ as

$$C_B(u_1, \dots, u_d) = \sum_{v_1=0}^{M_1} \dots \sum_{v_d=0}^{M_d} \alpha \left(\frac{v_1}{M_1}, \dots, \frac{v_d}{M_d} \right) P_{v_1, M_1}(u_1) \dots P_{v_d, M_d}(u_d)$$

where $\alpha \left(\frac{v_1}{M_1}, \dots, \frac{v_d}{M_d} \right)$ are some constants $\forall v_j = 0, \dots, M_j, j = 1, \dots, d$. C_B is a Bernstein copula if it satisfies (Theorem 1 in Sancetta and Satchell (2004))

$$\sum_{l_1=0}^1 \dots \sum_{l_d=0}^1 (-1)^{l_1+\dots+l_d} \alpha \left(\frac{v_1+l_1}{M_1}, \dots, \frac{v_d+l_d}{M_d} \right) \geq 0$$

$$\min \left(0, \frac{v_1}{M_1} + \dots + \frac{v_d}{M_d} - (d-1) \right) \leq \alpha \left(\frac{v_1}{M_1}, \dots, \frac{v_d}{M_d} \right) \leq \min \left(\frac{v_1}{M_1}, \dots, \frac{v_d}{M_d} \right)$$

Moreover, the α parameters have to satisfy the doubly stochastic matrix condition, *i.e.* $\sum_{v_1}^M M \beta \left(\frac{v_1}{M}, \dots, \frac{v_d}{M} \right) = 1 \forall j = 1, \dots, d$, where $\beta \left(\frac{v_1}{M}, \dots, \frac{v_d}{M} \right) \equiv (M+1)^d \Delta_{1,\dots,m} \alpha \left(\frac{v_1}{M+1}, \dots, \frac{v_d}{M+1} \right)$.

This copula has a well-defined density that can be expressed in terms of Bernstein

polynomials. Its coefficients are a function of the α coefficients. It is well-known that Bernstein polynomials can uniformly approximate any continuous function on $[0, 1]^d$ if the degree of the polynomial is high enough. Consequently, one can use the Bernstein copula to approximate any continuous multivariate copula. Sancetta and Satchell (2004) showed that the Bernstein copula and its approximand converge to an arbitrary limit at a different speed. Hence, although it can capture increasing dependence as one moves to the tails, it is not the appropriate choice to model copulas with extreme tail behavior.

The total number of parameters to estimate a d -variate Bernstein copula with polynomials of degree M equals M^d , and hence is subject to the curse of dimensionality, making it impractical to work with it when the dimension of the data is large. However, implementation when the dimension is small is relatively straightforward, since it just depends on a finite and small number of parameters. This, coupled with the flexibility of the copula makes it an attractive choice when the underlying copula is unknown.

S8 Extra Results

Figure 3: Copula contour of Table 6 $\hat{\rho}$ estimates

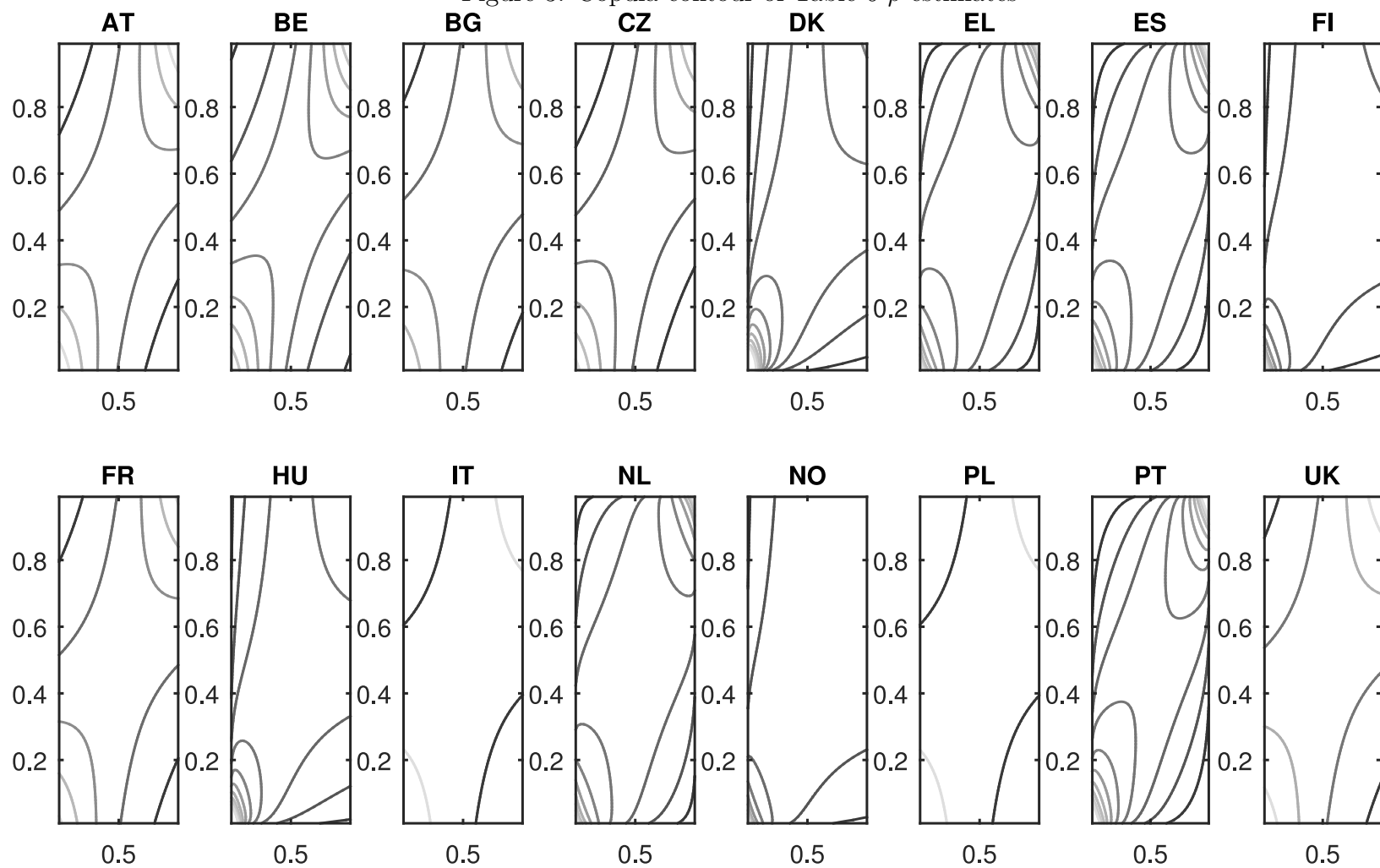


Table 8: Difference in the covariates between women married to employed and unemployed husbands

