

# Supplementary Materials for Bayesian spatial clustering of extremal behaviour for hydrological variables

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## **Abstract**

This supplementary material provides a graphical representation of our model and additional plots for the simulation studies and applications considered in the main paper.

# 1 Average rainfall in South Norway

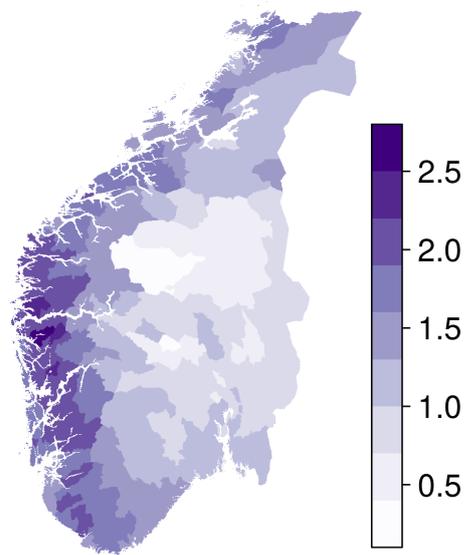


Figure 1: Average daily amount of precipitation (mm) on log-scale for 1997-2006 across the 343 municipalities in South Norway.

## 2 Illustration of the Bayesian cluster model

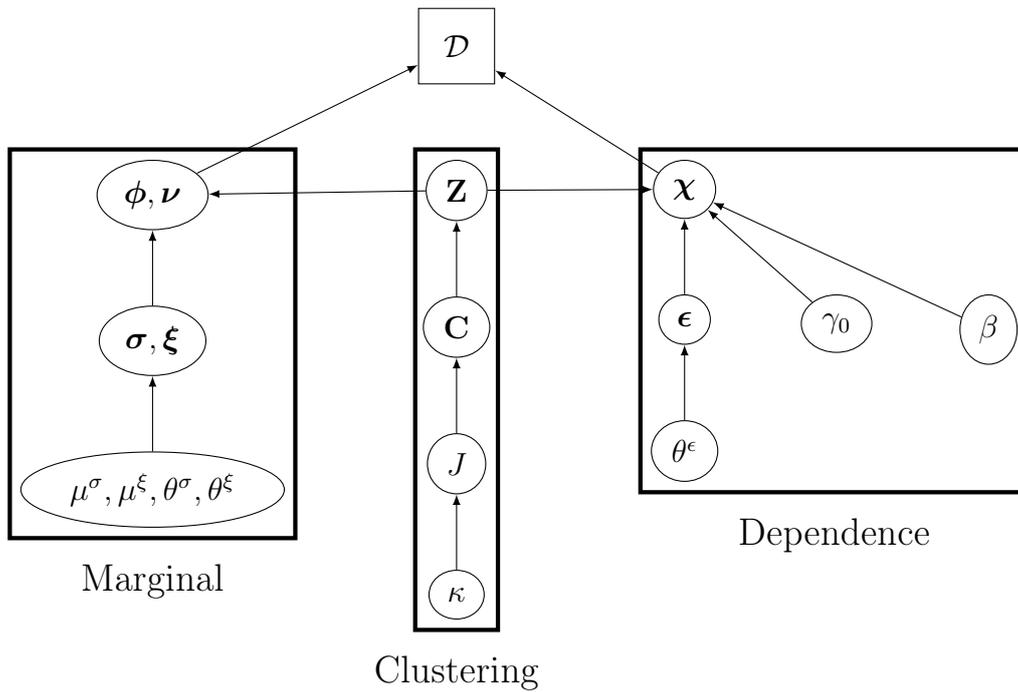


Figure 2: Directed acyclic graph representation to illustrate dependencies amongst the model parameters defined in Sections 3 and 4.

