# Appendix A

***A.1 Proof of Lemma 1***

A Stackelberg game model in which a leader exists in a supply chain needs to be solved by the backward induction method.

The optimal solution of the third party is solved first. The first partial derivative of third party’s profit function $π\_{t}^{B}$ with respect to the unit offline collection price $r\_{2}$ is $\frac{∂π\_{t}^{B}}{∂r\_{2}}=h\left(b-2r\_{2}\right)-Q$, and the second partial derivative is $\frac{∂^{2}π\_{t}^{B}}{∂\left(r\_{2}\right)^{2}}=-2h<0$*.* The optimal solution can be obtained through the equation $\frac{∂π\_{t}^{B}}{∂r\_{2}}=0$ because the second partial derivative is always negative, in which the optimal offline collection price is $r\_{2}^{B\*}=\frac{bh-Q}{2h}$*.*

Then, the optimal solutions of the remanufacturer are concluded based on the optimal solution of the third party. We substitute the optimal decision expression $r\_{2}^{B\*}$ into the profit function of the remanufacturer $π\_{r}^{B}=\left(P-C-r\_{1}\right)h\left(r\_{1}-r\_{2}\right)+\left(P-C-b\right)\left(Q+hr\_{2}\right)$. Next, the first and second partial derivatives of $π\_{r}^{B}$ with respect to $r\_{1}$ and $b$ are solved as follows respectively:

$\frac{∂π\_{r}^{B}}{∂r\_{1}}=h\left(P-C-2r\_{1}\right)+\frac{bh-Q}{2}$, $\frac{∂π\_{r}^{B}}{∂b}=\frac{h\left(r\_{1}-2b\right)-Q}{2}$; $\frac{∂^{2}π\_{r}^{B}}{∂\left(r\_{1}\right)^{2}}=-2h$, $\frac{∂^{2}π\_{r}^{B}}{∂\left(b\right)^{2}}=-h$, $\frac{∂^{2}π\_{r}^{B}}{∂r\_{1}∂b}=\frac{h}{2}$.

Thus, the Hessian matrix of it is below:

$$H=\left[\begin{matrix}-2h&\frac{h}{2}\\\frac{h}{2}&-h\end{matrix}\right]$$

It can be found that $π\_{r}^{B}$ is the joint concave function of $r\_{1}$ and $b$ due to $H\_{11}<0$ and det$\left(H\right)=\frac{7}{4}h^{2}>0.$ Therefore, the optimal solutions of $r\_{1}$ and $b$ can be solved as follows according to $\frac{∂π\_{r}^{B}}{∂r\_{1}}=0$ and $\frac{∂π\_{r}^{B}}{∂b}=0$: $r\_{1}^{B\*}=\frac{4h\left(P-C\right)-3Q}{7h}$, $b^{B\*}=\frac{2h\left(P-C\right)-5Q}{7h}$.

Finally, we substitute $r\_{1}^{B\*}$ and $b^{B\*}$ into the expression of $r\_{2}^{B\*}$ to figure out the optimal unit offline collection pricing decision which is $r\_{2}^{B\*}=\frac{h\left(P-C\right)-6Q}{7h}$.

The optimal decisions obtained above are substituted into equations (4), (5) and (6) to obtain the optimal profits as follows: $π\_{r}^{B\*}=\frac{2\left(Ah+Q\right)^{2}}{7h}$, $π\_{t}^{B\*}=\frac{\left(Ah+Q\right)^{2}}{49h}$, $π\_{sc}^{B\*}=\frac{15\left(Ah+Q\right)^{2}}{49h}$.

***A.2 Proof of Lemma 2***

When the offline channel is disrupted and closed, the profit of the remanufacturer is $π\_{r}^{Off}=\left(P-C-r\_{1}\right)\left(Q+hr\_{1}\right)$. Therefore, we set the first partial derivatives of $π\_{r}^{Off}$ with respect to $r\_{1}$ equal to 0 to solve the optimal decision $r\_{1}^{Off\*}=\frac{Ah-Q}{2h}. $

***A.3 Proof of Lemma 3***

The profit functions of the third party and the remanufacturer when the online channel is disrupted and closed are $π\_{t}^{On}=\left(b-r\_{2}\right)\left(Q+hr\_{2}\right)$ and $π\_{r}^{On}=\left(P-C-b\right)\left(Q+hr\_{2}\right)$. We solve the first partial derivatives of $π\_{t}^{On}$ with respect to $r\_{2}$ and set it equal to 0 to get the expression of $r\_{2}$. Then, we will substitute $r\_{2}$ into the profit function of the remanufacturer to get the optimal $b$. Therefore, we obtain $b^{On\*}=\frac{Ah-Q}{2h}$ and $r\_{1}^{On\*}=\frac{Ah-3Q}{4h}$.

***A.4 Proof of Lemma 4***

Similar to the proof of Lemma 1, we calculate the first and second partial derivatives of $π\_{t}^{Offd}$ with respect to $r\_{2}$ in the online disruption scenario, $\frac{∂π\_{t}^{Offd}}{∂r\_{2}}=h\left(b-2r\_{2}\right)-Q$, $\frac{∂^{2}π\_{t}^{Offd}}{∂\left(r\_{2}\right)^{2}}=-2h<0$. From $\frac{∂π\_{t}^{Offd}}{∂r\_{2}}=0$, the optimal solution of the third party is $r\_{2}^{Offd\*}=\frac{bh-Q}{2h}$.

For the profit function of the remanufacturer $π\_{r}^{Offd}$, the Hessian matrix of it can be obtained as below:

$$H=\left[\begin{matrix}-2h&\frac{h}{2}\\\frac{h}{2}&-h\end{matrix}\right]$$

It can be found that $π\_{r}^{Offd}$ is also the joint concave function of $r\_{1}$ and $b$ due to $H\_{11}<0$ and det$\left(H\right)=\frac{7}{4}h^{2}>0.$ Therefore, the optimal solutions of $r\_{1}$ and $b$ can be solved as follows according to $\frac{∂π\_{r}^{Offd}}{∂r\_{1}}=h\left(P-C-2r\_{1}\right)+\frac{bh-Q}{2}=0$ and $\frac{∂π\_{r}^{Offd}}{∂b}=\frac{h\left(r\_{1}-2b-∆\_{C}\right)-Q}{2}=0$, then $r\_{1}^{Offd\*}=\frac{h\left(4P-4C-∆\_{C}\right)-3Q}{7h}$, $b^{Offd\*}=\frac{2h\left(P-C-2∆\_{C}\right)-5Q}{7h}$.

Finally, we substitute the optimal decisions into the expression of $r\_{2}^{Offd\*}$ to figure out the optimal unit offline collection price in scenario Offd, which is $r\_{2}^{Offd\*}=\frac{h\left(P-C-2∆\_{C}\right)-6Q}{7h}$. The optimal decisions obtained above are substituted into equations (7), (8) and (9) to obtain the optimal profits as follows: $π\_{r}^{Offd\*}=\frac{\left[\left(2A-∆\_{C}\right)A+∆\_{C}^{2}\right]h^{2}+\left(4A-∆\_{C}\right)Qh+2Q^{2}}{7h}$, $π\_{t}^{Offd\*}=\frac{\left[\left(A-2∆\_{C}\right)h+Q\right]^{2}}{49h}$, $π\_{sc}^{Offd\*}=\frac{\left[\left(15A-11∆\_{C}\right)A+11∆\_{C}^{2}\right]h^{2}+\left(30A-11∆\_{C}\right)Qh+15Q^{2}}{49h}$.

