A Asymptotic Variance-Covariance Matrix of Mistreated Distributions

The matrices A and B in Section 3.1 have the following forms:

$$\mathbf{A} = n\sum_{j=1}^{m}\pi_j \begin{pmatrix} a_{\gamma_0\gamma_0} & x_j a_{\gamma_0\gamma_0} & a_{\gamma_0\sigma} \\ x_j a_{\gamma_0\gamma_0} & x_j^2 a_{\gamma_0\gamma_0} & x_j a_{\gamma_0\sigma} \\ a_{\gamma_0\sigma} & x_j a_{\gamma_0\sigma} & a_{\sigma\sigma} \end{pmatrix},$$

$$\mathbf{B} = n\sum_{j=1}^{m}\pi_j \begin{pmatrix} b_{\gamma_0\gamma_0} & x_j b_{\gamma_0\gamma_0} & a_{\gamma_0\sigma} \\ x_j b_{\gamma_0\gamma_0} & x_j^2 b_{\gamma_0\gamma_0} & x_j b_{\gamma_0\sigma} \\ b_{\gamma_0\sigma} & x_j b_{\gamma_0\sigma} & b_{\sigma\sigma} \end{pmatrix}.$$

Following the notations in Section 3.3, the elements of matrices A and B for the cases of complete and censored data are derived in as follows.

A.1 The Case of Complete Data

A.1.1 M_{LG} is true distribution, but wrongly treated as M_{SEV}

$$a_{\gamma_{0e}\gamma_{0e}} = \frac{1}{\sigma_e^2}\sum_{i=1}^{n} E_{LG}\left(-e^{Z_{ije}}\right)$$

$$a_{\gamma_{e0}\sigma_e} = \frac{1}{\sigma_e^2}\sum_{i=1}^{n} E_{LG}\left(\frac{-Z_{ije}e^{Z_{ije}}}{\sigma_e} - e^{Z_{ije}} + 1\right)$$

$$a_{\sigma_e\sigma_e} = \frac{1}{\sigma_e^2}\sum_{i=1}^{n} E_{LG}\left(\frac{-Z_{ije}^2 e^{Z_{ije}}}{\sigma_e} - 2Z_{ije}e^{Z_{ije}} + 2Z_{ije} + 1\right)$$

$$b_{\gamma_{0e}\gamma_{0e}} = \frac{1}{\sigma_e^2}\sum_{i=1}^{n} E_{LG}\left(e^{Z_{ije}} - 1\right)^2$$

$$b_{\gamma_{0e}\sigma_e} = \frac{1}{\sigma_e^2}\sum_{i=1}^{n} E_{LG}\left(Z_{ije}e^{Z_{ije}} - Z_{ije} - 1\right)\left(e^{Z_{ije}} - 1\right)$$

$$b_{\sigma_e\sigma_e} = \frac{1}{\sigma_e^2}\sum_{i=1}^{n} E_{LG}\left(Z_{ije}e^{Z_{ije}} - Z_{ije} - 1\right)$$

A.1.2 M_{LG} is true distribution, but wrongly treated as M_{Nor}

$$a_{\gamma_{0N}\gamma_{0N}} = -\frac{1}{\sigma_N^2}$$

$$a_{\gamma_{0N}\sigma_N} = \frac{1}{\sigma_N^2}\sum_{i=1}^{n} -\frac{2}{\sigma_N} E_{LG}\left(\log t_{ij} - \mu_N(x_j)\right)$$

$$a_{\sigma_N\sigma_N} = \frac{1}{\sigma_N^2}\left(1 - \sum_{i=1}^{n}\frac{3}{\sigma_N^2} E_{LG}\left(\log t_{ij} - \mu_N(x_j)\right)^2\right)$$

$$b_{\gamma_{0N}\gamma_{0N}} = \frac{1}{\sigma_N^2}\sum_{i=1}^{n} E_{LG}(Z_{ijN})^2$$

$$b_{\gamma_{0N}\sigma_N} = \frac{1}{\sigma_N^2}\sum_{i=1}^{n} E_{LG}\left(Z_{ijN}\left(Z_{ijN}^2 - 1\right)\right)$$

$$b_{\sigma_N\sigma_N} = \frac{1}{\sigma_N^2}\sum_{i=1}^{n} E_{LG}\left(Z_{ijN}^2 - 1\right)^2$$