### 3 Simulation Examples - Further Results

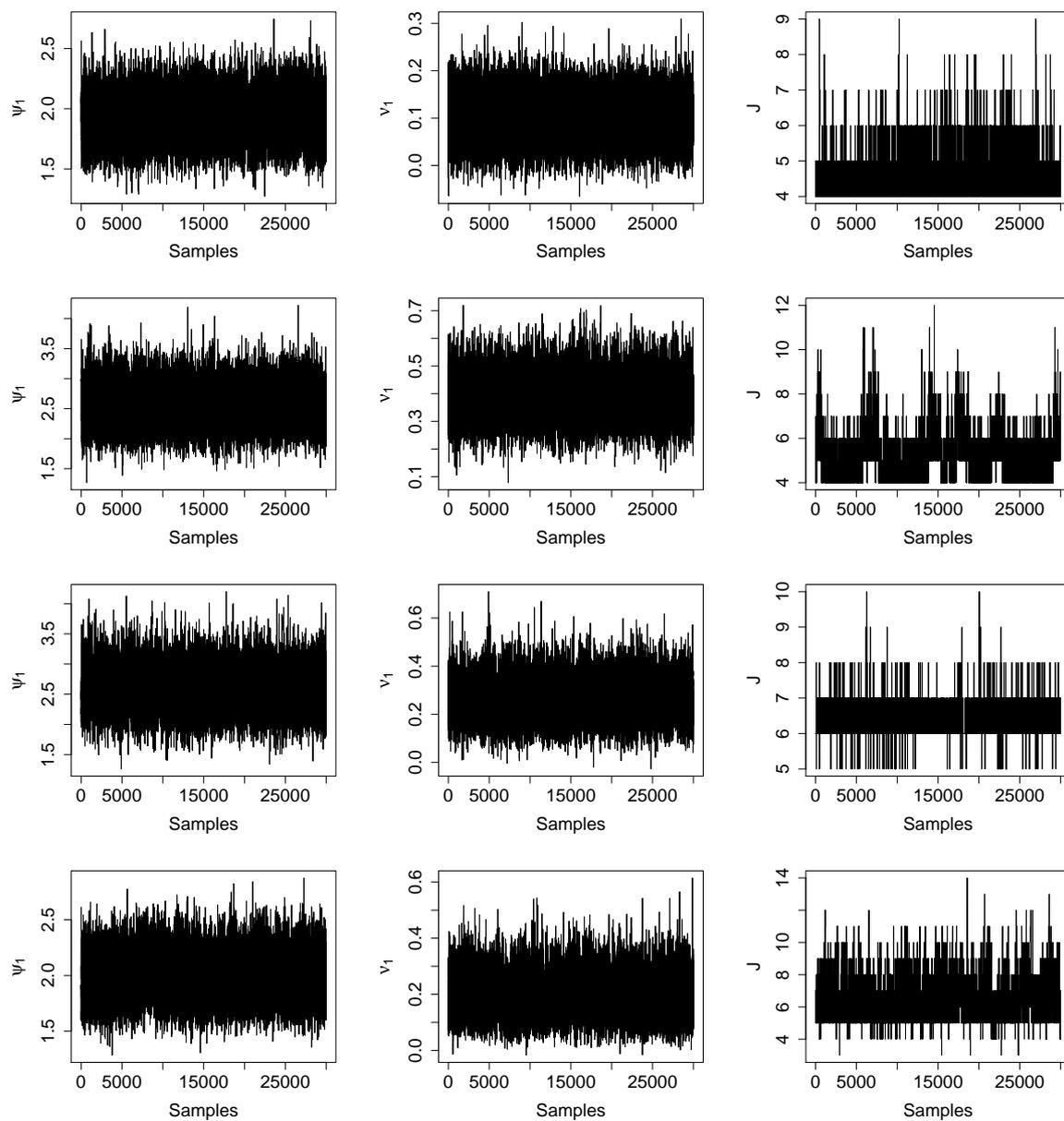


Figure 3: Posterior samples of  $\psi_1$ ,  $\nu_1$  and  $J$  (left–right) for the 25th, 50th, 75th and 100th generated data set in Study 3 (top–bottom).

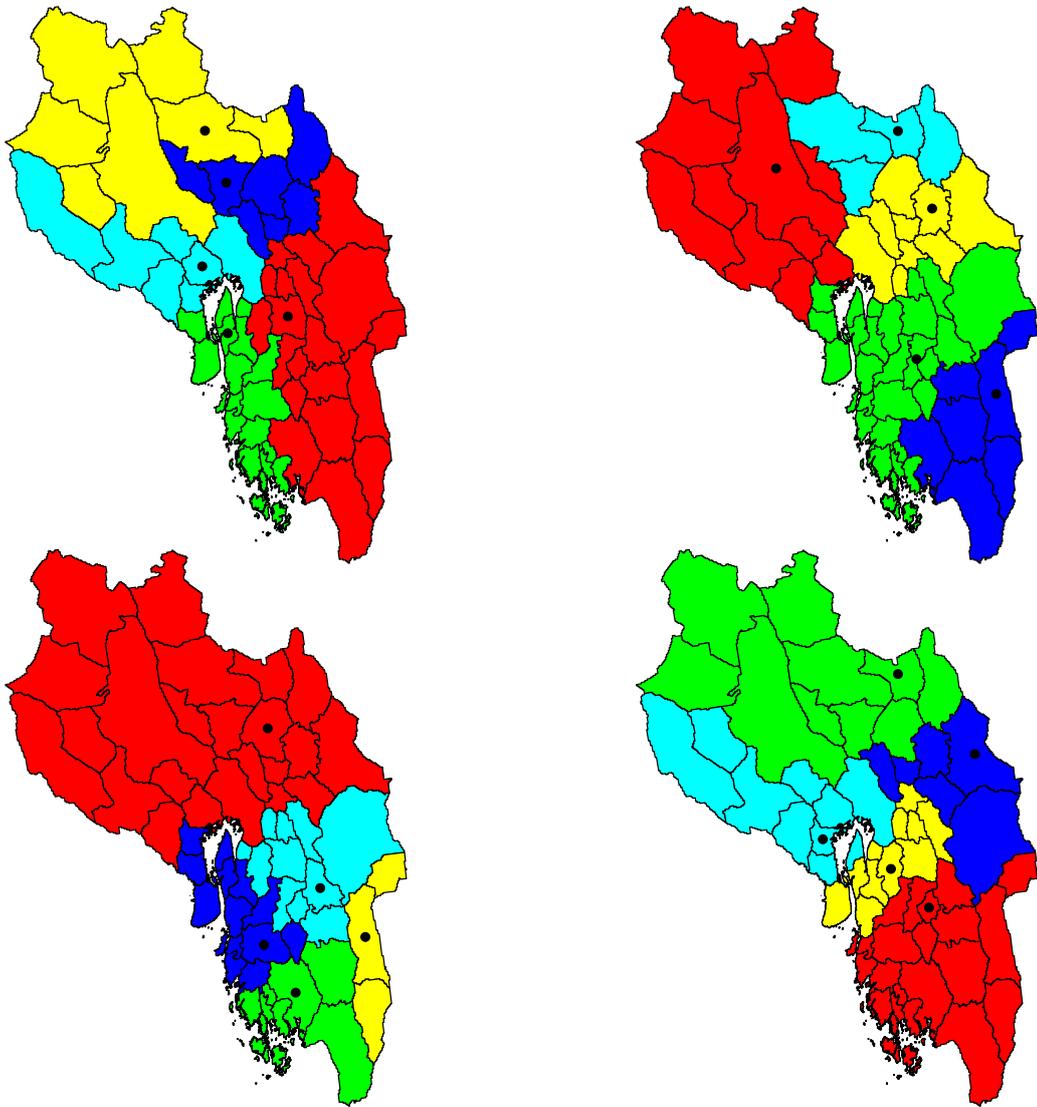


Figure 4: Four of the 32 sampled clusterings for which true and estimated clusterings are perfectly matched. The black dots in the true clusterings highlight the underlying cluster centres.