***A.5 Proof of Corollary 1***

In order to explore the impacts of the price coefficient and the penalty cost to optimal profits in the supply chain, the corresponding first partial derivatives are obtained as follows:

(1) $\frac{∂π\_{r}^{Offd\*}}{∂h}=\frac{h^{2}∆\_{C}^{2}-Ah^{2}∆\_{C}+2\left(A^{2}h^{2}-Q\right)}{7h^{2}}>0$, $\frac{∂π\_{t}^{Offd\*}}{∂h}=\frac{\left(A-2∆\_{C}\right)^{2}h^{2}-Q^{2}}{49h^{2}}>0$,

 $\frac{∂π\_{sc}^{Offd\*}}{∂h}=\frac{11h^{2}∆\_{C}^{2}-11Ah^{2}∆\_{C}+15\left(A^{2}h^{2}-Q^{2}\right)}{49h^{2}}>0$;

(2) $\frac{∂π\_{r}^{Offd\*}}{∂∆\_{C}}=\frac{-\left[\left(A-2∆\_{C}\right)h+Q\right]}{7}<0$, $\frac{∂π\_{t}^{Offd\*}}{∂∆\_{C}}=\frac{-4\left[\left(A-2∆\_{C}\right)h+Q\right]}{49}<0$, $\frac{∂π\_{sc}^{Offd\*}}{∂∆\_{C}}=\frac{-11\left[\left(A-2∆\_{C}\right)h+Q\right]}{49}<0$.

***A.6 Proof of Lemma 5 and Corollary 2***

Optimal results could be obtained through the backward induction method, which is similar to the proofs in Lemmas 1 and 2 and we therefore omit the details. We also omit the proof of Corollary 2 due to the similarity with Corollary 1.

***A.7 Proof of Theorems 1-3***

First, we take the difference of the decision and profits between the scenario B and the scenario Off. The corresponding results are as follows:

$r\_{1}^{B\*}-r\_{1}^{Off\*}=\frac{Ah+Q}{14h}>0$; $π\_{r}^{B\*}-π\_{r}^{Off\*}=\frac{\left(Ah+Q\right)^{2}}{28h}>0$; $π\_{sc}^{B\*}-π\_{sc}^{Off\*}=\frac{11\left(Ah+Q\right)^{2}}{196h}>0$.

Next, we take the difference of the decisions and profits between the scenario B and the scenario On. The corresponding results are as follows:

$r\_{1}^{B\*}-r\_{2}^{On\*}=\frac{9\left(Ah+Q\right)}{28h}>0$; $r\_{2}^{On\*}-r\_{2}^{B\*}=\frac{3\left(Ah+Q\right)}{28h}>0$;

$π\_{r}^{B\*}-π\_{r}^{On\*}=\frac{9\left(Ah+Q\right)^{2}}{56h}>0$; $π\_{t}^{On\*}-π\_{t}^{B\*}=\frac{33\left(Ah+Q\right)^{2}}{784h}>0$; $π\_{sc}^{B\*}-π\_{sc}^{Off\*}=\frac{93\left(Ah+Q\right)^{2}}{784h}>0$.

Finally, we take the difference of the decision and profits between the scenario Off and the scenario On. The corresponding results are as follows:

$r\_{1}^{Off\*}-r\_{2}^{On\*}=\frac{Ah+Q}{4h}>0$; $π\_{r}^{Off\*}-π\_{r}^{On\*}=\frac{\left(Ah+Q\right)^{2}}{8h}>0$; $π\_{sc}^{Off\*}-π\_{sc}^{On\*}=\frac{\left(Ah+Q\right)^{2}}{16h}>0$.

***A.8 Proof of Theorems 4-6***

First, the optimal decisions in scenario B and scenario Offd are concluded. Taking the difference of decisions, we have the following results.

$r\_{1}^{B\*}-r\_{1}^{Offd\*}=-\frac{∆\_{C}}{7}<0$; $r\_{2}^{B\*}-r\_{2}^{Offd\*}=-\frac{2∆\_{C}}{7}<0$; $b^{B\*}-b^{Offd\*}=-\frac{4∆\_{C}}{7}<0$*.*

Therefore, we can prove $r\_{1}^{Offd\*}<r\_{1}^{B\*}$, $ r\_{2}^{Offd\*}<r\_{2}^{B\*}$ and $b^{Offd\*}<b^{B\*}$*.*

The optimal profits in scenario B and scenario Offd are concluded. Taking the difference of profits, we have the following results in the case of $P-C-∆\_{C}>0$*.*

$π\_{r}^{B\*}-π\_{r}^{Offd\*}=\frac{\left[\left(P-C-∆\_{C}\right)+Q\right]∆\_{C}}{7}>0$; $π\_{t}^{B\*}-π\_{t}^{Offd\*}=\frac{4\left[\left(P-C-∆\_{C}\right)+Q\right]∆\_{C}}{49}>0$; $π\_{sc}^{B\*}-π\_{sc}^{Offd\*}=\frac{11\left[\left(P-C-∆\_{C}\right)+Q\right]∆\_{C}}{49}>0$.

Next, the optimal decisions in scenario B and scenario Ond are concluded. Taking the difference of decisions, we have the following results.

$r\_{1}^{B\*}-r\_{1}^{Ond\*}=\frac{21∆\_{C}h}{49h}>0$; $r\_{2}^{B\*}-r\_{2}^{Ond\*}=\frac{-7∆\_{C}h}{49h}<0$; $b^{B\*}-b^{Ond\*}=\frac{-14∆\_{C}h}{49h}<0$.

We obtain the following results after taking the difference of profits in the case of $P-C-∆\_{C}>0$ and $Ah>6Q$. $π\_{r}^{B\*}-π\_{r}^{Ond\*}=\frac{-14h^{2}∆\_{C}^{2}+21\left(Ah^{2}+Qh\right)∆\_{C}}{49h}>0$; $π\_{t}^{B\*}-π\_{t}^{Ond\*}=\frac{\left(7t\_{1}t\_{2}-196h^{4}K^{2}\right)}{1372h^{3}}>0$.

Finally, the comparisons between scenario Offd and scenario Ond are concluded. We have $r\_{1}^{Ond\*}-r\_{1}^{Offd\*}=\frac{-14h∆\_{C}}{49h}<0$, $r\_{2}^{Ond\*}-r\_{2}^{Offd\*}=\frac{21h∆\_{C}}{49h}>0$, $b^{Ond\*}-b^{Offd\*}=\frac{42h∆\_{C}}{49h}>0$, $π\_{t}^{Ond\*}-π\_{t}^{Offd\*}=\frac{\left[Q+h\left(A+∆\_{C}\right)\right]^{2}-\left[Q+h\left(A-2∆\_{C}\right)\right]^{2}}{49h}>0$, $π\_{r}^{Ond\*}-π\_{r}^{Offd\*}=\frac{7h^{2}∆\_{C}^{2}-14\left(Ah^{2}+Qh\right)∆\_{C}}{49h}<0$, $π\_{sc}^{Ond\*}-π\_{sc}^{Offd\*}=\frac{196h^{2}∆\_{C}^{2}-392\left(Ah^{2}+Qh\right)∆\_{C}}{2401h}<0$.