A.2 The Case of Censored Data

A.2.1 M_{LG} is true distribution, but wrongly treated as M_{SEV}

$$a_{\gamma_{0e}\gamma_{0e}} = \frac{1}{\sigma_e^2}\left\{-e^{\zeta_{je}}\left(1 - \Phi_{LG}(\zeta_{jg})\right) - \int_{-\infty}^{\eta_j} e^{Z_{ije}}\phi_{LG}(Z_{ijg})dt_{ij}\right\}$$

$$a_{\gamma_{0e}\sigma_e} = \frac{1}{\sigma_e^2}\left\{a_{\gamma_{0e}\gamma_{0e}} - \zeta_{je}e^{\zeta_{je}}\left(1 - \Phi_{LG}(\zeta_{jg})\right) + \Phi_{LG}(\zeta_{jg}) - \int_{-\infty}^{\eta_j} Z_{ije}e^{Z_{ije}}\phi_{LG}(Z_{ijg})dt_{ij}\right\}$$

$$a_{\sigma_e\sigma_e} = \frac{1}{\sigma_e^2}\left\{\zeta_{je}e^{\zeta_{je}}\left(1 - \Phi_{LG}(\zeta_{jg})\right)(2 - \zeta_{je}) - 3\Phi_{LG}(\zeta_{jg}) - \right.$$
$$\left.\int_{-\infty}^{\eta_j} Z_{ije}\left[Z_{ije}e^{Z_{ije}} - 2 + 2e^{Z_{ije}}\right]\phi_{LG}(Z_{ijg})dt_{ij}\right\}$$

$$b_{\gamma_{0e}\gamma_{0e}} = \frac{1}{\sigma_e^2}\left\{e^{2\zeta_{je}}\left(1 - \Phi_{LG}(\zeta_{jg})\right) + \int_{-\infty}^{\eta_j}(e^{Z_{ije}} - 1)^2\phi_{LG}(Z_{ijg})dt_{ij}\right\}$$

$$b_{\gamma_{0e}\sigma_e} = \frac{1}{\sigma_e^2}\left\{\zeta_{je}e^{2\zeta_{je}}\left(1 - \Phi_{LG}(\zeta_{jg})\right) + \Phi_{LG}(\zeta_{jg}) + \int_{-\infty}^{\eta_j}\left[Z_{ije}\left(e^{Z_{ije}} - 1\right)^2 - e^{Z_{ije}}\right]\phi_{LG}(Z_{ijg})dt_{ij}\right\}$$

$$b_{\sigma_e\sigma_e} = \frac{1}{\sigma_e^2}\left\{\left(\zeta_{je}e^{\zeta_{je}}\right)^2\left(1 - \Phi_{LG}(\zeta_{jg})\right) + \int_{-\infty}^{\eta_j}\left[Z_{ije}e^{Z_{ije}} - 1 - Z_{ije}\right]^2\phi_{LG}(Z_{ijg})dt_{ij}\right\}$$

A.2.2 M_{LG} is true distribution, but wrongly treated as M_{Nor}

$$a_{\gamma_{0N}\gamma_{0N}} = \frac{1}{\sigma_N^2}\left\{\left(1-\Phi_{LG}(\zeta_{jg})\right)\left(\frac{\zeta_{jN}\phi_{Nor}(\zeta_{jN})}{1-\Phi_{Nor}(\zeta_{jN})} - \left(\frac{\phi_{Nor}(\zeta_{jN})}{1-\Phi_{Nor}(\zeta_{jN})}\right)^2\right) - \Phi_{LG}(\zeta_{jg})\right\}$$

$$a_{\gamma_{0N}\sigma_N} = \frac{1}{\sigma_N^2}\Big\{\tfrac{\phi_{Nor}(\zeta_{jN})\left(1-\Phi_{LG}(\zeta_{jg})\right)}{1-\Phi_{Nor}(\zeta_{jN})}\Big(-\tfrac{\zeta_{jN}\phi_{Nor}(\zeta_{jN})}{1-\Phi_{Nor}(\zeta_{jN})} + \zeta_{jN}^2 - 1\Big) - 2\int_{-\infty}^{\eta_j} Z_{ijN}\phi_{LG}(Z_{ijg})dt_{ij}\Big\}$$

$$a_{\sigma_N\sigma_N} = \frac{1}{\sigma_N^2}\Big\{\tfrac{\zeta_{jN}\phi_{Nor}(\zeta_{jN})\left(1-\Phi_{LG}(\zeta_{jg})\right)}{1-\Phi_{Nor}(\zeta_{jN})}\Big(-\tfrac{\zeta_{jN}\phi_{Nor}(\zeta_{jN})}{1-\Phi_{Nor}(\zeta_{jN})} + \zeta_{jN}^2 - 2\Big) + \Phi_{LG}(\zeta_{jg}) - 3\int_{-\infty}^{\eta_j} Z_{ijN}^2\phi_{LG}(Z_{ijg})dt_{ij}\Big\}$$