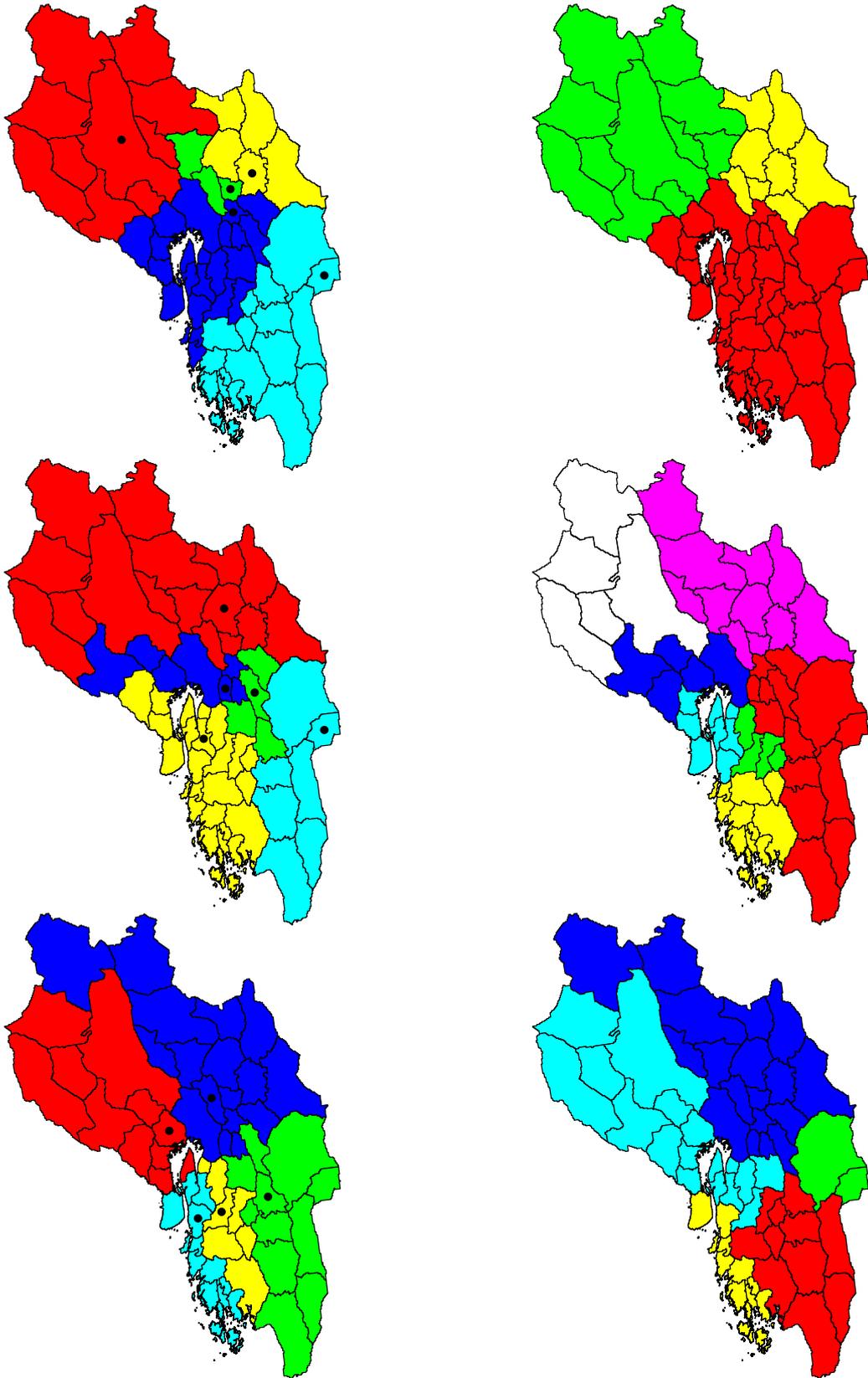


Figure 5: True and estimated clusterings for the three examples with lowest V-measure of 0.67, 0.69 and 0.7 (top–bottom). The black dots in the true clusterings highlight the underlying cluster centres.