For the profit of the remanufacturer, the numerator can view as a quadratic equation of one variable with respect to $∆\_{C}$. There are two roots, $∆\_{C}=0$ and $∆\_{C}=\frac{2(Ah+Q)}{h}>\frac{Ah-6Q}{2h}$. Therefore, $π\_{r}^{Ond\*}<π\_{r}^{Offd\*}$ always exists on the basis of $∆\_{C}<\frac{Ah-6Q}{2h}$.

For the profit of the supply chain, the numerator can view as a quadratic equation of one variable with respect to $∆\_{C}$. There are two roots, $∆\_{C}=0$ and $∆\_{C}=\frac{392(Ah+Q)}{196h}>\frac{Ah-6Q}{2h}$. Therefore, $π\_{sc}^{Ond\*}<π\_{sc}^{Offd\*}$ always exists on the basis of $∆\_{C}<\frac{Ah-6Q}{2h}$.

***A.9 Proof of Theorem 7***

To figure out the better strategy for the remanufacturer when the offline channel disruption is happened, we compare the highest price decision and the profit of the remanufacturer between two strategies. The results are as follows: $r\_{1}^{Offd\*}-r\_{1}^{Off\*}=\frac{\left(A-2∆\_{C}\right)h+Q}{14h}>0$; $π\_{r}^{Offd\*}-π\_{r}^{Off\*}=\frac{\left[\left(A-2∆\_{C}\right)h+Q\right]^{2}}{28h}>0.$ Therefore, the delay strategy with dual collection channels is better for the profit of the remanufacturer and the environment in the online channel disruption scenario.

To figure out the better strategy for the remanufacturer when the online channel disruption is happened, we compare the highest price decision and the profit of the remanufacturer between two strategies. The results are as follows: $r\_{1}^{Ond\*}-r\_{2}^{On\*}=\frac{(9A-12∆\_{C})h+18Q}{14h}>0$; $∆\_{π}=π\_{r}^{On\*}-π\_{r}^{Ond\*}=\frac{\left(Ah+Q\right)^{2}}{8h}-\frac{(2∆\_{C}^{2}-3A∆\_{C}+A^{2})h^{2}+\left(4A-3∆\_{C}\right)Qh+Q^{2}}{7h}<0$.

Among these, $∆\_{π}>0$ is true due to $S=\left(Ah+Q\right)^{2}h\left(8h^{2}-7\right)+8h\left[Qh\left(4A-3∆\_{C}\right)+Q^{2}\right]>0$, in which the $∆\_{π}<0$ if $S>0$.

# Appendix B

***Extension 5.1***

***B.1 Proof of Theorem 8***

(1) Optimal Results

Through the backward induction method, we obtain the optimal results of extension 5.1(1), which are shown below.

The optimal results in scenario Ond are: $r\_{1}^{1Ond\*}=\frac{\left(1-β\right)h\left(4A-3∆\_{C}\right)-\left(3-4β\right)Q}{\left(7-8β\right)h}$, $b^{1Ond\*}=\frac{2\left(1-2β\right)Ah-\left(5-4β\right)Q+\left(2-β\right)h∆\_{C}}{\left(7-8β\right)h}$, $r\_{2}^{1Ond\*}=\frac{\left(1-4β\right)Ah-2\left(3-2β\right)Q+\left(1+β\right)h∆\_{C}}{\left(7-8β\right)h}$; $π\_{r}^{1Ond\*}=\frac{\left(1-β\right)\left\{\left[\left(2-β\right)∆\_{C}^{2}-3A∆\_{C}+A^{2}\right]h^{2}+\left(4A-3∆\_{C}\right)Qh+Q^{2}\right\}}{h\left(7-8β\right)}$, $π\_{t}^{1Ond\*}=\frac{\left(1-β\right)\left\{\left[A-\left(2β-1\right)∆\_{C}\right]h+Q\right\}^{2}}{h\left(7-8β\right)^{2}}$, $π\_{sc}^{1Ond\*}=\frac{\left(1-β\right)\left\{\left[\left(15-16β\right)A^{2}+\left(12β^{2}-27β+15\right)∆\_{C}^{2}+\left(20β-19\right)A∆\_{C}\right]h^{2}+\left[\left(30-32β\right)A+\left(20β-19\right)∆\_{C}\right]Qh+\left(15-16β\right)Q^{2}\right\}}{h\left(7-8β\right)^{2}}$.

It shows that a higher $Q$ will reduce the offline pricing decisions in scenario Ond, which is also applicable for the online collection price when the transfer ratio is not too high. At the same time, a higher price coefficient will improve both the collection prices and wholesale price.

(2) Sensitivity Analysis

For exploring the impacts of some important parameters to optimal decisions and profits in the supply chain, the corresponding first partial derivatives are obtained and simplified as follows:

$ \frac{∂r\_{1}^{1Ond\*}}{∂P}=\frac{4\left(1-β\right)}{\left(7-8β\right)}>0$, $\frac{∂r\_{2}^{1Ond\*}}{∂P}=\frac{1-4β}{\left(7-8β\right)}>0 \left(β<\frac{1}{4}\right)$, $ \frac{∂b\_{ }^{1Ond\*}}{∂P}=\frac{2\left(1-2β\right)}{\left(7-8β\right)}>0 \left(β<\frac{1}{2}\right)$;

$ \frac{∂r\_{1}^{1Ond\*}}{∂C}=\frac{-4\left(1-β\right)}{\left(7-8β\right)}<0$, $ \frac{∂r\_{2}^{1Ond\*}}{∂C}=-\frac{1-4β}{\left(7-8β\right)}<0 \left(β<\frac{1}{4}\right)$, $ \frac{∂b\_{ }^{1Ond\*}}{∂C}=-\frac{2\left(1-2β\right)}{\left(7-8β\right)}<0 \left(β<\frac{1}{2}\right)$;

$\frac{∂r\_{1}^{1Ond\*}}{∂∆\_{C}}=\frac{-3\left(1-β\right)}{\left(7-8β\right)}<0$, $ \frac{∂r\_{2}^{1Ond\*}}{∂∆\_{C}}=\frac{1+β}{\left(7-8β\right)}>0$, $ \frac{∂b\_{ }^{1Ond\*}}{∂∆\_{C}}=\frac{1+β}{\left(7-8β\right)}>0$.

$ \frac{∂π\_{r}^{1Ond\*}}{∂h}=\frac{\left(1-β\right)\left[h^{2}\left(\frac{3}{2\sqrt{2-β}}A-\sqrt{2-β}∆\_{C}\right)^{2}+\frac{251-280β}{8-4β}Q^{2}\right]}{\left(7-8β\right)h^{2}}>0$, $\frac{∂π\_{t}^{1Ond\*}}{∂h}=\frac{\left(1-β\right)K\left[Ah-\left(2β-1\right)∆\_{C}h-Q\right]}{\left(7-8β\right)^{2}h^{2}}>0$,

$\frac{∂π\_{sc}^{1Ond\*}}{∂h}=\frac{\left[25\left(21-23β\right)A^{2}h^{2}-36\left(19-20β\right)Ah^{2}∆\_{C}+108\left(1-β\right)\left(5-4β\right)h^{2}∆\_{C}^{2}\right]}{36\left(7-8β\right)^{2}h^{2}}>0$;

$\frac{∂π\_{r}^{1Ond\*}}{∂∆\_{C}}=\frac{-\left(1-β\right)\left\{\left[3A-\left(4-2β\right)∆\_{C}\right]h+3Q\right\}}{\left(7-8β\right)}<0$, $\frac{∂π\_{t}^{1Ond\*}}{∂∆\_{C}}=\frac{-\left\{h\left[\left(19-20β\right)A-6\left(1-β\right)\left(5-4β\right)∆\_{C}\right]+\left(19-20β\right)Q\right\}}{\left(7-8β\right)}<0$,

