Appendix

Slice volume derivation

We calculate the relative volume of a slice through a p dimensional hypersphere of radius R. In general the volume of a full hypersphere with radius r in q dimensions is given as

$$V(r,q) = \frac{\pi^{q/2} r^q}{\frac{q}{2}\Gamma(q/2)}, \tag{1}$$

and its variation with the radius is

$$\frac{dV(r,q)}{dr} = 2\frac{\pi^{q/2} r^{q-1}}{\Gamma(q/2)}. \tag{2}$$

To calculate the slice volume we note that it is spherical in the $p-2$ dimensional orthogonal space on the projection plane, while capturing the full volume (i.e. area) inside the plane. To calculate the slice volume we therefore integrate the product of $\frac{dV(r,p-2)}{dr}$ and the area in the plane parametrised by r, from the origin to the full slice thickness h. The area in the plane is a circle with radius $\sqrt{R^2-r^2}$. The slice volume is thus calculated as

$$V_{slice}(R,h,p) = \int_0^h \frac{dV(r,p-2)}{dr} V(\sqrt{R^2-r^2},2)dr = \frac{\pi^{p/2}}{\Gamma(p/2)} \frac{h^{p-2}}{p}[pR^2-(p-2)h^2]. \tag{3}$$

The relative volume of the slice is then simply given as

$$V_{rel}(R,h,p) = \frac{V_{slice}(R,h,p)}{V(R,p)} = \frac{h^{p-2}}{2R^p}[pR^2-(p-2)h^2]. \tag{4}$$