## 4 Additional plots for Section 6.1

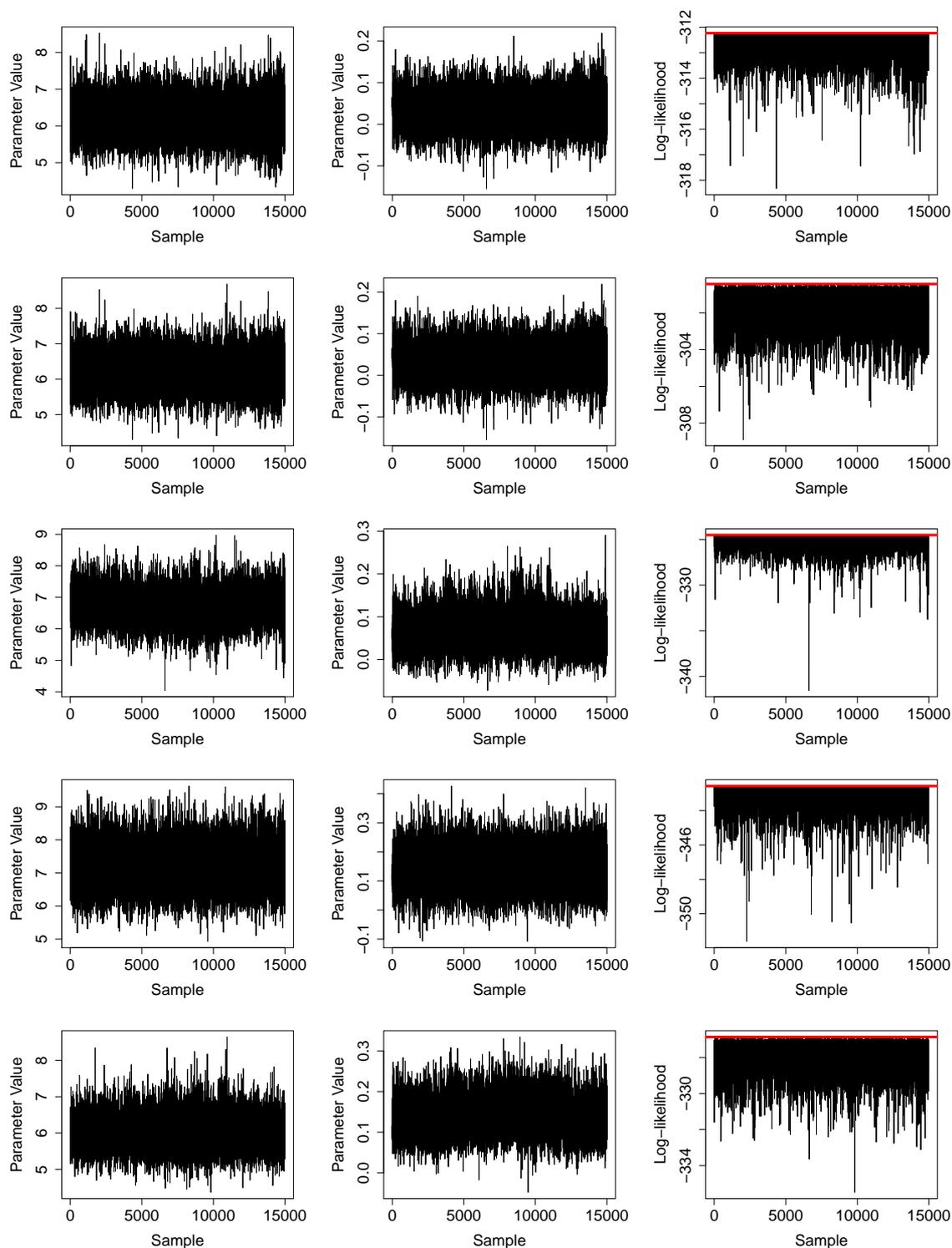


Figure 6: Trace plots of  $\psi_k$  (left),  $\nu_k$  (middle) and the log-likelihood (right) for Sarpsborg, Fredrikstad, Lillehammer, Stavanger and Trondheim (top-bottom). In the right plots, the red line illustrates the log-likelihood value for the maximum likelihood estimates for  $\psi_k$  and  $\nu_k$ .

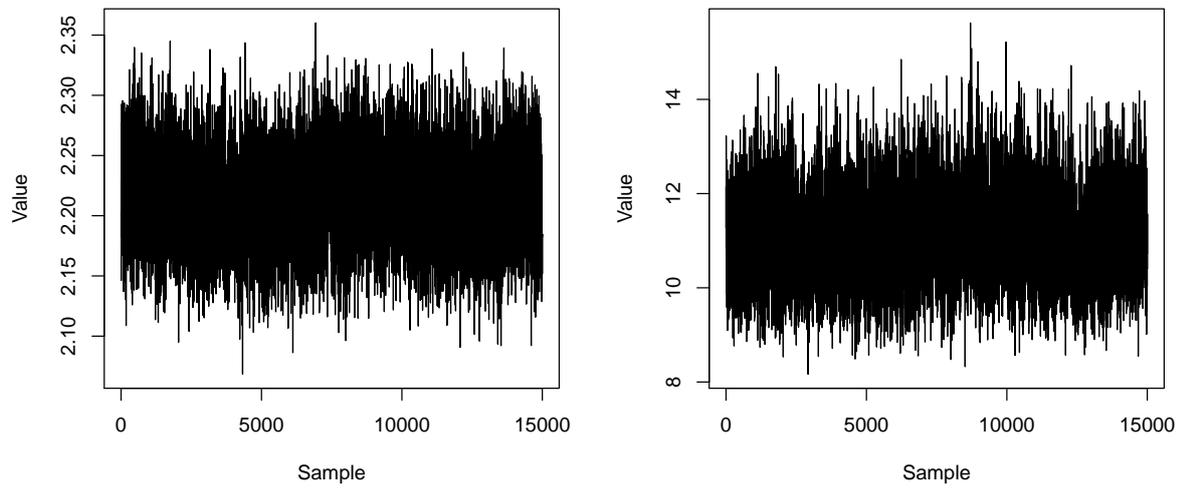


Figure 7: Trace plots of  $\gamma_0$  (left) and  $\beta$  (right) for the precipitation data in Section 6.1.