	AT	BE	BG	CZ	DK	EL	ES	FI	FR	HU	IT	NL	NO	PL	PT	UK
C5	0.12 (0.03)	0.12 (0.03)	0.03 (0.02)	0.16 (0.02)	0.01 (0.04)	0.17 (0.03)	0.08 (0.02)	0.06 (0.03)	0.15 (0.01)	0.13 (0.02)	0.18 (0.02)	0.15 (0.03)	0.08 (0.03)	0.15 (0.02)	0.10 (0.02)	0.13 (0.04)
AGE	-8.19 (0.77)	-7.69 (0.82)	-5.60 (0.69)	-9.68 (0.62)	-3.50 (0.95)	-8.36 (0.62)	-3.31 (0.50)	-5.65 (0.73)	-9.30 (0.34)	-7.46 (0.51)	-7.42 (0.42)	-8.63 (0.78)	-4.49 (0.82)	-8.97 (0.44)	-6.90 (0.64)	-5.54 (1.02)
SE	0.07 (0.04)	0.07 (0.04)	-0.06 (0.04)	0.05 (0.03)	-0.07 (0.06)	0.05 (0.04)	0.08 (0.02)	-0.02 (0.04)	0.02 (0.02)	0.08 (0.03)	0.13 (0.02)	0.00 (0.05)	-0.11 (0.04)	-0.07 (0.02)	0.04 (0.03)	-0.12 (0.05)
TE	0.09 (0.03)	0.20 (0.05)	0.17 (0.03)	0.11 (0.02)	0.09 (0.05)	0.09 (0.03)	0.17 (0.02)	0.20 (0.04)	0.25 (0.02)	0.15 (0.02)	0.12 (0.01)	0.09 (0.04)	0.30 (0.04)	0.14 (0.02)	0.11 (0.02)	0.11 (0.05)
IN	0.87 (0.69)	-1.11 (1.23)	-0.05 (0.05)	0.18 (0.07)	0.49 (2.20)	0.48 (0.36)	0.15 (0.16)	0.92 (0.92)	1.86 (1.30)	-0.03 (0.04)	0.27 (0.21)	0.84 (0.46)	3.40 (1.61)	0.19 (0.09)	0.18 (0.15)	-0.49 (0.68)

Notes: Standard errors in parentheses. C5, AGE, SE, TE, and IN respectively denote female, number of children smaller than 5 years old, age, secondary education, tertiary education, and non-labor income (expressed in thousands of euros). The estimates shown are those of Austria, Belgium, Bulgaria, Czech Republic, Denmark, Greece, Spain, Finland, Hungary, Italy, Netherlands, Norway, Poland, Portugal, and the United Kingdom. The estimates of the remaining countries (Cyprus, Estonia, Iceland, Lithuania, Luxemburg, Latvia, Malta, and Slovenia) are available upon request.

Table 9: RE estimates

	AT	BE	BG	CZ	DK	EL	ES	FI	FR	HU	IT	NL	NO	PL	PT	UK
FE	-1.63 (0.60)	-1.51 (0.50)	-0.96 (0.60)	-1.63 (0.65)	-0.98 (0.32)	-4.36 (0.44)	-4.41 (0.53)	-0.82 (0.43)	-2.35 (0.43)	-0.46 (0.64)	-4.03 (0.29)	-5.07 (0.44)	-2.09 (0.39)	-2.33 (0.34)	-3.40 (0.84)	-0.42 (0.42)
C5	-0.79 (1.00)	-0.20 (1.59)	-3.65 (1.87)	0.45 (0.71)	-0.68 (1.10)	0.98 (0.94)	0.49 (0.47)	-2.18 (0.91)	-0.29 (0.48)	-0.19 (0.68)	0.87 (0.58)	0.23 (0.43)	-1.23 (0.82)	0.70 (0.62)	-0.15 (1.10)	0.80 (0.49)
C5*FE	-0.78 (1.48)	-0.13 (2.15)	2.50 (1.74)	-4.50 (2.81)	0.79 (1.62)	-1.27 (1.02)	-1.42 (0.56)	-0.39 (2.23)	-1.23 (0.73)	-2.49 (1.44)	-1.89 (0.72)	-1.79 (1.42)	0.52 (1.01)	-1.88 (1.03)	0.19 (1.36)	-2.27 (0.62)
AGE	-0.29 (0.05)	-0.33 (0.03)	-0.17 (0.05)	-0.27 (0.03)	-0.11 (0.03)	-0.20 (0.03)	-0.15 (0.04)	-0.19 (0.06)	-0.42 (0.03)	-0.21 (0.04)	-0.20 (0.02)	-0.36 (0.03)	-0.10 (0.03)	-0.36 (0.02)	-0.26 (0.07)	-0.07 (0.04)
SE	1.16 (0.61)	1.51 (0.46)	1.71 (0.55)	2.68 (1.24)	0.50 (0.67)	-0.45 (0.51)	1.69 (0.47)	2.00 (0.96)	1.34 (0.36)	2.11 (0.40)	2.48 (0.32)	2.39 (0.55)	2.08 (0.60)	2.08 (0.43)	1.52 (0.60)	0.03 (0.29)
TE	2.33 (1.16)	3.14 (0.68)	4.81 (0.67)	4.54 (1.47)	2.36 (0.78)	3.58 (0.43)	3.62 (0.59)	3.04 (0.90)	4.05 (0.39)	4.11 (1.26)	4.69 (0.34)	4.38 (0.76)	3.42 (0.65)	6.02 (0.84)	4.80 (3.15)	0.57 (0.35)
IN	0.00 (0.01)	-0.01 (0.01)	0.04 (0.16)	0.05 (0.07)	0.00 (0.00)	0.03 (0.05)	-0.01 (0.03)	0.00 (0.01)	0.00 (0.00)	-0.01 (0.09)	0.00 (0.02)	0.03 (0.01)	0.00 (0.00)	-0.01 (0.02)	0.03 (0.40)	0.00 (0.01)
$\overline{C5}$	-2.09 (1.83)	-0.88 (1.77)	4.21 (2.42)	-1.42 (1.36)	-1.76 (1.94)	-0.28 (1.09)	-1.64 (0.95)	0.50 (2.27)	-3.38 (0.87)	0.45 (0.88)	-0.42 (1.18)	-0.09 (1.58)	0.82 (1.19)	-2.61 (1.26)	-1.71 (1.72)	-1.09 (2.02)
$\overline{C5 * FE}$	-4.01 (2.09)	-4.35 (2.38)	-7.08 (2.11)	-5.56 (2.94)	-0.25 (3.09)	-0.32 (1.30)	1.53 (0.93)	-5.70 (2.80)	-3.92 (1.01)	-6.51 (2.19)	-1.81 (1.20)	-2.61 (2.06)	-3.00 (1.40)	-4.48 (1.85)	-0.87 (3.42)	0.10 (2.42)
\overline{IN}	0.04 (0.01)	0.01 (0.02)	-0.47 (0.39)	0.05 (0.26)	-0.01 (0.01)	0.00 (0.05)	0.12 (0.23)	0.00 (0.03)	0.01 (0.00)	-0.02 (0.28)	0.12 (0.04)	-0.02 (0.01)	0.00 (0.00)	0.01 (0.04)	-0.02 (0.97)	-0.07 (0.04)
$\hat{\sigma}$	4.25 (0.53)	6.95 (0.68)	4.98 (0.48)	4.16 (0.32)	3.31 (0.33)	5.50 (0.40)	4.46 (0.23)	3.81 (0.49)	5.56 (0.22)	4.52 (0.63)	4.48 (0.21)	6.00 (0.42)	3.89 (0.33)	5.40 (0.31)	5.10 (0.86)	2.52 (0.21)
PW	73.5 (2.4)	66.9 (1.6)	76.9 (1.5)	74.7 (1.3)	86.6 (1.8)	55.7 (1.6)	55.8 (3.7)	76.0 (1.6)	71.8 (1.2)	64.7 (1.7)	55.0 (1.2)	76.3 (1.2)	81.8 (1.2)	64.9 (1.4)	65.5 (2.3)	78.2 (3.4)
$CP1$	75.4 (2.3)	69.6 (1.6)	78.8 (1.5)	75.7 (1.3)	87.0 (1.7)	58.3 (1.6)	57.5 (3.6)	77.2 (1.5)	75.7 (1.2)	66.9 (1.9)	57.7 (1.2)	77.4 (1.2)	82.3 (1.2)	68.7 (1.4)	68.4 (2.3)	78.4 (3.4)
$CP0$	63.4 (4.7)	56.6 (1.7)	70.0 (2.2)	67.5 (2.1)	83.4 (2.1)	45.8 (1.9)	48.1 (4.2)	70.0 (2.9)	56.2 (1.7)	58.6 (2.0)	43.3 (1.4)	62.8 (1.6)	75.6 (2.0)	52.5 (1.6)	54.8 (2.4)	76.6 (3.4)
N	764	604	780	1404	696	930	1906	882	3370	1422	2186	1360	1364	1848	814	640