$\frac{∂π\_{sc}^{1Ond\*}}{∂∆\_{C}}=\frac{-\left\{h\left[\left(22-23β\right)A-\left(26β^{2}-60β+34\right)∆\_{C}\right]+\left(22-23β\right)Q\right\}}{\left(7-8β\right)}<0$;

$\frac{∂π\_{r}^{1Ond\*}}{∂β}=\frac{K\left\{h\left[A-\left(5-4β\right)∆\_{C}\right]+Q\right\}}{h\left(7-8β\right)^{2}}>0 \left(β>\frac{5h∆\_{C}-\left(Ah+Q\right)}{4h∆\_{C}} or ∆\_{C}<\frac{Ah+Q}{\left(5-4β\right)h}\right)$,

$\frac{∂π\_{t}^{1Ond\*}}{∂β}=\frac{\left(9-8β\right)Q+\left[\left(9-8β\right)A-\left(16β^{2}-34β+19\right)∆\_{C}\right]h}{h\left(7-8β\right)^{3}}>0 \left(β>\frac{9-\sqrt{65}}{16}\right)$,

$\frac{∂π\_{sc}^{1Ond\*}}{∂β}=\frac{K\left\{\left(23-24β\right)Q+\left[\left(23-24β\right)A-6\left(8β^{2}-17β+9\right)∆\_{C}\right]h\right\}}{h\left(7-8β\right)^{3}}>0 \left(β>\frac{27-\sqrt{345}}{48}\right)$.

For $\frac{∂π\_{t}^{1Ond\*}}{∂h}$, if $2β-1<0$ exists, then we can get $Ah-\left(2β-1\right)∆\_{C}h-Q>0$ on the basis of $∆h-Q>0$. However, if $2β-1>0$ is true, then we can rewrite the expression $Ah-\left(2β-1\right)∆\_{C}h-Q$ in the numerator into $ \left(2β-1\right)h\left(A-∆\_{C}\right)+\left(2-2β\right)Ah-Q$, where $\left(2β-1\right)h\left(A-∆\_{C}\right)>0$and $\left(2-2β\right)Ah-Q>\left(11-12β\right)Q>0$ are always true. Therefore, $Ah-\left(2β-1\right)∆\_{C}h-Q>0$ and $\frac{∂π\_{t}^{1Ond\*}}{∂h}>0$ always exist.

For $\frac{∂π\_{sc}^{1Ond\*}}{∂h}$, when $25\left(21-23β\right)>\frac{36\left(19-20β\right)}{2}$, that is, $β<\frac{183}{215}≈0.8511$, which is close to $β<\frac{7}{8}=0.875$, there are $25\left(21-23β\right)A^{2}h^{2}-36\left(19-20β\right)Ah^{2}∆\_{C}>0$and $A-2∆\_{C}>0$. Therefore, we consider $\frac{∂π\_{sc}^{1Ond\*}}{∂h}>0$ approximatively.

For $\frac{∂π\_{r}^{1Ond\*}}{∂∆\_{C}}$, $\frac{∂π\_{t}^{1Ond\*}}{∂∆\_{C}}$, and $\frac{∂π\_{sc}^{1Ond\*}}{∂∆\_{C}}$, inequalities $3>\frac{4-2β}{2}$, $19-20β>\frac{6\left(1-β\right)\left(5-4β\right)}{2}$, and $22-23β>\frac{26β^{2}-60β+34}{2}$ can be proved respectively because of $A>2∆\_{C}$, which indicate $\frac{∂π\_{r}^{1Ond\*}}{∂∆\_{C}}<0$, $\frac{∂π\_{t}^{1Ond\*}}{∂∆\_{C}}<0$*,* and $\frac{∂π\_{sc}^{1Ond\*}}{∂∆\_{C}}<0$*.*

Finally, for $\frac{∂π\_{r}^{1Ond\*}}{∂β}$, $\frac{∂π\_{t}^{1Ond\*}}{∂β}$, and $\frac{∂π\_{sc}^{1Ond\*}}{∂β}$, the value ranges of $β$ are $β>\frac{9-\sqrt{65}}{16}≈0.0586$ and $β>\frac{27-\sqrt{345}}{48}≈0.175$according to inequalities $9-8β>\frac{6β^{2}-34β+19}{2}$ and $23-24β>\frac{6\left(8β^{2}-17β+9\right)}{2}$ based on $A>2∆\_{C}$. Therefore, $\frac{∂π\_{t}^{1Ond\*}}{∂β}>0$ and $\frac{∂π\_{sc}^{1Ond\*}}{∂β}>0$ are proved approximatively in most conditions.

Therefore, the impacts of parameters on results in scenario Ond with customer transferring are as follows:

$\frac{∂r\_{1}^{1Ond\*}}{∂∆\_{C}}<0$, $\frac{∂r\_{2}^{1Ond\*}}{∂∆\_{C}}>0$, $\frac{∂b\_{ }^{1Ond\*}}{∂∆\_{C}}>0$; $\frac{∂r\_{1}^{1Ond\*}}{∂P}>0$, $\left\{\begin{matrix}\frac{∂r\_{2}^{1Ond\*}}{∂P}>0, if β\leq \frac{1}{4}\\\frac{∂r\_{2}^{1Ond\*}}{∂P}<0, if \frac{7}{8}>β>\frac{1}{4}\end{matrix}\right.$, $\left\{\begin{matrix}\frac{∂b\_{ }^{1Ond\*}}{∂P}>0, if β\leq \frac{1}{2}\\\frac{∂b\_{ }^{1Ond\*}}{∂P}<0, if \frac{7}{8}>β>\frac{1}{2}\end{matrix}\right.$; $\frac{∂r\_{1}^{1Ond\*}}{∂C}<0$, $\left\{\begin{matrix}\frac{∂r\_{2}^{1Ond\*}}{∂C}<0, if β\leq \frac{1}{4}\\\frac{∂r\_{2}^{1Ond\*}}{∂C}>0, if \frac{7}{8}>β>\frac{1}{4}\end{matrix}\right.$, $\left\{\begin{matrix}\frac{∂b\_{ }^{1Ond\*}}{∂C}<0, if β\leq \frac{1}{2}\\\frac{∂b\_{ }^{1Ond\*}}{∂C}>0, if \frac{7}{8}>β>\frac{1}{2}\end{matrix}\right.$; $\frac{∂π\_{r}^{1Ond\*}}{∂h}>0$, $\frac{∂π\_{t}^{1Ond\*}}{∂h}>0$, $\frac{∂π\_{sc}^{1Ond\*}}{∂h}>0$; $\frac{∂π\_{r}^{1Ond\*}}{∂∆\_{C}}<0$, $\frac{∂π\_{t}^{1Ond\*}}{∂∆\_{C}}<0$, $\frac{∂π\_{sc}^{1Ond\*}}{∂∆\_{C}}<0$.

It shows the influence of parameters on decisions and profits when the online disruption happens with the delay strategy. The impacts on online collection price and profits are the same as Corollaries 1 and 2. However, the offline pricing decisions increase with higher penalty cost. Besides, the impacts of sales prices and production cost on offline decisions depend on customer transfer ratio. For example, higher sales price generates higher pricing decisions when $β\leq \frac{1}{4}$, which has the opposite effect when $\frac{7}{8}>β>\frac{1}{4}$. Because with high transfer ratio, the offline channel has more customers from online channel, which enables members to appropriately reduce offline prices.