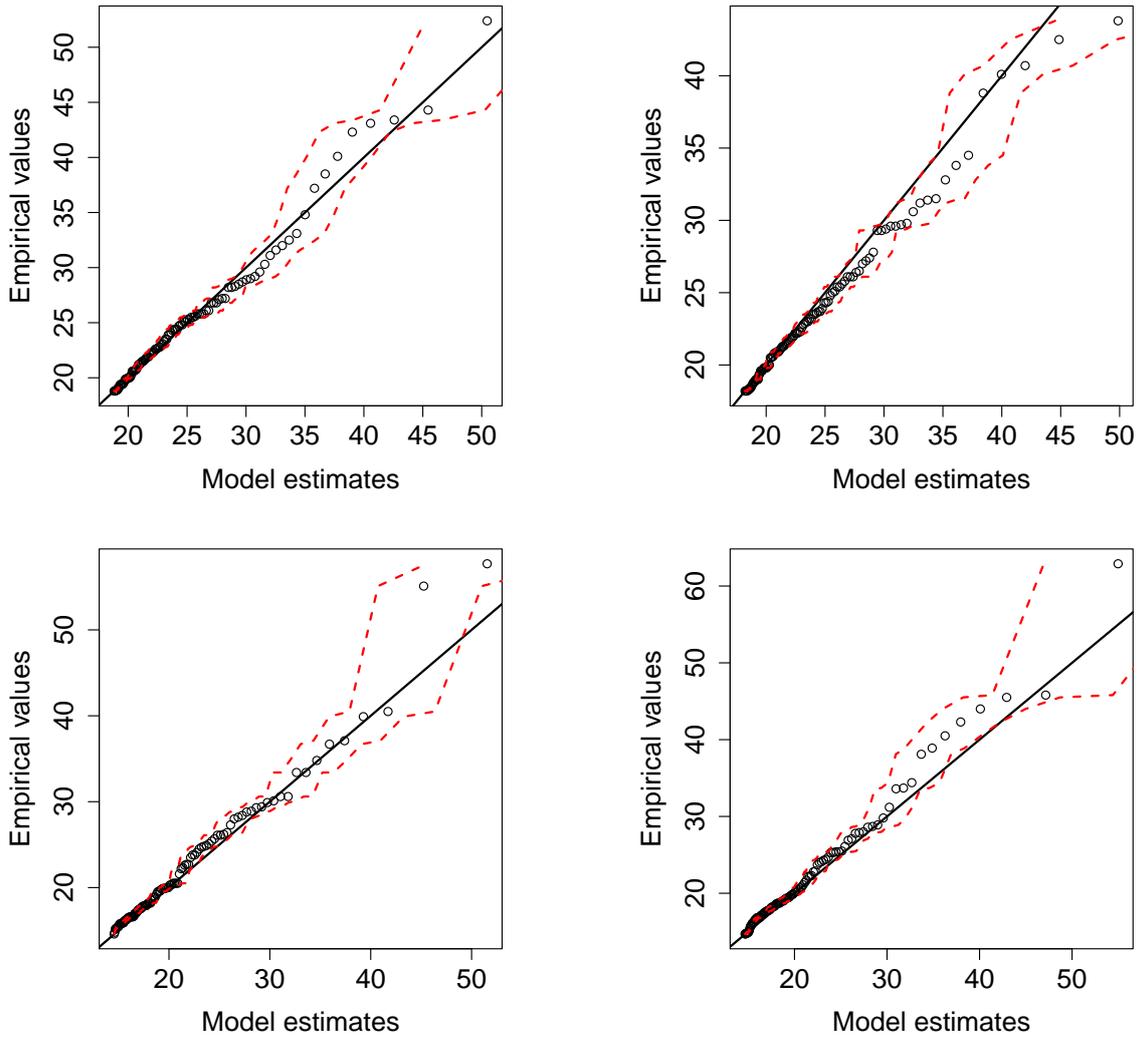


Figure 8: Quantile-Quantile plots for the municipalities Sarpsborg, Fredrikstad, Lillehammer and Trondheim. The red dashed lines in the QQ plots correspond to the central 95% credible interval.

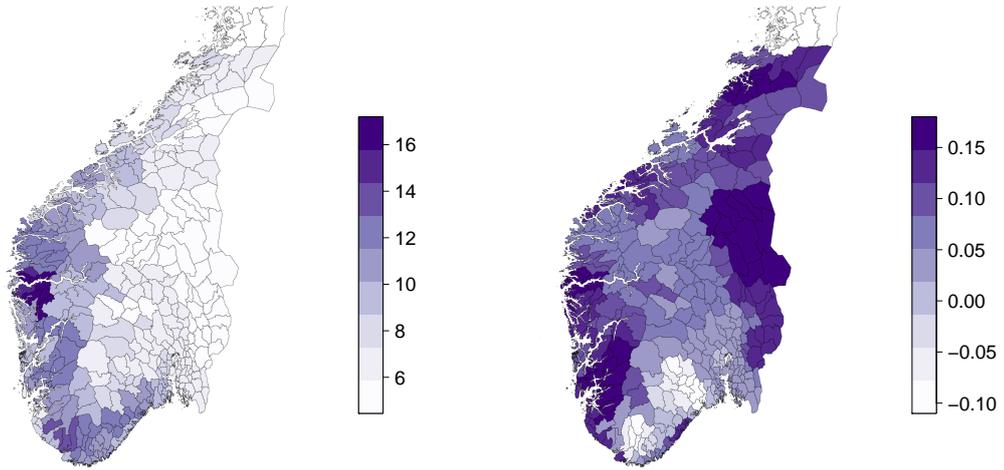


Figure 9: Posterior mean estimates for scale parameter  $\psi_k$  (left) and the shape parameter  $\nu_k$  (right).