Notes: Standard errors in parentheses. FE, C5, AGE, SE, TE, and IN respectively denote female, number of children smaller than 5 years old, age, secondary education, tertiary education, and non-labor income (expressed in thousands of euros); the σ symbol is used to denote the coefficient of the correlated random effect; PW denotes the unconditional probability that a wife is employed, $CP1$ and $CP0$ respectively denote the probability that a wife is employed conditional on her husband being employed, and conditional on her husband being unemployed; the selected model (logit/probit) for each country is the same as in Table 6; N is the sample size of each country. The estimates shown are those of Austria, Belgium, Bulgaria, Czech Republic, Denmark, Greece, Spain, Finland, Hungary, Italy, Netherlands, Norway, Poland, Portugal, and the United Kingdom. The estimates of the remaining countries (Cyprus, Estonia, Iceland, Lithuania, Luxemburg, Latvia, Malta, and Slovenia) are available upon request.

Table 10: Bernstein copula CBRE estimates

	AT	BE	BG	CZ	DK	EL	ES	FI	FR	HU	IT	NL	NO	PL	PT	UK
FE	-1.97 (0.55)	-3.42 (0.90)	-0.93 (0.48)	-1.56 (0.37)	-1.20 (0.44)	-4.96 (0.61)	-4.36 (0.33)	-0.76 (0.43)	-1.74 (0.33)	-0.50 (0.33)	-4.14 (0.31)	-5.33 (0.69)	-2.09 (0.37)	-2.57 (0.40)	-2.95 (0.54)	-1.10 (0.52)
C5	-0.79 (0.92)	0.11 (0.79)	-3.65 (2.94)	0.36 (1.77)	-0.68 (1.16)	1.00 (0.67)	0.49 (0.38)	-2.13 (0.67)	-0.35 (0.55)	-0.20 (0.51)	0.89 (0.46)	0.49 (3.81)	-1.22 (0.64)	0.71 (0.51)	-0.10 (1.32)	1.52 (2.21)
C5*FE	-0.84 (1.17)	-0.39 (1.35)	2.57 (4.11)	-4.54 (1.88)	0.83 (1.70)	-1.25 (0.91)	-1.42 (0.54)	-0.47 (0.89)	-1.24 (0.67)	-2.58 (0.54)	-1.90 (0.59)	-1.93 (3.88)	0.51 (0.76)	-1.84 (0.61)	0.05 (1.55)	-4.21 (2.40)
AGE	-0.30 (0.04)	-0.47 (0.07)	-0.20 (0.03)	-0.28 (0.02)	-0.14 (0.03)	-0.25 (0.03)	-0.15 (0.02)	-0.21 (0.02)	-0.43 (0.02)	-0.23 (0.02)	-0.21 (0.02)	-0.41 (0.05)	-0.12 (0.02)	-0.36 (0.03)	-0.29 (0.03)	-0.12 (0.04)
SE	0.95 (0.41)	1.38 (0.45)	1.73 (0.61)	2.67 (0.63)	0.44 (0.43)	-0.16 (0.31)	2.02 (0.29)	1.88 (0.53)	1.18 (0.26)	1.85 (0.28)	2.54 (0.29)	2.18 (0.65)	2.00 (0.40)	2.25 (0.69)	1.71 (0.49)	-0.04 (0.38)
TE	1.87 (0.62)	2.52 (0.48)	4.46 (0.80)	4.77 (0.81)	2.25 (0.50)	3.85 (0.55)	3.87 (0.31)	2.73 (0.60)	4.00 (0.39)	3.85 (0.42)	4.79 (0.46)	4.33 (0.64)	3.33 (0.43)	5.86 (0.80)	4.46 (0.83)	0.83 (0.43)
IN	0.00 (0.01)	-0.01 (0.01)	0.04 (0.65)	0.04 (0.05)	0.00 (0.01)	0.03 (0.05)	-0.01 (0.03)	0.00 (0.02)	0.00 (0.00)	-0.01 (0.10)	0.00 (0.02)	0.03 (0.02)	0.00 (0.01)	-0.01 (0.18)	0.03 (0.11)	0.00 (0.04)
$\overline{C5}$	-1.58 (1.52)	-3.94 (2.36)	3.60 (3.46)	-2.49 (1.85)	-2.13 (1.61)	-1.27 (1.31)	-1.53 (0.74)	-0.07 (1.17)	-2.67 (0.83)	1.29 (1.16)	-0.50 (0.79)	-3.46 (4.34)	0.06 (1.13)	-2.29 (1.07)	-0.98 (1.96)	-1.41 (2.40)
$\overline{C5 * FE}$	-4.50 (1.74)	-3.26 (2.48)	-8.63 (4.82)	-4.98 (2.13)	-0.06 (2.04)	-1.43 (1.46)	0.99 (0.88)	-5.90 (1.51)	-4.64 (1.07)	-8.07 (1.41)	-1.88 (0.93)	-1.25 (4.37)	-2.73 (1.25)	-4.96 (1.20)	-2.46 (2.18)	0.09 (2.63)
\overline{IN}	0.05 (0.03)	0.00 (0.06)	-0.48 (1.41)	0.01 (0.13)	0.00 (0.01)	-0.03 (0.08)	0.10 (0.07)	0.00 (0.05)	0.01 (0.01)	0.07 (0.58)	0.12 (0.04)	-0.03 (0.03)	0.00 (0.01)	-0.05 (0.12)	0.11 (0.22)	-0.11 (0.09)
$\hat{\sigma}$	4.82 (0.36)	7.40 (0.78)	5.22 (0.38)	4.92 (0.29)	3.57 (0.29)	5.76 (0.41)	4.78 (0.21)	4.19 (0.31)	6.69 (0.26)	4.98 (0.25)	4.82 (0.20)	6.81 (0.50)	4.15 (0.25)	6.24 (0.32)	5.55 (0.39)	4.57 (0.40)
Model	Log	Log	Log	Log	Log	Log	Log	Log	Log	Log	Log	Log	Log	Log	Log	Log
Order	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	2
τ	0.21	0.22	0.16	0.21	0.22	0.16	0.11	0.16	0.17	0.20	0.10	0.08	0.16	0.11	0.14	0.17
N	764	604	780	1404	696	930	1906	882	3370	1422	2186	1360	1364	1848	814	640