$$b_{\gamma_{0N}\gamma_{0N}} = \frac{1}{\sigma_N^2}\left\{\left(1-\Phi_{LG}(\zeta_{jg})\right)\left(\frac{\phi_{Nor}(\zeta_{jN})}{1-\Phi_{Nor}(\zeta_{jN})}\right)^2 + \int_{-\infty}^{\eta_j} Z_{ijN}^2\phi_{LG}(Z_{ijg})dt_{ij}\right\}$$

$$b_{\gamma_{0N}\sigma_N} = \frac{1}{\sigma_N^2}\left\{\zeta_{jN}\left(1-\Phi_{LG}(\zeta_{jg})\right)\left(\frac{\phi_{Nor}(\zeta_{jN})}{1-\Phi_{Nor}(\zeta_{jN})}\right)^2 + \int_{-\infty}^{\eta_j} Z_{ijN}\left(Z_{ijN}^2-1\right)\phi_{LG}(Z_{ijg})dt_{ij}\right\}$$

$$b_{\sigma_N\sigma_N} = \frac{1}{\sigma_N^2}\left\{\left(1-\Phi_{LG}(\zeta_{jg})\right)\left(\frac{\zeta_{jN}\phi_{Nor}(\zeta_{jN})}{1-\Phi_{Nor}(\zeta_{jN})}\right)^2 + \int_{-\infty}^{\eta_j} \left(Z_{ijN}^2-1\right)^2\phi_{LG}(Z_{ijg})dt_{ij}\right\}$$

B The Derivation of P_2 and P_3 in Table 2

B.1 The Case of Censored data

To prove P_3 in Table 2 following results based on the definition of δ_i for the model M_k in (31) are required:

$$\begin{aligned} E_k(\delta_i) &= \Phi_k(\zeta_j) \\ E_k(\delta_i) &= E_k(\delta_i^2), \\ E_k(g(\log t_{ij})\delta_i) &= \int_{-\infty}^{\log\eta_j} g(\log t_{ij})\phi_k(\log t_{ij})dt_{ij}, \\ E_k(g(\log t_{ij})\delta_i^2) &= E_k(g(\log t_{ij})\delta_i). \end{aligned}$$

B.1.1 M_{LG} is true distribution, but wrongly treated as M_{SEV}

Consider P_1 in Table 2, when the wrong model is Weibull the expectation of the $(-\Im_{SEV})$ under the GG_3 distribution is given by:

$$\begin{aligned} E(-\Im_{SEV}) = n\sum_{j=1}^{m}\pi_j \Big\{ & E_{LG}\left(\delta_i \log\sigma_e\right) - E_{LG}\left(\delta_i Z_{ije}\right) + E_{LG}\left(\delta_i e^{Z_{ije}}\right) \\ & + E_{LG}\left(e^{\zeta_{ije}}\right) - E_{LG}\left(\delta_i e^{\zeta_{ije}}\right)\Big\} \end{aligned} \tag{1}$$

B.1.2 M_{LG} is true distribution, but wrongly treated as M_{Nor}

Consider P_1 in Table 2, when the wrong model is Lognormal the expectation of the $(-\Im_{Nor})$ under the GG_3 distribution is given by:

$$\begin{aligned} E(-\Im_{Nor}) = n\sum_{j=1}^{m}\pi_j \Bigg\{ & E_{LG}\left(\delta_i \log\sigma_N\right) - E_{LG}\left(\delta_i \frac{\log 2\pi}{2}\right) + E_{LG}\left(\delta_i \frac{Z_{ijN}^2}{2}\right) - \\ & \log\left(1-\Phi_{Nor}(\zeta_{ijN})\right) + E_{LG}\left(\delta_i \log\left(1-\phi_{Nor}(\zeta_{ijN})\right)\right)\Bigg\} \end{aligned} \tag{2}$$

B.2 The Case of Complete Data

To prove P_2 in Table 2 the results in Section B.1 and following results are required:

$$E_{LG}(\log t_{ij}) = \mu_g(x_j) + \sigma_g \varphi(k),$$

$$E_{LG}(e^{a\log t_{ij}}) = \frac{e^{a\mu_g(x_j)}\Gamma(a\sigma_g + k)}{\Gamma(k)},$$

$$E_{LG}(\log t_{ij}^2) = [\mu_g(x_j) + \sigma_g\varphi(k)]^2 + \sigma_g^2\varphi'(k).$$

B.2.1 M_{LG} is true distribution, but wrongly treated as M_{SEV}

Consider $\eta \to \infty$, Equation (1) can be simplified as:

$$E(-\Im_{SEV}) = n\sum_{j=1}^{m}\pi_j \left\{\log\sigma_e - E_{LG}\left(Z_{ije}\right) + E_{LG}\left(e^{Z_{ije}}\right)\right\} \tag{3}$$