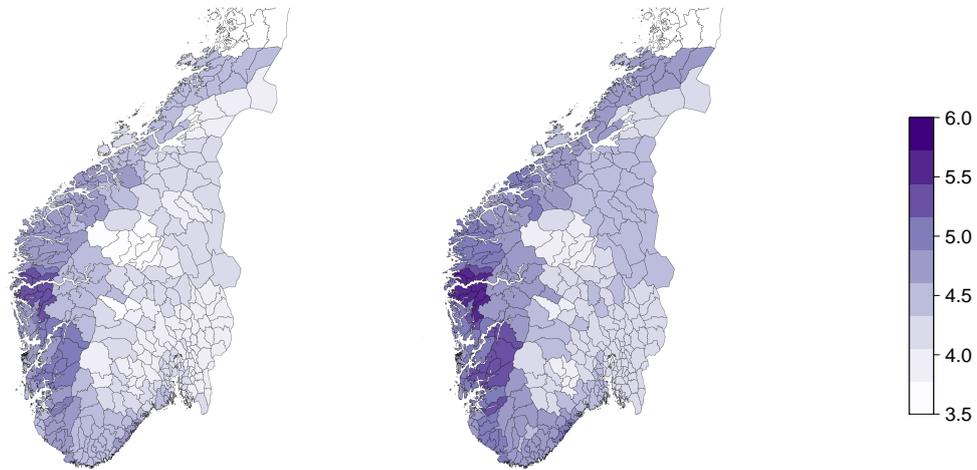


Figure 10: Posterior mean estimates for the 25 (left) and 100 (right) year return levels on logarithmic scale.

## 5 Additional plots for Section 6.2

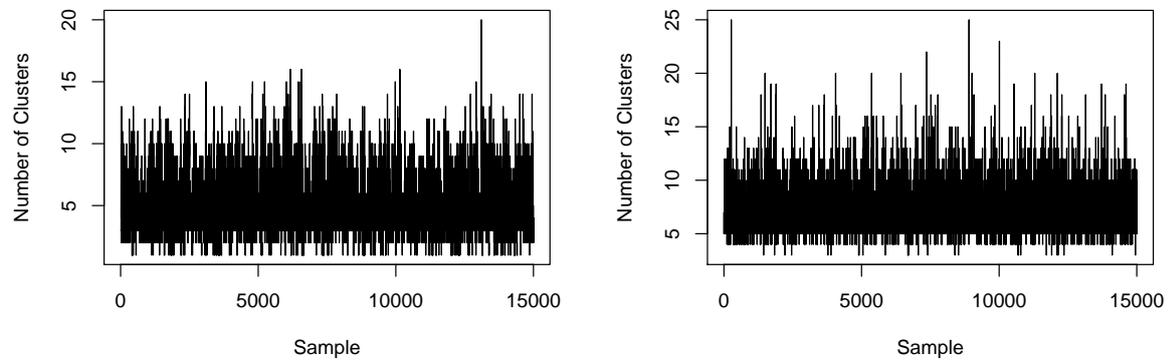


Figure 11: Trace plots of the sampled number of clusters for the periods November-March (left) and May-September (right).