Notes: Standard errors in parentheses. FE, C5, AGE, SE, TE, and IN respectively denote female, number of children smaller than 5 years old, age, secondary education, tertiary education, and non-labor income (expressed in thousands of euros); the σ symbol is used to denote the coefficient of the correlated random effect; the contour plots of the copula estimates are shown in Figure 4; Model denotes the best fitting binary choice model: logit (Log) or probit (Pro); N is the sample size of each country. The estimates shown are those of Austria, Belgium, Bulgaria, Czech Republic, Denmark, Greece, Spain, Finland, Hungary, Italy, Netherlands, Norway, Poland, Portugal, and the United Kingdom. The estimates of the remaining countries (Cyprus, Estonia, Iceland, Lithuania, Luxemburg, Latvia, Malta, and Slovenia) are available upon request.

Figure 4: Contour of estimated Bernstein copulas

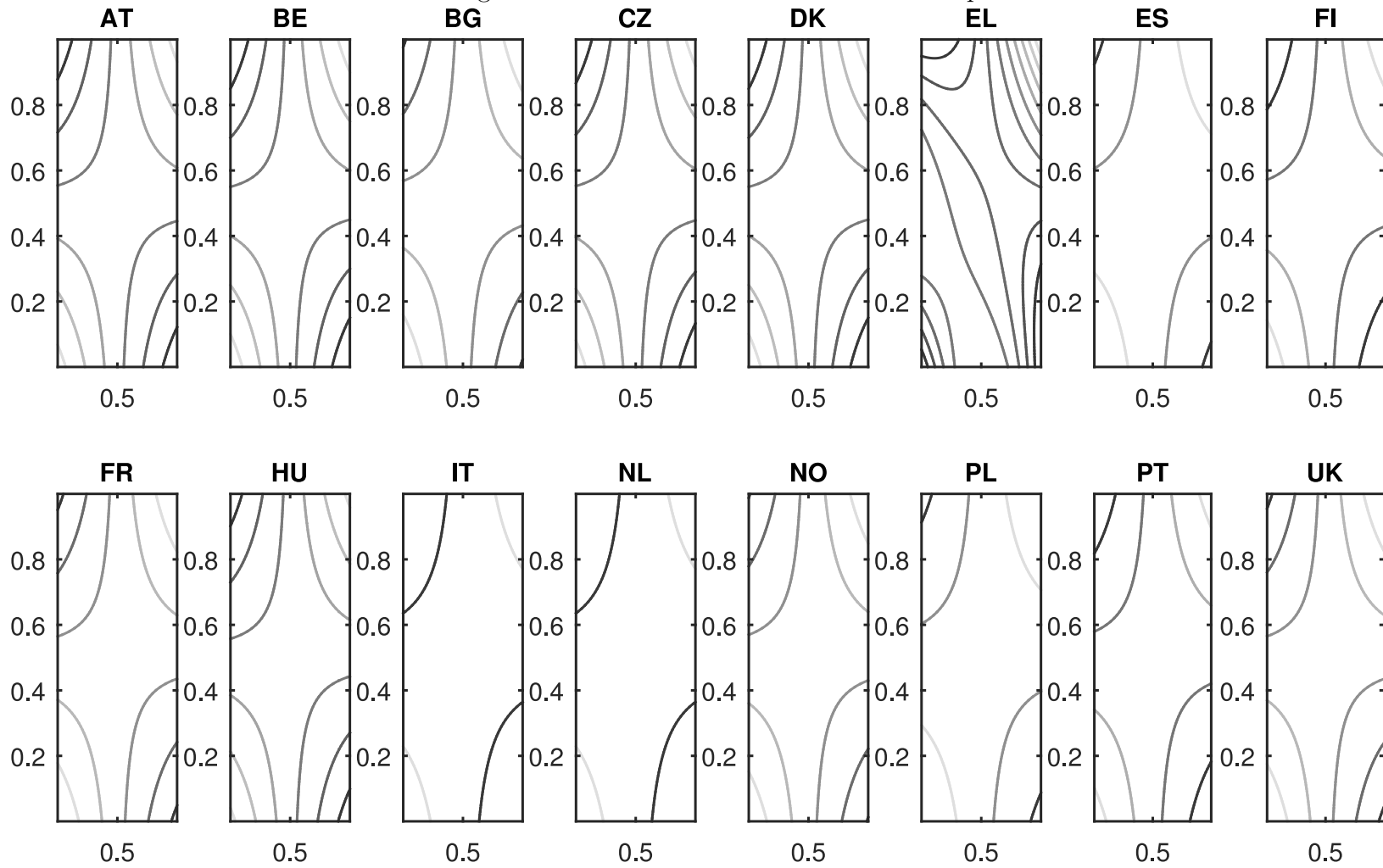


Table 11: Counterfactuals with Bernstein copulas

	AT	BE	BG	CZ	DK	EL	ES	FI	FR	HU	IT	NL	NO	PL	PT	UK
PW_ρ	72.6 (1.9)	67.0 (2.3)	76.9 (1.8)	74.3 (1.3)	86.4 (1.4)	54.2 (1.9)	55.3 (1.3)	75.9 (1.6)	72.8 (0.9)	65.1 (1.4)	54.6 (1.2)	77.2 (1.3)	82.7 (1.2)	63.7 (1.3)	65.0 (1.9)	77.8 (2.0)
$CP1_\rho$	76.3 (1.9)	73.7 (2.4)	80.8 (1.7)	76.7 (1.3)	87.8 (1.4)	57.9 (2.1)	58.6 (1.4)	78.3 (1.7)	78.3 (0.9)	70.4 (1.5)	58.8 (1.2)	78.7 (1.3)	83.9 (1.2)	68.9 (1.3)	70.3 (2.0)	79.4 (1.9)
$CP0_\rho$	51.4 (4.3)	41.9 (4.8)	61.5 (3.9)	55.7 (3.5)	74.2 (3.9)	37.4 (3.4)	41.2 (2.5)	63.1 (3.5)	49.5 (2.0)	49.7 (2.5)	37.1 (2.2)	58.9 (4.9)	68.1 (3.8)	46.6 (2.5)	46.1 (3.8)	66.7 (4.9)
$\Delta p(y_{2ct} x_g; \hat{\theta}_{CBRE})$	24.9 (4.3)	31.8 (5.0)	19.3 (3.9)	21.0 (3.5)	13.5 (3.9)	20.5 (3.8)	17.4 (2.6)	15.2 (3.5)	28.8 (2.1)	20.7 (2.7)	21.7 (2.3)	19.8 (4.9)	15.7 (3.7)	22.3 (2.7)	24.2 (4.0)	12.7 (4.8)
Endowment effect	12.1 (3.8)	13.1 (1.1)	8.8 (1.9)	8.2 (1.9)	3.6 (0.8)	12.5 (1.2)	9.4 (0.9)	7.3 (2.7)	19.5 (1.1)	8.3 (1.6)	14.5 (1.0)	14.6 (1.0)	6.7 (1.4)	16.1 (1.0)	13.6 (1.0)	1.8 (0.7)