B.2.2 M_{LG} is true distribution, but wrongly treated as M_{Nor}

Consider $\eta \to \infty$, Equation (2) can be simplified as:

$$E(-\Im_{Nor}) = n \sum_{j=1}^{m} \pi_j \left\{ \log \sigma_N - \frac{\log 2\pi}{2} + E_{LG}\left(\frac{Z_{ijN}^2}{2} \right) \right\} \tag{4}$$

C The Pairwise Plots of RB and RV for Weibull and Lognormal for Complete Data

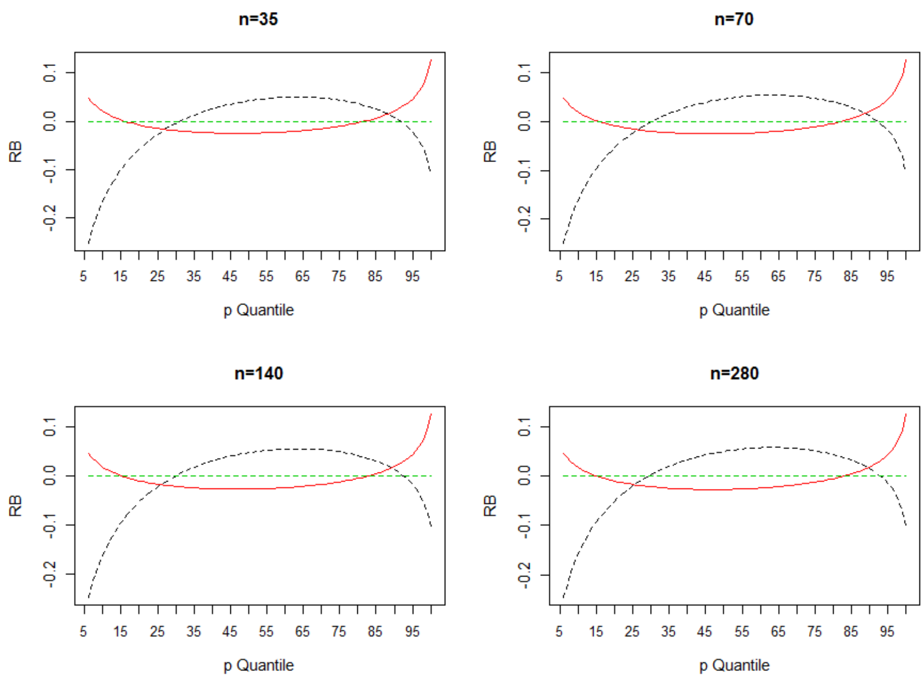

Figure 1: Comparison of RB between Lognormal (black/dash) and Weibull (red/solid) base on the total sample size.

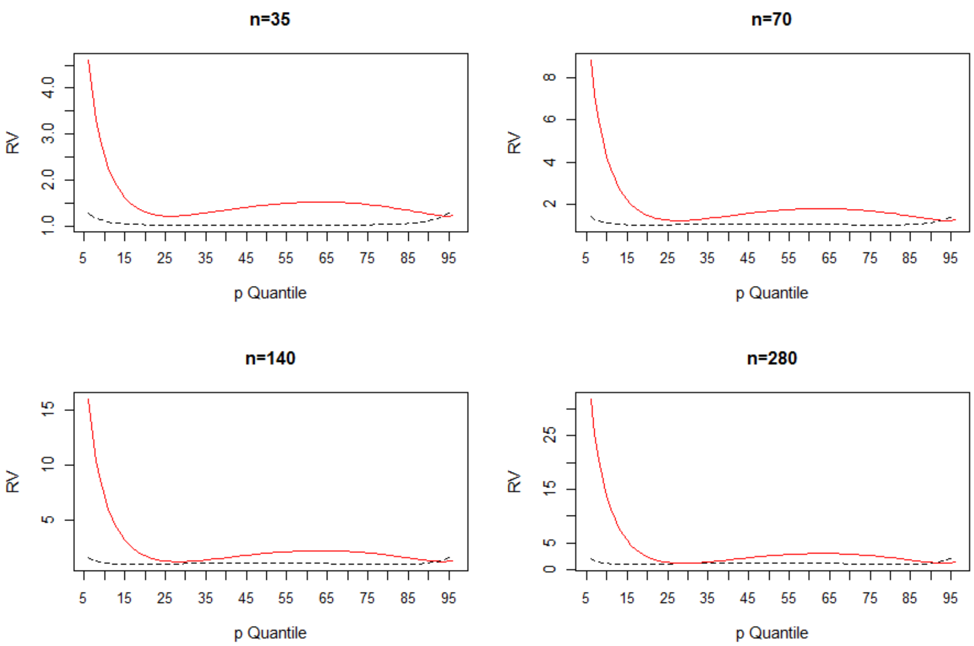

Figure 2: Comparison of *RV* between Lognormal (black/dash) and Weibull (red/solid) base on the total sample size.