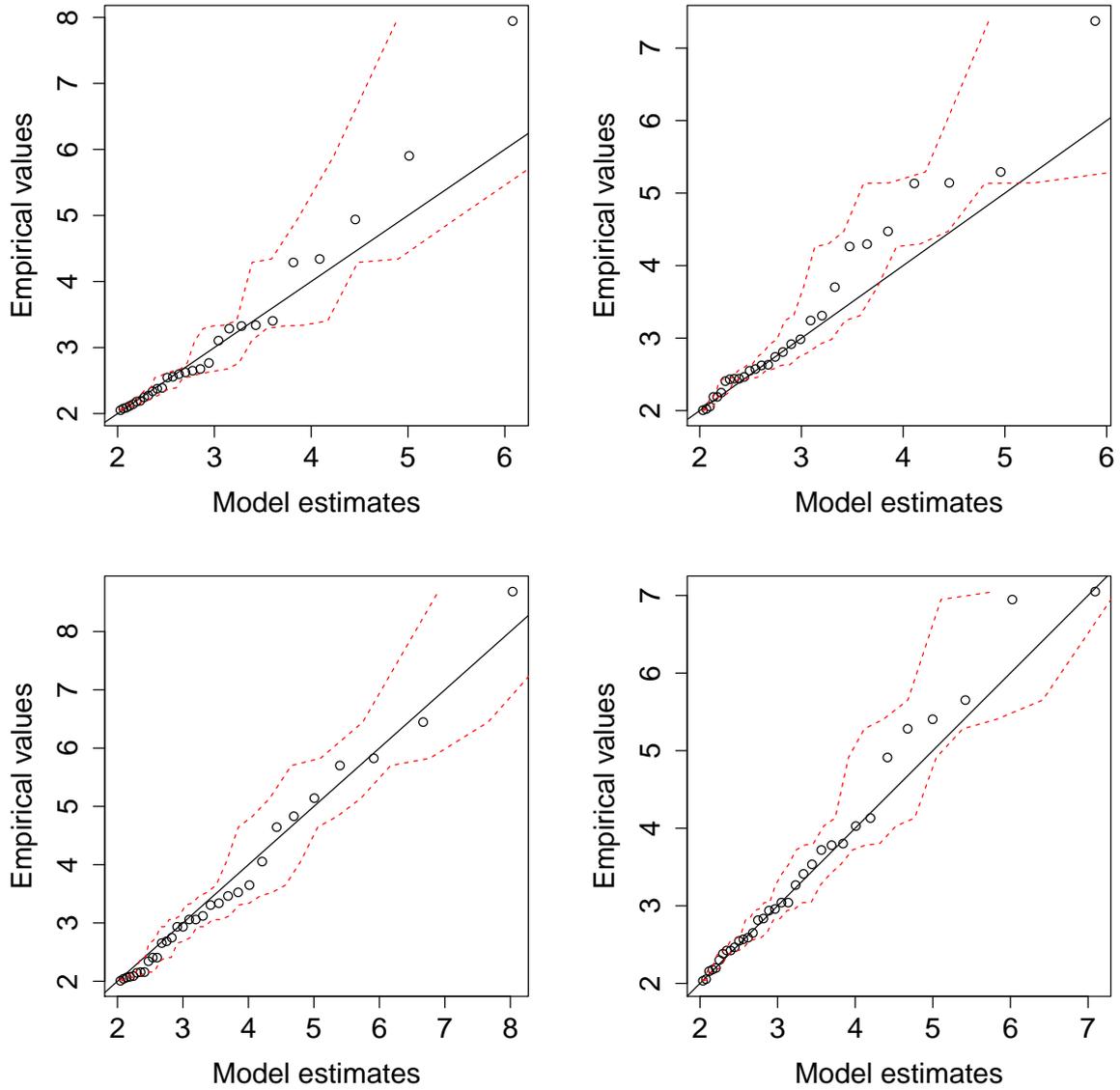


Figure 12: Quantile-Quantile plots for Marple Bridge (left) and Kirkby Stephen (right) for November-March (top) and May-September (bottom). The red dashed lines in the QQ plots correspond to the central 95% credible interval.

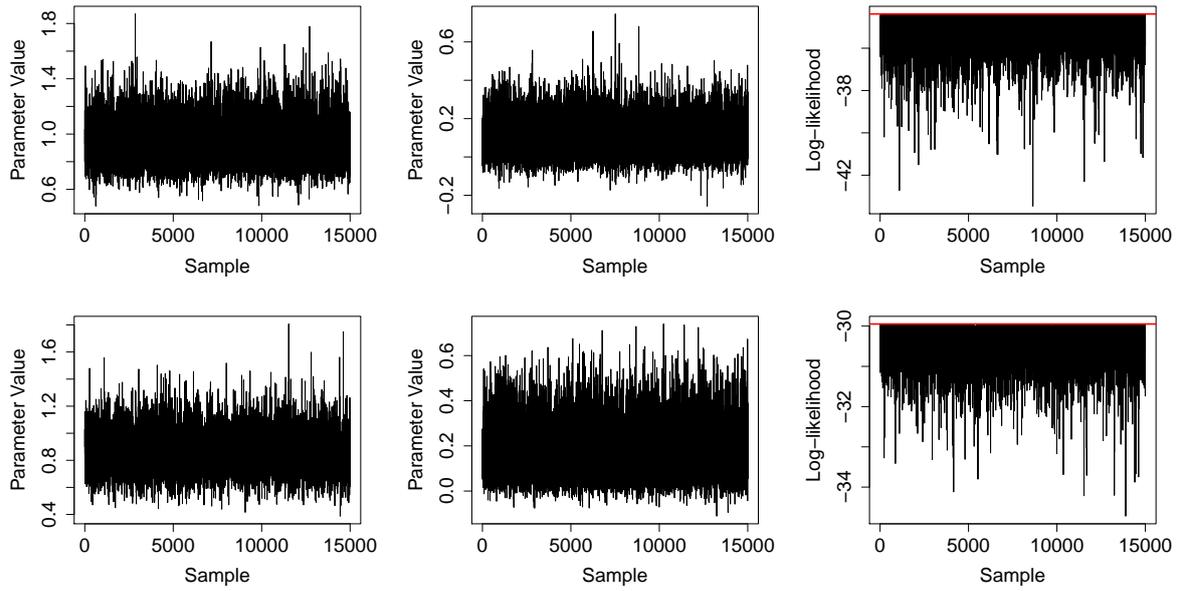


Figure 13: Trace plots of  $\psi_k$  (left),  $\nu_k$  (middle) and the log-likelihood (right) for Kirkby Stephen (top) and Marple Bridge (bottom) for the period November-March. In the right plots, the red line illustrates the log-likelihood value for the maximum likelihood estimates for  $\psi_k$  and  $\nu_k$