Homophily effect	12.8 (5.7)	18.8 (5.1)	10.6 (4.3)	12.8 (4.0)	10.0 (4.0)	7.9 (3.9)	8.0 (2.8)	8.0 (4.4)	9.3 (2.4)	12.5 (3.2)	7.3 (2.5)	5.2 (5.0)	9.0 (4.0)	6.2 (2.9)	10.6 (4.1)	10.9 (4.9)
P_ρ	77.6 (2.6)	65.2 (3.4)	75.2 (2.9)	82.6 (1.8)	90.1 (1.7)	64.5 (3.3)	66.2 (1.9)	80.8 (2.4)	72.5 (1.4)	62.2 (2.3)	65.0 (1.8)	89.9 (1.6)	91.9 (1.2)	64.8 (1.9)	65.3 (2.9)	84.7 (2.9)
P_I	83.0 (1.9)	72.3 (2.7)	80.2 (2.0)	86.9 (1.2)	93.3 (1.0)	66.0 (2.4)	69.5 (1.5)	84.5 (1.6)	77.5 (1.0)	68.8 (1.8)	68.0 (1.5)	91.2 (1.0)	94.1 (0.7)	68.3 (1.5)	70.0 (2.2)	88.7 (1.8)
Δ_P	-5.4 (1.6)	-7.1 (2.1)	-5.0 (1.8)	-4.3 (1.1)	-3.2 (1.3)	-1.5 (2.1)	-3.3 (1.0)	-3.8 (1.5)	-5.0 (0.8)	-6.6 (1.3)	-2.9 (0.9)	-1.4 (1.1)	-2.2 (0.9)	-3.5 (1.1)	-4.7 (1.7)	-4.1 (1.8)
N	764	604	780	1404	696	930	1906	882	3370	1422	2186	1360	1364	1848	814	640

Notes: Standard errors of the estimated probabilities were computed using the delta method. PW_ρ denotes the unconditional probability that a wife is employed, $CP1_\rho$ and $CP0_\rho$ respectively denote the probability that a wife is employed conditional on her husband being employed, and conditional on her husband being unemployed; $\Delta p(y_{2gt}|x_g; \hat{\theta}_{CBRE})$ and the homophily and endowment effects are defined in the text; P_ρ and P_I respectively denote the probability (in %) that at least one member of the couple was employed in every period when the parameter of the copula is the estimated one and when the copula is independent, and Δ_P denotes the difference between the two; N is the sample size of each country. The estimates shown are those of Austria, Belgium, Bulgaria, Czech Republic, Denmark, Greece, Spain, Finland, Hungary, Italy, Netherlands, Norway, Poland, Portugal, and the United Kingdom. The estimates of the remaining countries (Cyprus, Estonia, Iceland, Lithuania, Luxemburg, Latvia, Malta, and Slovenia) are available upon request.