(3) Decisions Comparisons

Next, the optimal decisions in scenario B and scenario 1Ond are compared. Taking the difference of decisions, we have the following results. $r\_{1}^{B\*}-r\_{1}^{1Ond\*}=-\frac{4β\left(Ah+Q\right)-21\left(1-β\right)∆\_{C}h}{7h\left(7-8β\right)}$; $r\_{2}^{B\*}-r\_{2}^{1Ond\*}=\frac{20β\left(Ah+Q\right)-7\left(1+β\right)∆\_{C}h}{7h\left(7-8β\right)}$; $b^{B\*}-b^{1Ond\*}=\frac{12β\left(Ah+Q\right)-7\left(2-β\right)∆\_{C}h}{7h\left(7-8β\right)}$.

We set up $β\_{1}=\frac{14h∆\_{C}}{40\left(Ah+Q\right)-14h∆\_{C}}$, $β\_{2}=\frac{21h∆\_{C}}{21h∆\_{C}+4\left(Ah+Q\right)}$, and $β\_{3}=\frac{14h∆\_{C}}{7h∆\_{C}+12\left(Ah+Q\right)}$ to simplify the expression under the background of $β<\frac{7}{8}$. For the analysis of $r\_{2}$, $r\_{2}^{B\*}-r\_{2}^{Ond\*}>0$ exists if $β>β\_{1}$, and $r\_{2}^{B\*}-r\_{2}^{Ond\*}<0$ is true when $β\leq β\_{1}$. For $r\_{1}$, $r\_{1}^{B\*}-r\_{1}^{Ond\*}>0$ is true if $β<β\_{2}$, or $r\_{1}^{B\*}-r\_{1}^{Ond\*}<0$. For the wholesale pricing decision $b$, there is $b^{B\*}-b^{Ond\*}>0$ when $β>β\_{3}$, and $b^{B\*}-b^{Ond\*}<0$ is true on the contrary.

To figure out the changes of decisions with different transfer ratio values, the relationships among $β\_{1}$, $β\_{2}$, and $β\_{3}$ are compared. It is found that $β\_{2}-β\_{3}=\frac{2h\left(4A-7∆\_{C}\right)+8Q}{\left[21h∆\_{C}+4\left(Ah+Q\right)\right]\left[40\left(Ah+Q\right)-14h∆\_{C}\right]}>0$ and $β\_{3}-β\_{1}=\frac{7h\left(4A-3∆\_{C}\right)+28Q}{\left[40\left(Ah+Q\right)-14h∆\_{C}\right]\left[40\left(Ah+Q\right)-14h∆\_{C}\right]}>0$ on the basis of $∆\_{C}<\frac{A}{2}$. Therefore, we can get $β\_{1}<β\_{3}<β\_{2}$ because of $β\_{2}>β\_{3}$ and $β\_{3}>β\_{1}$.

(4) Profits Comparisons

For profit comparisons, we obtain the following results after taking the difference of profits in the case of $P-C-∆\_{C}>0$, $Ah>6Q$,and $β<\frac{7}{8}$; $π\_{r}^{B\*}-π\_{r}^{1Ond\*}=\frac{-7\left(1-β\right)\left(2-β\right)h^{2}∆\_{C}^{2}+21\left(1-β\right)\left(Ah^{2}+Qh\right)∆\_{C}-2β\left(A^{2}h^{2}+2AQh+Q^{2}\right)}{7h\left(7-8β\right)}$; $π\_{t}^{B\*}-π\_{t}^{1Ond\*}=\frac{\left(7t\_{1}t\_{2}-196h^{4}K^{2}\right)-\left(8t\_{1}t\_{2}-196h^{4}K^{2}\right)β}{196\left(7-8β\right)h^{3}}$; $π\_{sc}^{B\*}-π\_{sc}^{1Ond\*}=\frac{-B\_{1}\left(2-β\right)h^{2}∆\_{C}^{2}+3B\_{1}h\left(∆h+Q\right)∆\_{C}+B\_{2}-2B\_{1}\left(Ah+Q\right)^{2}}{196h^{3}\left(7-8β\right)^{2}}$.

For the profit of the remanufacturer, the denominator of their difference is always positive, so the sign of the difference depends on the numerator. The expression in the numerator is a quadratic equation of one variable with respect to $∆\_{C}$, in which the quadratic coefficient and the constant are negative, and the discriminant is $56\left(1-β\right)\left(β-9\right)\left(β-\frac{7}{8}\right)>0$. Therefore, two roots exist when $∆\_{C}>0$, where the expression of the axis of symmetry is $D\_{1}=\frac{21\left(Ah^{2}+Qh\right)}{2\left(2-β\right)}$. Besides, $D\_{1}-\left(\frac{A}{2}-\frac{3Q}{h}\right)=\frac{Ah\left[21h^{2}-\left(2-β\right)\right]+21Qh^{2}+6Q\left(2-β\right)}{2h\left(2-β\right)}>0$ is always true based on $∆\_{C}<\frac{A}{2}-\frac{3Q}{h}$. There exists a threshold value $∆\_{C1}=\frac{21\left(1-β\right)\left(Ah+Q\right)-\left(Ah+Q\right)\sqrt{56\left(1-β\right)\left(β-9\right)\left(β-\frac{7}{8}\right)}}{14h\left(1-β\right)\left(2-β\right)}$, which makes $π\_{r}^{B\*}-π\_{r}^{1Ond\*}\leq 0$ when $0<∆\_{C}\leq ∆\_{C1}$, but $π\_{r}^{B\*}>π\_{r}^{1Ond\*}$ on the contrary.

For the profit of the third party, the analysis is similar to above. The expression in the numerator is a linear equation of one variable with respect to $β$, in which $\left(7t\_{1}t\_{2}-196h^{4}K^{2}\right)-\left(8t\_{1}t\_{2}-196h^{4}K^{2}\right)β>0$makes $π\_{t}^{B\*}-π\_{t}^{Ond\*}>0$ exist. If $7t\_{1}t\_{2}-196h^{4}K^{2}>0$, then $8t\_{1}t\_{2}-196h^{4}K^{2}>7t\_{1}t\_{2}-196h^{4}K^{2}$ is obviously true, which makes$β<β\_{4}=\frac{7t\_{1}t\_{2}-196h^{4}K^{2}}{8t\_{1}t\_{2}-196h^{4}K^{2}}<1$always true. However, $β>β\_{4}>1$ always exists due to the existence of $7t\_{1}t\_{2}-196h^{4}K^{2}<8t\_{1}t\_{2}-196h^{4}K^{2}<0$ when $8t\_{1}t\_{2}-196h^{4}K^{2}<0$. In conclusion, $π\_{t}^{B\*}<π\_{t}^{1Ond\*}$ exists if $8t\_{1}t\_{2}-196h^{4}K^{2}<0$, or in the condition of $7t\_{1}t\_{2}-196h^{4}K^{2}>0$and $β\_{4}<β<1$. Otherwise, $π\_{t}^{B\*}>π\_{t}^{1Ond\*}$ happens.