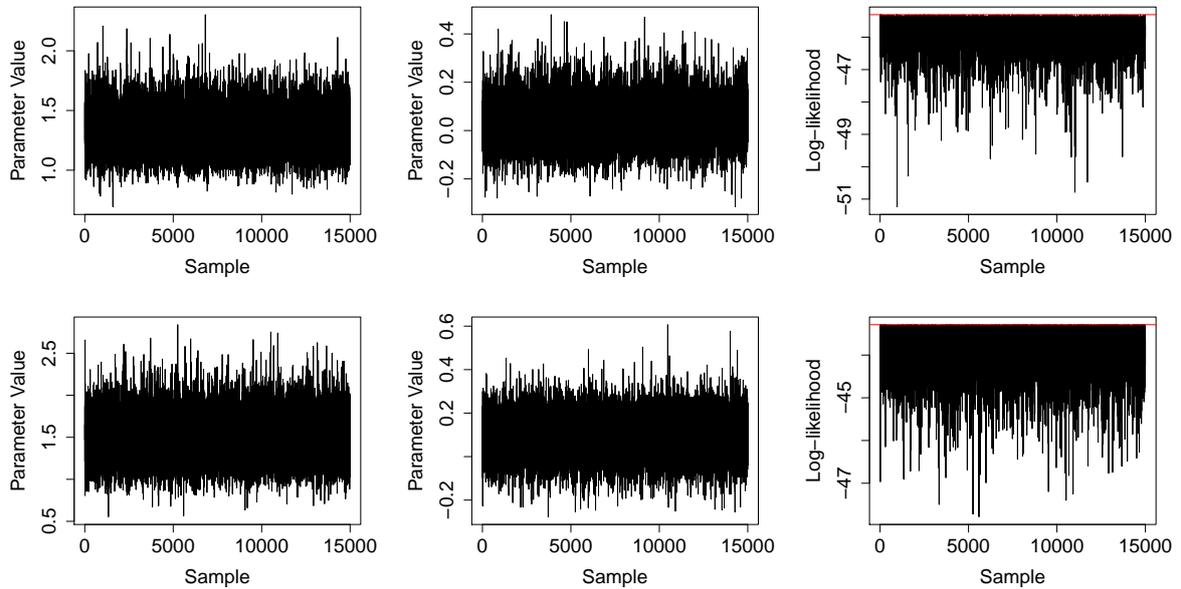


Figure 14: Trace plots of  $\psi_k$  (left),  $\nu_k$  (middle) and the log-likelihood (right) for Kirkby Stephen (top) and Marple Bridge (bottom) for the period May-September. In the right plots, the red line illustrates the log-likelihood value for the maximum likelihood estimates for  $\psi_k$  and  $\nu_k$

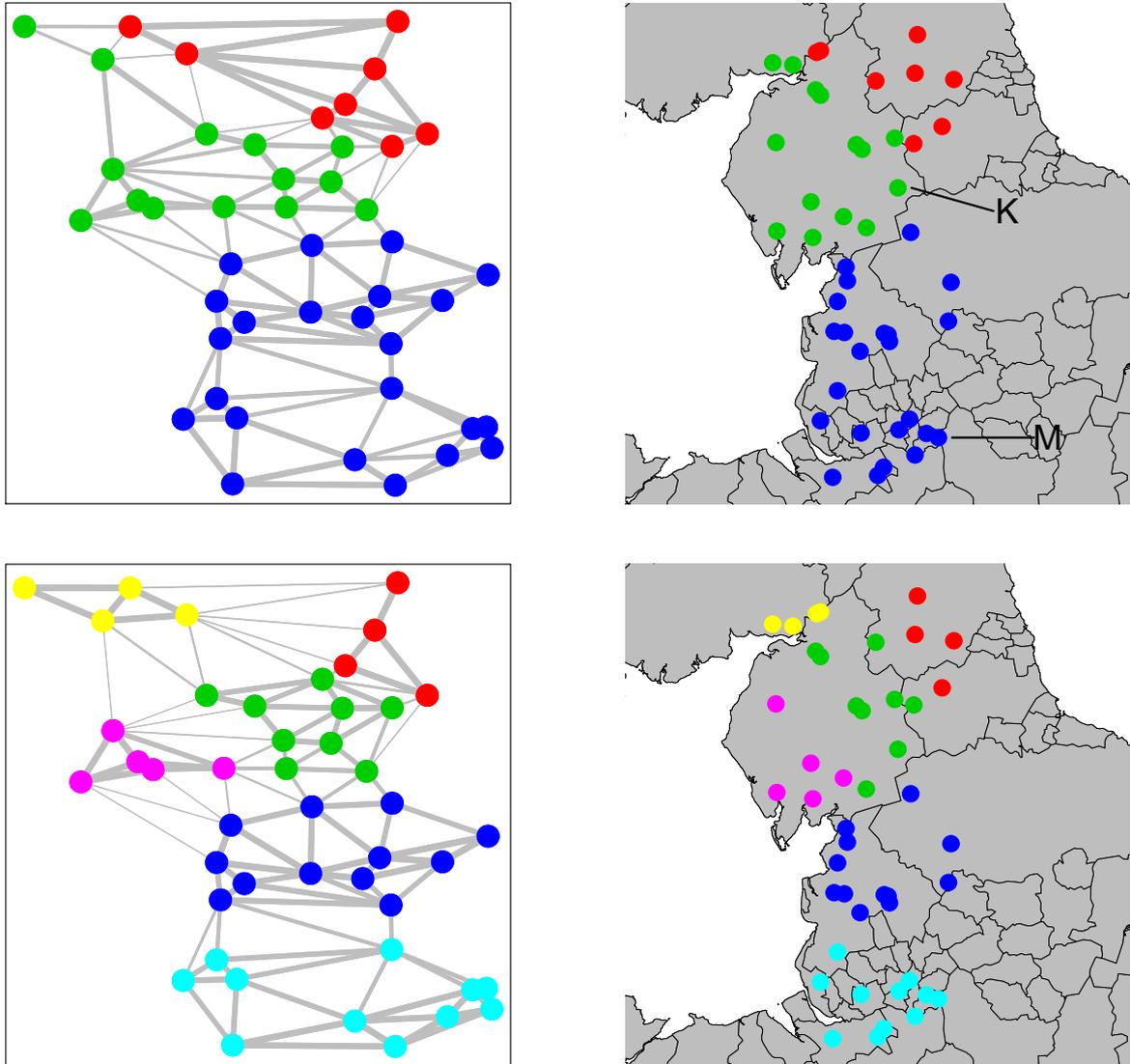


Figure 15: Point estimates for the underlying spatial cluster structure for November-March (top) and May-September (bottom). The left plots show the cluster allocation on the space of hydrological coordinates. A line between two sites indicates that they are considered adjacent and the line width corresponds to the posterior probability of them being in the same cluster. The right plots illustrate the derived clusters with respect to their latitude and longitude coordinates. The solid black lines are the boundaries of the metropolitan and non-metropolitan counties, and the highlighted gauges are Kirkby Stephen (K) and Marple Bridge (M).