Table 12: Akaike Information Criterion across specifications

			AT	BE	BG	CZ	DK	EL	ES	FI	FR	HU	IT	NL	NO	PL	PT	UK
RE	-	Lo	1932.8	1385.0	2014.7	1278.7	3068.2	1506.7	1381.9	2629.4	5461.7	2283.3	7507.3	4096.1	707.4	6303.5	2104.4	5632.3
RE	-	Pr	1933.7	1388.8	2017.5	1277.3	3073.1	1510.9	1383.3	2629.2	5463.4	2286.1	7522.0	4098.7	708.8	6306.6	2102.2	5637.4
CRE	-	Lo	1909.5	1372.3	2012.4	1281.8	3023.1	1509.5	1378.1	2634.8	5457.4	2264.8	7418.1	4052.0	711.4	6290.9	2107.2	5611.1
CRE	-	Pr	1910.5	1370.9	2015.7	1280.0	3020.1	1513.6	1377.4	2634.1	5461.0	2264.9	7444.2	4049.8	712.5	6293.0	2104.4	5615.6
CBRE	Cl	Lo	1915.4	1365.5	2016.2	1280.2	3057.5	1483.9	1385.6	2630.8	5443.0	2278.2	7440.4	4065.8	710.6	6300.2	2109.5	5611.8
CBRE	Fr	Lo	1915.3	1359.7	2012.6	1281.0	3054.6	1487.6	1385.6	2629.0	5442.9	2279.8	7438.5	4066.9	709.9	6294.9	2108.8	5608.1
CBRE	Ga	Lo	1915.8	1361.6	2014.5	1280.6	3056.7	1485.9	1385.6	2628.4	5441.0	2280.4	7437.2	4066.9	710.1	6296.5	2109.0	5609.1
CBRE	Cl	Pr	1918.5	1365.9	2018.9	1280.8	3064.2	1488.2	1386.2	2631.2	5446.9	2281.4	7456.2	4069.0	710.8	6304.2	2109.4	5616.4
CBRE	Fr	Pr	1918.7	1360.1	2015.4	1281.3	3061.5	1492.8	1386.2	2629.2	5446.7	2283.2	7454.2	4070.3	710.0	6301.7	2108.6	5612.8
CBRE	Ga	Pr	1919.2	1362.0	2017.2	1281.1	3063.6	1490.4	1386.2	2628.7	5444.9	2283.7	7452.7	4070.3	710.3	6300.5	2108.9	5613.7
CBCRE	Cl	Lo	1896.1	1357.9	2011.1	1280.9	3014.8	1487.8	1378.0	2630.3	5444.7	2257.7	7368.4	4017.4	713.8	6288.5	2112.3	5595.2
CBCRE	Fr	Lo	1895.4	1352.8	2008.1	1281.4	3012.7	1491.7	1378.1	2628.6	5444.6	2258.4	7360.1	4017.7	712.9	6283.7	2111.7	5591.6
CBCRE	Ga	Lo	1895.7	1354.7	2009.6	1281.0	3013.7	1489.6	1378.0	2628.2	5442.8	2258.9	7363.5	4017.5	713.4	6285.1	2111.8	5592.9
CBCRE	Cl	Pr	1898.4	1358.3	2013.4	1281.5	3014.8	1492.0	1377.9	2630.8	5448.6	2259.0	7389.0	4018.6	714.1	6292.3	2112.3	5599.6
CBCRE	Fr	Pr	1898.0	1353.2	2010.2	1282.0	3013.0	1496.0	1377.9	2629.0	5448.6	2259.7	7380.2	4018.8	713.1	6287.6	2111.6	5596.5
CBCRE	Ga	Pr	1898.1	1355.3	2011.9	1281.6	3013.7	1493.9	1377.9	2628.6	5446.7	2260.2	7383.5	4018.6	713.6	6288.8	2111.7	5597.3
CBRE	B2	Lo	1916.6	1361.6	2013.2	1280.8	3064.0	1492.9	1385.7	2627.7	5443.2	2282.4	7440.2	4069.7	709.4	6293.5	2108.9	5606.2
CBRE	B3	Lo	1920.9	1365.8	2017.5	1285.9	3068.4	1495.0	1390.6	2623.8	5445.9	2283.9	7443.1	4072.2	715.1	6298.2	2113.1	5608.9
CBRE	B4	Lo	1930.7	1375.4	2026.4	1295.0	3077.5	1503.3	1399.5	2632.6	5451.4	2291.3	7450.6	4080.8	724.6	6303.8	2122.4	5614.5
CBRE	B2	Pr	1920.7	1362.2	2016.5	1281.3	3071.4	1498.1	1386.7	2628.5	5449.0	2287.0	7456.8	4074.1	709.8	6300.0	2109.4	5611.7
CBRE	B3	Pr	1926.1	1366.3	2021.5	1286.4	3075.5	1500.5	1391.8	2624.7	5452.2	2288.5	7458.1	4076.5	715.7	6304.2	2113.9	5614.2
CBRE	B4	Pr	1934.8	1375.9	2029.6	1295.8	3084.5	1509.1	1401.1	2633.7	5457.5	2295.6	7465.1	4083.3	725.2	6309.7	2123.1	5619.6
CBCRE	B2	Lo	1896.7	1354.8	2008.6	1281.4	3024.4	1496.3	1378.1	2627.7	5445.2	2261.4	7364.2	4021.1	712.5	6282.7	2111.9	5590.9
CBCRE	B3	Lo	1901.7	1359.6	2012.9	1286.7	3029.8	1498.5	1382.8	2624.3	5447.9	2264.4	7366.1	4023.7	718.2	6287.1	2115.9	5593.1
CBCRE	B4	Lo	1911.1	1369.0	2021.8	1296.0	3038.7	1507.1	1391.6	2633.0	5453.6	2272.5	7373.9	4031.9	727.6	6292.7	2125.0	5600.7
CBCRE	B2	Pr	1900.1	1355.4	2011.6	1282.2	3025.4	1501.9	1378.4	2628.8	5451.2	2264.1	7385.8	4023.6	713.2	6289.5	2112.4	5596.6
CBCRE	B3	Pr	1905.6	1360.3	2016.7	1287.5	3030.9	1504.4	1383.1	2625.4	5454.5	2266.8	7388.0	4026.0	719.0	6293.3	2116.8	5598.8
CBCRE	B4	Pr	1914.3	1369.7	2025.1	1297.0	3039.7	1513.1	1391.9	2634.4	5460.0	2275.1	7394.7	4034.0	728.5	6298.2	2125.8	5606.5
N			764	604	780	568	1404	696	498	930	1906	882	3370	1422	386	2186	774	2294

Notes: CBCRE and CRE respectively denote the CBRE and RE estimators with correlated random effects; Lo and Pr stand for logit and probit; Cl, Fr, Ga, and Bx stand for independent, Clayton, Frank, Gaussian, and Bernstein copula of order x; N is the sample size of each country. The estimates shown are those of Austria, Belgium, Bulgaria, Czech Republic, Denmark, Greece, Spain, Finland, Hungary, Italy, Netherlands, Norway, Poland, Portugal, and the United Kingdom. The estimates of the remaining countries (Cyprus, Estonia, Iceland, Lithuania, Luxemburg, Latvia, Malta, and Slovenia) are available upon request.