Finally, similar comparison is made for the profit in the whole supply chain. The expression in the numerator is a quadratic equation of one variable with respect to $∆\_{C}$, in which the quadratic coefficient is negative, and the discriminant is $9B\_{1}^{2}h\left(Ah+Q\right)^{2}+4B\_{1}h^{2}\left(2-β\right)\left[B\_{2}-2B\_{1}\left(Ah+Q\right)^{2}\right]>0$. The sign of constant $B\_{2}-2B\_{1}\left(Ah+Q\right)^{2}$ depends on the conditions of the recycled and remanufactured product market. Besides, the expression of the axis of symmetry is $D\_{2}=\frac{Ah+Q}{\left(5-4β\right)h}$, which makes $D\_{2}-\left(\frac{A}{2}-\frac{3Q}{h}\right)=\frac{\left(4β-3\right)Ah+8\left(4-3β\right)Q}{2h\left(5-4β\right)}>\frac{14Q}{2h\left(5-4β\right)}>0$ exist with the assumption $∆\_{C}<\frac{A}{2}-\frac{3Q}{h}$. Therefore, $π\_{sc}^{B\*}-π\_{sc}^{Ond\*}>0$ always exists if $\left(Ah+Q\right)^{2}<\frac{B\_{2}}{2B\_{1}}$ is true, which indicates the market is poor. Otherwise, if $\left(Ah+Q\right)^{2}>\frac{B\_{2}}{2B\_{1}}$ is true（the market is good enough）, a threshold value $∆\_{C2}=\frac{3B\_{1}h\left(Ah+Q\right)-\sqrt{9B\_{1}^{2}h\left(Ah+Q\right)^{2}+4B\_{1}h^{2}\left(2-β\right)\left[B\_{2}-2B\_{1}\left(Ah+Q\right)^{2}\right]}}{2B\_{1}\left(2-β\right)h^{2}}$ exists to make$π\_{sc}^{B\*}-π\_{sc}^{1Ond\*}<0$ in the condition of $0<∆\_{C}\leq ∆\_{C2}$, or make$π\_{sc}^{B\*}>π\_{sc}^{1Ond\*}$when $∆\_{C}>∆\_{C2}$.

***B.2 Proof of Theorem 9***

The optimal decisions in scenario Offd and scenario 1Ond are compared. With assumption of $7-8β>0$, there is $r\_{1}^{1Ond\*}-r\_{1}^{Offd\*}=\frac{4β\left(Ah+Q\right)-\left(14-13β\right)h∆\_{C}}{7h\left(7-8β\right)}\geq 0$if $∆\_{C}\leq ∆\_{C3}=\frac{4β\left(Ah+Q\right)}{\left(14-13β\right)h}$, but $r\_{1}^{1Ond\*}<r\_{1}^{Offd\*}$ exists when $∆\_{C}>∆\_{C3}$. Next, $r\_{2}^{1Ond\*}-r\_{2}^{Offd\*}=\frac{\left(21-9β\right)h∆\_{C}-20β\left(Ah+Q\right)}{7h\left(7-8β\right)}>0$ is always true when $∆\_{C}>∆\_{C4}=\frac{20β\left(Ah+Q\right)}{\left(21-9β\right)h}$, but $r\_{2}^{1Ond\*}\leq r\_{2}^{Offd\*}$ exists on the contrary. Finally, we can obtain $b^{1Ond\*}-b^{Offd\*}=\frac{\left(42-39β\right)h∆\_{C}-12β\left(Ah+Q\right)}{7h\left(7-8β\right)}>0$ when $∆\_{C}>∆\_{C5}=\frac{12β\left(Ah+Q\right)}{\left(42-39β\right)h}$, otherwise $b^{1Ond\*}\leq b^{Offd\*}$.

The optimal profits in scenario Offd and scenario 1Ond are also concluded. For the profit of the third party, there is $π\_{t}^{1Ond\*}-π\_{t}^{Offd\*}=\frac{\left(1-β\right)}{\left(7-8β\right)^{2}h}\left\{Q+h\left[A-\left(2β-1\right)∆\_{C}\right]\right\}^{2}-\frac{1}{49h}\left[Q+h\left(A-2∆\_{C}\right)\right]^{2}$, in which $8β-63<0$is always right. Therefore, $\frac{\left(1-β\right)}{\left(7-8β\right)^{2}h}>\frac{1}{49h}$ and $\left\{Q+h\left[A-\left(2β-1\right)∆\_{C}\right]\right\}^{2}>\left[Q+h\left(A-2∆\_{C}\right)\right]^{2}$ always exist, which make $\frac{\left(1-β\right)}{\left(7-8β\right)^{2}h}\left\{Q+h\left[A-\left(2β-1\right)∆\_{C}\right]\right\}^{2}>\frac{1}{49h}\left[Q+h\left(A-2∆\_{C}\right)\right]^{2}$*.* Thus, we can conclude $π\_{t}^{1Ond\*}-π\_{t}^{Offd\*}>0$.

For the profit of the remanufacturer, there is $π\_{r}^{1Ond\*}-π\_{r}^{Offd\*}=\frac{\left(7β^{2}-13β+7\right)h^{2}∆\_{C}^{2}+\left(13β-14\right)\left(Ah^{2}+Qh\right)∆\_{C}+\left(4βAhQ+2βA^{2}h^{2}+2βQ^{2}\right)}{7h\left(7-8β\right)}$, where the denominator is always positive and the numerator can view as a quadratic equation of one variable with respect to $∆\_{C}$. We know $7β^{2}-13β+7>0$, $13β-14<0$, and $4βAhQ+2βA^{2}h^{2}+2βQ^{2}>0$according to the value range of $β$, which means the value of the quadratic coefficient, the axis of symmetry, and the constant are all positive. Besides, the discriminant of it is $-56β^{3}+273β^{2}-420β+196$, and $-56β^{3}+273β^{2}-420β+196>0$ exists because the first derivative of the discriminant is $-β^{2}-420β+196<0$. Moreover, the expression of the axis of symmetry is $D\_{3}=\frac{\left(13β-14\right)\left(Ah^{2}+Qh\right)}{2\left(7β^{2}-13β+7\right)h}$ which satisfies $D\_{3}-\frac{Ah-6Q}{2h}=\frac{7\left(1-β^{2}\right)Ah+7\left(6β^{2}-13β+8\right)Q}{2\left(7β^{2}-13β+7\right)h}>0$ on the basis of $∆\_{C}<\frac{Ah-6Q}{2h}$. Therefore, there is a threshold value $∆\_{C6}=\frac{\left[\left(14-13β\right)-\sqrt{-56β^{3}+273β^{2}-420β+196}\right]\left(Ah+Q\right)}{2\left(7β^{2}-13β+7\right)h}$ for the remanufacturer. There is $π\_{r}^{1Ond\*}-π\_{r}^{Offd\*}>0$ when the penalty cost is not higher than the threshold value, but $π\_{r}^{1Ond\*}<π\_{r}^{Offd\*}$ always exists when $∆\_{C}>∆\_{C6}$.