Table 13: Bayesian Information Criterion across specifications

			AT	BE	BG	CZ	DK	EL	ES	FI	FR	HU	IT	NL	NO	PL	PT	UK
RE	-	Lo	2005.1	1454.5	2087.2	1347.5	3147.8	1577.8	1449.1	2704.1	5544.9	2357.3	7597.4	4175.8	771.5	6388.4	2176.9	5717.8
RE	-	Pr	2006.0	1458.3	2090.1	1346.1	3152.7	1582.1	1450.5	2703.9	5546.6	2360.2	7612.1	4178.4	773.0	6391.5	2174.7	5722.9
CRE	-	Lo	1999.9	1459.2	2103.1	1367.7	3122.6	1598.4	1462.0	2728.1	5561.5	2357.3	7530.7	4151.7	791.5	6397.1	2197.7	5718.0
CRE	-	Pr	2000.9	1457.8	2106.3	1365.9	3119.6	1602.6	1461.3	2727.4	5565.1	2357.4	7556.8	4149.5	792.6	6399.2	2195.0	5722.4
CBRE	Cl	Lo	1993.8	1440.8	2094.8	1354.6	3143.7	1561.0	1458.4	2711.7	5533.2	2358.4	7538.0	4152.2	780.0	6392.1	2188.0	5704.4
CBRE	Fr	Lo	1993.7	1435.0	2091.2	1355.5	3140.8	1564.7	1458.3	2709.8	5533.2	2360.0	7536.1	4153.3	779.3	6386.8	2187.3	5700.7
CBRE	Ga	Lo	1994.1	1436.9	2093.1	1355.1	3142.9	1563.0	1458.4	2709.3	5531.2	2360.6	7534.8	4153.3	779.6	6388.5	2187.4	5701.7
CBRE	Cl	Pr	1996.9	1441.2	2097.5	1355.3	3150.5	1565.3	1459.0	2712.1	5537.1	2361.5	7553.9	4155.4	780.3	6396.2	2187.9	5709.0
CBRE	Fr	Pr	1997.0	1435.3	2094.0	1355.8	3147.7	1569.9	1458.9	2710.1	5536.9	2363.4	7551.9	4156.7	779.4	6393.7	2187.1	5705.4
CBRE	Ga	Pr	1997.5	1437.2	2095.7	1355.6	3149.8	1567.5	1459.0	2709.6	5535.1	2363.9	7550.3	4156.7	779.8	6392.5	2187.4	5706.3
CBCRE	Cl	Lo	1992.5	1450.5	2107.8	1372.6	3120.9	1582.8	1467.6	2729.8	5555.8	2356.4	7488.5	4123.7	799.2	6401.7	2208.9	5709.2
CBCRE	Fr	Lo	1991.8	1445.5	2104.8	1373.0	3118.8	1586.6	1467.6	2728.1	5555.6	2357.1	7480.3	4124.0	798.4	6396.9	2208.3	5705.6
CBCRE	Ga	Lo	1992.1	1447.3	2106.3	1372.7	3119.8	1584.5	1467.5	2727.7	5553.8	2357.6	7483.7	4123.8	798.8	6398.3	2208.4	5706.9
CBCRE	Cl	Pr	1994.8	1450.9	2110.2	1373.2	3120.9	1586.9	1467.4	2730.3	5559.7	2357.7	7509.2	4124.9	799.6	6405.6	2208.9	5713.6
CBCRE	Fr	Pr	1994.4	1445.9	2106.9	1373.6	3119.1	1590.9	1467.4	2728.5	5559.6	2358.4	7500.3	4125.1	798.6	6400.8	2208.2	5710.5
CBCRE	Ga	Pr	1994.5	1448.0	2108.6	1373.3	3119.9	1588.9	1467.4	2728.2	5557.7	2358.9	7503.6	4125.0	799.1	6402.0	2208.4	5711.3
CBRE	B2	Lo	1994.9	1436.9	2091.8	1355.3	3150.3	1570.0	1458.5	2708.6	5533.4	2362.6	7537.8	4156.1	778.8	6385.5	2187.4	5698.8
CBRE	B3	Lo	2017.3	1458.5	2114.2	1377.5	3174.5	1589.9	1480.1	2723.3	5556.9	2382.6	7563.2	4178.6	800.6	6411.4	2209.7	5722.9
CBRE	B4	Lo	2057.2	1497.0	2153.3	1415.3	3216.8	1627.9	1517.1	2763.3	5597.2	2420.8	7608.3	4220.4	836.8	6452.4	2249.2	5764.2
CBRE	B2	Pr	1999.0	1437.4	2095.1	1355.8	3157.6	1575.2	1459.4	2709.4	5539.2	2367.2	7554.5	4160.5	779.3	6391.9	2187.8	5704.3
CBRE	B3	Pr	2022.5	1459.0	2118.2	1378.0	3181.6	1595.4	1481.4	2724.3	5563.2	2387.2	7578.3	4182.8	801.2	6417.5	2210.5	5728.2
CBRE	B4	Pr	2061.3	1497.5	2156.6	1416.1	3223.8	1633.7	1518.6	2764.3	5603.2	2425.1	7622.7	4222.9	837.4	6458.3	2249.9	5769.2
CBCRE	B2	Lo	1993.1	1447.4	2105.3	1373.0	3130.5	1591.2	1467.6	2727.3	5556.3	2360.1	7484.3	4127.4	797.9	6395.9	2208.5	5704.8
CBCRE	B3	Lo	2016.2	1469.7	2127.7	1395.5	3155.8	1611.2	1489.2	2742.5	5579.7	2381.6	7508.8	4149.9	819.7	6421.5	2230.6	5728.5
CBCRE	B4	Lo	2055.7	1508.0	2166.9	1433.5	3197.9	1649.5	1525.9	2782.3	5620.1	2420.6	7554.1	4191.5	855.8	6462.5	2269.9	5771.7
CBCRE	B2	Pr	1996.5	1448.1	2108.3	1373.9	3131.5	1596.8	1467.9	2728.4	5562.3	2362.8	7506.0	4129.9	798.6	6402.7	2209.0	5710.6
CBCRE	B3	Pr	2020.1	1470.3	2131.5	1396.3	3156.9	1617.1	1489.4	2743.6	5586.3	2384.0	7530.7	4152.3	820.5	6427.7	2231.5	5734.1
CBCRE	B4	Pr	2058.9	1508.7	2170.1	1434.5	3198.9	1655.5	1526.3	2783.7	5626.6	2423.1	7574.9	4193.5	856.7	6468.1	2270.7	5777.5
N			764	604	780	568	1404	696	498	930	1906	882	3370	1422	386	2186	774	2294

Notes: CBCRE and CRE respectively denote the CBRE and RE estimators with correlated random effects; Lo and Pr stand for logit and probit; Cl, Fr, Ga, and Bx stand for independent, Clayton, Frank, Gaussian, and Bernstein copula of order x; N is the sample size of each country. The estimates shown are those of Austria, Belgium, Bulgaria, Czech Republic, Denmark, Greece, Spain, Finland, Hungary, Italy, Netherlands, Norway, Poland, Portugal, and the United Kingdom. The estimates of the remaining countries (Cyprus, Estonia, Iceland, Lithuania, Luxemburg, Latvia, Malta, and Slovenia) are available upon request.