Finally, from the perspective of the profit in the supply chain, we have $π\_{sc}^{1Ond\*}-π\_{sc}^{Offd\*}=\frac{\left(-588β^{3}+1207β^{2}-826β+196\right)h^{2}∆\_{C}^{2}+\left(-276β^{2}+679β-392\right)\left(Ah^{2}+Qh\right)∆\_{C}+β\left(161-176β\right)\left(\frac{352}{176}AhQ+A^{2}h^{2}+Q^{2}\right)}{49h\left(7-8β\right)^{2}}$ which can be analyzed similarly. We know $-588β^{3}+1207β^{2}-826β+196>0$, $-276β^{2}+679β-392<0$, and $161-176β>0$ according to the value range of $β$, which also indicates the value of the quadratic coefficient, the axis of symmetry, and the constant are all positive. Besides, the function of the discriminant of it is $f\left(β\right)=-8448β^{5}+26624β^{4}-35380β^{3}+27497β^{2}-13440β+3136$, which is analyzed more complex. The derivatives of the first order to the third order in the value range of $β$ are obtained as follows:

$f\left(β\right)^{'''}=-506880β^{2}+638976β-212280<0$, $f\left(β\right)^{''}$ is monotone decreasing;

$f\left(β\right)^{''}=-168960β^{3}+319488β^{2}-212280β+54994>f\left(\frac{7}{8}\right)^{''}>0$, $f\left(β\right)^{'}$ is monotone increasing;

$f\left(β\right)^{'}=-42240β^{4}+106496β^{3}-106140β^{2}+54994β-13440<f\left(\frac{7}{8}\right)^{'}<0$, $f\left(β\right)$ is monotone decreasing.

Therefore, $f\left(β\right)>f\left(\frac{7}{8}\right)>0$ exists. There are two roots for this quadratic equation of one variable. Because the axis of symmetry is $D\_{4}=\frac{\left(276β^{2}-679β+392\right)\left(Ah^{2}+Qh\right)}{2\left(-588β^{3}+1207β^{2}-826β+196\right)h}$ which satisfies $D\_{4}-\frac{Ah-6Q}{2h}=\frac{Ah\left(588β^{3}-931β^{2}+147β+196\right)+Q\left[\left(276β^{2}-679β+392\right)+6\left(-588β^{3}+1207β^{2}-826β+196\right)\right]}{2\left(-588β^{3}+1207β^{2}-826β+196\right)h}>0$ based on $∆\_{C}<\frac{Ah-6Q}{2h}$, $∆\_{C7}=\frac{\left[\left(-276β^{2}+679β-392\right)-7\sqrt{-8448β^{5}+26624β^{4}-35380β^{3}+27497β^{2}-13440β+3136}\right]\left(Ah+Q\right)}{2\left(588β^{3}-1207β^{2}+826β-196\right)h}$ as a threshold exists in the supply chain. There is $π\_{sc}^{1Ond\*}-π\_{sc}^{Offd\*}>0$ when the unit remanufacturing cost is not higher than the threshold value, otherwise $π\_{sc}^{1Ond\*}<π\_{sc}^{Offd\*}$ is true.

***B.3 Proof of Theorem 10***

(1) Optimal Results

Through the backward induction method, we obtain the optimal results of extension 5.1(2), which are shown below:$r\_{1}^{1Offd\*}=\frac{\left[4A-\left(1-β\right)∆\_{C}\right]h-\left(3+β\right)Q}{\left(7+β\right)h}$, $b^{1Offd\*}=\frac{2h\left(A-2∆\_{C}\right)-\left(5+β\right)Q}{\left(7+β\right)h}$, $r\_{2}^{1Offd\*}=\frac{h\left(A-2∆\_{C}\right)-\left(6+β\right)Q}{\left(7+β\right)h}$.

(2) Results Comparisons

The difference between them is as follows: $r\_{1}^{Offd\*}-r\_{1}^{1Offd\*}=\frac{4\left[\left(A-2∆\_{C}\right)h+Q\right]β}{7\left(7+β\right)h}>0$, $r\_{2}^{Offd\*}-r\_{2}^{1Offd\*}=\frac{\left[\left(A-2∆\_{C}\right)h+Q\right]β}{7\left(7+β\right)h}>0$, $b^{Offd\*}-b^{1Offd\*}=\frac{2\left[\left(A-2∆\_{C}\right)h+Q\right]β}{7\left(7+β\right)h}$. Therefore, $r\_{1}^{Offd\*}>r\_{1}^{1Offd\*}$, $r\_{2}^{Offd\*}>r\_{2}^{1Offd\*}$, and $b^{Offd\*}>b^{1Offd\*}$ are proved, which means the optimal decisions are lower when the offline disruption between the third party and customers compared with the disruption between the third party and the remanufacturer.

We calculate the first partial derivatives for the difference with respect to $β$, and obtain $\frac{∂\left(r\_{1}^{Offd\*}-r\_{1}^{1Offd\*}\right)}{∂β}=\frac{196h\left[\left(A-2∆\_{C}\right)h+Q\right]}{\left[7\left(7+β\right)h\right]^{2}}>0$, $\frac{∂\left(r\_{2}^{Offd\*}-r\_{2}^{1Offd\*}\right)}{∂β}=\frac{49h\left[\left(A-2∆\_{C}\right)h+Q\right]}{\left[7\left(7+β\right)h\right]^{2}}>0$, and $\frac{∂\left(b^{Offd\*}-b^{1Offd\*}\right)}{∂β}=\frac{98h\left[\left(A-2∆\_{C}\right)h+Q\right]}{\left[7\left(7+β\right)h\right]^{2}}>0$. Therefore, the greater the transfer ratio is, the greater the difference between the two offline disruption conditions will be.

Next, the comparison of profits is conducted. The optimal profits of the second offline disruption are obtained by introducing the optimal solutions into the profit functions. Besides, the difference between these profits is as follows: $π\_{t}^{Offd\*}-π\_{t}^{1Offd\*}=\frac{\left(β+63\right)\left[\left(A-2∆\_{C}\right)+Q\right]^{2}β}{49h\left(7+β\right)^{2}}>0$, $π\_{r}^{Offd\*}-π\_{r}^{1Offd\*}=\frac{2\left[\left(A-2∆\_{C}\right)+Q\right]^{2}β}{7h\left(7+β\right)}>0$.

***Extension 5.2***

***B.4 Proof of Corollary 3***

According to the backward inducive method, the optimal decisions with partial disruptions are obtained as follows:

$r\_{1}^{2Offd\*}=\frac{h\left(4A-θ∆\_{C}\right)-3Q}{7h}$, $b^{2Offd\*}=\frac{2h\left(A-2θ∆\_{C}\right)-5Q}{7h}$, $r\_{2}^{2Offd\*}=\frac{h\left(A-2θ∆\_{C}\right)-6Q}{7h}$; $r\_{1}^{2Ond\*}=\frac{\left\{4A-\left[\left(4A-3∆\_{C}\right)β+3∆\_{C}\right]θ\right\}h-\left(3-4βθ\right)Q}{\left(7-8θβ\right)h}$, $b^{2Ond\*}=\frac{\left\{\left[\left(4A+∆\_{c}\right)β-∆\_{c}\right]h-4βQ\right\}βQ^{2}+\left\{9Qβ+h\left[2∆\_{c}-\left(6A+2∆\_{c}\right)β\right]\right\}θ+2Ah-5Q}{\left(1-βθ\right)\left(7-8βθ\right)h}$, $r\_{2}^{2Ond\*}=\frac{\left\{\left[\left(4A-∆\_{c}\right)β+∆\_{c}\right]h-4βQ\right\}βQ^{2}+\left\{10Qβ+h\left[∆\_{c}-\left(5A+∆\_{c}\right)β\right]\right\}θ+Ah-6Q}{\left(1-βθ\right)\left(7-8βθ\right)h}$.