Table 14: 10-fold cross validated log-likelihood value across specifications

			AT	BE	BG	CZ	DK	EL	ES	FI	FR	HU	IT	NL	NO	PL	PT	UK
RE	-	Lo	-974.3	-719.5	-1011.7	-644.0	-1542.8	-756.5	-691.3	-1316.7	-2732.6	-1147.6	-3762.6	-2059.0	-354.2	-3153.0	-1054.2	-2827.6
RE	-	Pr	-973.1	-743.5	-1014.0	-642.9	-1543.5	-758.1	-692.2	-1316.7	-2732.8	-1149.3	-3770.5	-2057.5	-357.3	-3155.2	-1049.6	-2826.6
CRE	-	Lo	-986.1	-699.7	-1008.7	-654.4	-1522.0	-765.3	-696.2	-1329.3	-2730.9	-1138.4	-3718.4	-2031.1	-358.9	-3150.8	-1067.1	-2817.4
CRE	-	Pr	-988.7	-699.6	-1010.4	-651.4	-1515.2	-766.2	-694.6	-1326.3	-2732.0	-1137.7	-3738.5	-2028.1	-360.9	-3150.1	-1080.1	-2820.4
CBRE	Cl	Lo	-960.2	-701.7	-1008.4	-641.2	-1533.0	-744.2	-691.9	-1317.0	-2722.9	-1144.0	-3725.4	-2037.5	-355.8	-3151.5	-1054.4	-2805.9
CBRE	Fr	Lo	-959.6	-696.9	-1005.8	-642.1	-1527.8	-739.8	-691.3	-1313.6	-2723.5	-1145.2	-3724.1	-2036.6	-356.4	-3147.8	-1053.3	-2802.7
CBRE	Ga	Lo	-960.8	-698.4	-1007.4	-641.7	-1532.7	-745.0	-691.5	-1315.6	-2722.1	-1146.2	-3723.6	-2037.7	-356.3	-3149.7	-1054.1	-2804.5
CBRE	Cl	Pr	-962.0	-703.5	-1009.8	-641.8	-1537.0	-746.1	-692.9	-1317.3	-2724.9	-1145.7	-3734.9	-2039.5	-357.8	-3153.9	-1054.5	-2808.2
CBRE	Fr	Pr	-962.2	-696.9	-1007.4	-642.1	-1533.1	-736.3	-692.2	-1309.8	-2720.5	-1145.8	-3733.3	-2039.2	-356.5	-3147.9	-1053.6	-2804.2
CBRE	Ga	Pr	-962.7	-701.3	-1009.0	-642.1	-1536.8	-747.1	-692.5	-1315.9	-2724.2	-1148.0	-3733.3	-2040.0	-357.4	-3152.0	-1053.9	-2806.6
CBCRE	Cl	Lo	-970.8	-697.9	-1004.8	-645.2	-1511.3	-751.6	-690.1	-1316.8	-2723.6	-1133.8	-3689.3	-2012.7	-358.7	-3146.6	-1065.1	-2803.0
CBCRE	Fr	Lo	-970.6	-694.5	-1003.3	-645.8	-1499.4	-750.8	-689.6	-1315.0	-2724.1	-1133.0	-3683.9	-2012.2	-359.4	-3144.2	-1063.3	-2799.4
CBCRE	Ga	Lo	-969.8	-695.7	-1004.2	-645.0	-1511.2	-751.8	-690.0	-1315.4	-2723.0	-1135.0	-3686.3	-2012.6	-358.9	-3144.9	-1064.5	-2800.3
CBCRE	Cl	Pr	-979.5	-700.1	-1006.5	-645.7	-1511.4	-753.7	-690.4	-1316.3	-2725.8	-1134.5	-3704.2	-2013.8	-359.9	-3148.7	-1072.3	-2805.1
CBCRE	Fr	Pr	-979.3	-697.5	-1004.0	-646.6	-1499.7	-743.3	-689.6	-1315.3	-2725.7	-1133.6	-3699.7	-2012.9	-359.6	-3145.5	-1073.4	-2801.7
CBCRE	Ga	Pr	-979.6	-698.2	-1005.6	-645.9	-1511.0	-753.9	-690.2	-1316.3	-2725.1	-1135.7	-3700.8	-2013.2	-360.5	-3146.9	-1072.3	-2802.5
CBRE	B2	Lo	-960.6	-697.5	-1006.4	-641.9	-1539.8	-746.4	-691.3	-1315.3	-2723.2	-1147.0	-3725.4	-2038.5	-355.7	-3148.1	-1053.9	-2802.8
CBRE	B3	Lo	-960.2	-697.3	-1005.0	-642.5	-1539.3	-745.2	-692.6	-1310.4	-2721.7	-1143.1	-3722.5	-2036.4	-355.9	-3149.4	-1052.8	-2802.1
CBRE	B4	Lo	-960.2	-698.7	-1004.9	-642.6	-1538.7	-745.3	-691.2	-1311.0	-2718.3	-1142.2	-3722.6	-2036.0	-355.9	-3145.4	-1053.5	-2799.4
CBRE	B2	Pr	-963.1	-699.6	-1008.2	-641.8	-1543.5	-748.0	-692.7	-1315.5	-2725.6	-1148.6	-3733.9	-2040.3	-355.6	-3150.5	-1053.8	-2804.8
CBRE	B3	Pr	-964.2	-699.1	-1007.0	-642.8	-1542.8	-746.9	-694.3	-1310.7	-2723.9	-1145.5	-3731.5	-2038.0	-356.3	-3151.8	-1053.3	-2803.9
CBRE	B4	Pr	-962.3	-700.9	-1005.9	-642.8	-1542.3	-746.9	-693.3	-1311.7	-2720.0	-1143.5	-3728.7	-2036.4	-356.3	-3147.7	-1053.6	-2800.8
CBCRE	B2	Lo	-972.6	-695.6	-1003.9	-645.1	-1523.4	-753.4	-690.1	-1315.8	-2724.2	-1136.4	-3685.5	-2014.4	-358.1	-3143.9	-1064.5	-2799.6
CBCRE	B3	Lo	-972.2	-695.5	-1002.2	-646.3	-1523.5	-752.6	-691.1	-1310.9	-2722.9	-1134.8	-3682.6	-2012.2	-358.7	-3145.3	-1062.3	-2798.7
CBCRE	B4	Lo	-972.9	-696.6	-1002.2	-646.4	-1522.0	-753.1	-689.4	-1311.2	-2720.0	-1135.2	-3683.3	-2011.1	-358.6	-3142.0	-1063.6	-2797.8
CBCRE	B2	Pr	-1636.7	-698.0	-1004.3	-645.4	-1521.8	-756.3	-690.4	-1316.6	-2727.7	-1136.5	-3698.5	-2015.5	-358.8	-3145.2	-1074.5	-2800.7
CBCRE	B3	Pr	-1636.7	-697.8	-1004.2	-646.9	-1522.4	-755.7	-690.5	-1311.4	-2726.3	-1136.9	-3696.4	-2013.3	-359.1	-3146.4	-1079.1	-2800.0
CBCRE	B4	Pr	-1634.7	-698.5	-1002.9	-646.9	-1520.8	-756.8	-689.7	-1312.6	-2722.9	-1135.3	-3694.7	-2012.4	-359.5	-3142.8	-1079.1	-2800.1
N			764	604	780	568	1404	696	498	930	1906	882	3370	1422	386	2186	774	2294

Notes: CBCRE and CRE respectively denote the CBRE and RE estimators with correlated random effects; Lo and Pr stand for logit and probit; Cl, Fr, Ga, and Bx stand for independent, Clayton, Frank, Gaussian, and Bernstein copula of order x; N is the sample size of each country. The estimates shown are those of Austria, Belgium, Bulgaria, Czech Republic, Denmark, Greece, Spain, Finland, Hungary, Italy, Netherlands, Norway, Poland, Portugal, and the United Kingdom. The estimates of the remaining countries (Cyprus, Estonia, Iceland, Lithuania, Luxemburg, Latvia, Malta, and Slovenia) are available upon request.

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