Then we substitute the optimal results into the profit function of each members to gain their optimal profits:

$π\_{r}^{2Offd\*}=\frac{\left[\left(2A-θ∆\_{C}\right)A+(θ∆\_{C})^{2}\right]h^{2}+\left(4A-θ∆\_{C}\right)Qh+2Q^{2}}{7h}$, $π\_{t}^{2Offd\*}=\frac{\left[\left(A-2θ∆\_{C}\right)h+Q\right]^{2}}{49h}$; $π\_{r}^{2Ond\*}=\frac{\left\{\begin{array}{c}-2∆\_{c}^{2}β\left(1-β\right)^{2}θ^{3}+\left[\left(2∆\_{c}^{2}-3A∆\_{c}+2A^{2}\right)β^{2}+∆\_{c}\left(3A-4∆\_{c}\right)β+2∆\_{c}^{2}\right]θ^{2}\\-\left\{\left[\left(4A-3∆\_{c}\right)β+3∆\_{c}\right]Aθ+2A^{2}\right\}h^{2}-Q\left(1-βθ\right)\left\{\left[\left(4A-3∆\_{c}\right)β+3∆\_{c}\right]θ-4a\right\}h+2Q^{2}(1-βθ)^{2}\end{array}\right\}^{2}}{(7-15βθ+8β^{2}θ^{2})h}$, $π\_{t}^{2Ond\*}=\frac{\{2∆\_{c}hβ\left(1-β\right)θ^{2}+\left[\left(A+∆\_{c}\right)β-∆\_{c}\right]hθ+Qβθ-Ah-Q\}^{2}}{\left(1-βθ\right)\left(7-8βθ\right)^{2}h}$.

For the partial offline disruption, it is obviously to find the decisions are all decreasing with the increase of the penalty cost $∆\_{c}$. Therefore, the pricing decisions with partial disruption are higher than these with complete disruptions while still lower than these without disruptions respectively.

As for profits, we have the following comparisons: $π\_{r}^{Offd\*}-π\_{r}^{2Offd\*}=-\frac{\left\{\left[A-\left(1-θ\right)∆\_{C}\right]h+Q\right\}∆\_{C}\left(1-θ\right)}{7}<0$, $π\_{t}^{Offd\*}-π\_{t}^{2Offd\*}=-\frac{4\left\{\left[A-\left(1-θ\right)∆\_{C}\right]h+Q\right\}∆\_{C}\left(1-θ\right)}{49}<0$; $π\_{r}^{B\*}-π\_{r}^{2Offd\*}=\frac{[(A-θ∆\_{C})h+Q]∆\_{C}θ}{7}>0$, $π\_{t}^{B\*}-π\_{t}^{2Offd\*}=\frac{4[(A-θ∆\_{C})h+Q]∆\_{C}θ}{49}>0$.

***B.5 Proof of Corollary 4***

According to the identify of $θ$, it represents the condition without disruption when $θ=0$ while $θ=1$ means the complete disruption. Therefore, we can analyze the relationships of optimal solutions between partial online channel disruption and other two conditions.

As for the online collection price $r\_{1}^{2ond\*}$, we have $\frac{∂r\_{1}^{2ond\*}}{∂θ}=\frac{\left[\left(4A+21∆\_{c}\right)h+4Q\right]β-21∆\_{c}h}{\left(7-8βθ\right)^{2}h}$, in which $\frac{∂r\_{1}^{2ond\*}}{∂θ}>0$ when $β>β\_{2}=\frac{21h∆\_{C}}{21h∆\_{C}+4\left(Ah+Q\right)}$ and $\frac{∂r\_{1}^{2ond\*}}{∂θ}\leq 0$ when $β\leq β\_{2}$. Coincidently, this value threshold is exactly the same as the one for the online collection price comparison between complete online disruption and no disruption. Therefore, when $β>β\_{2}$, there is $r\_{1}^{1ond\*}>r\_{1}^{2ond\*}>r\_{1}^{B\*}$; when $β\leq β\_{2}$, $r\_{1}^{1ond\*}\leq r\_{1}^{2ond\*}\leq r\_{1}^{B\*}$ is always true.

However, for the offline pricing decisions and profits, we only adopt the numerical comparison through figures due to the complexity of the results.

***Extension 5.3***

***B.6 Proof of Corollary 6***

Similar to the previous Lemmas, the optimal solutions are as follows:

(1) $r\_{1}^{3B\*}=\frac{\left[4P-\left(4-t\right)C-tC\_{1}\right]h-3Q}{7h}$, $b^{3B\*}=\frac{\left[2P-2C+\left(3C\_{1}+4C-7f\right)t\right]h-5Q}{7h}$, $r\_{2}^{3B\*}=\frac{\left[P-\left(1-2t\right)C-2tC\_{1}\right]h-6Q}{7h}$;

(2) $r\_{1}^{3Offd\*}=\frac{h\left(4A-∆\_{C}\right)+\left(C+∆\_{C}-C\_{1}\right)ht-3Q}{7h}$, $b^{3Offd\*}=\frac{2h\left(A-2∆\_{C}\right)+\left(4C+4∆\_{C}+3C\_{1}-7f\right)ht-5Q}{7h}$,$r\_{2}^{3Offd\*}=\frac{h\left(A-2∆\_{C}\right)+2\left(C+∆\_{C}-C\_{1}\right)ht-6Q}{7h}$;

(3) $r\_{1}^{3Ond\*}=\frac{h\left(1-β\right)\left[4P-3∆\_{C}-\left(4-t\right)C-tC\_{1}\right]-\left(3-4β\right)Q}{\left(7-8β\right)h}$,

$b^{3Ond\*}=\frac{h\left\{\left[2P+2∆\_{C}-2C+\left(3C\_{1}+4C-7f\right)t\right]-β\left[4P+∆\_{C}-4C+\left(3C\_{1}+5C-8f\right)t\right]\right\}-\left(6-4β\right)Q}{\left(7-8β\right)h}$,

$r\_{2}^{3Ond\*}=\frac{h\left\{\left[P+∆\_{C}-2tC\_{1}-\left(1-2t\right)C\right]-β\left[4P-∆\_{C}-\left(4-2t\right)C-3tC\_{1}\right]\right\}-\left(6-4β\right)Q}{\left(7-8β\right)h}$.

Next, the optimal decisions are compared on the basis of $∆\_{C}>f>C\_{1}>C$. Firstly, we compare the optimal decisions between the offline disruption situation with the hybrid remanufacturing and the normal scenario in the basic model. There are $r\_{1}^{B\*}-r\_{1}^{3Offd\*}=\frac{\left(1-t\right)∆\_{C}+t\left(C\_{1}-C\right)}{7}>0$, $r\_{2}^{B\*}-r\_{2}^{3Offd\*}=\frac{\left(1-t\right)∆\_{C}+t\left(C\_{1}-C\right)}{7}>0$, and $b^{B\*}-b^{3Offd\*}=\frac{4\left(1-t\right)∆\_{C}+t\left(7f-3C\_{1}-4C\right)}{7}>0$, which indicates $r\_{1}^{B\*}>r\_{1}^{3Offd\*}$, $r\_{2}^{B\*}>r\_{2}^{3Offd\*}$, and $b^{B\*}>b^{3Offd\*}$. Therefore, the optimal decisions are still decreasing compared to that of no disruption although the third party will remanufacture together.

Secondly, we analyze the difference of decisions when the offline disruption occurs. There is $f>\frac{3C\_{1}+4C+4∆\_{C}}{7}$ before the analysis, which means the agency fee will not be too high and is consistent with the reality. Therefore, we get $r\_{1}^{Offd\*}-r\_{1}^{3Offd\*}=-\frac{\left(C+∆\_{C}-C\_{1}\right)ht}{7h}<0$, $r\_{2}^{Offd\*}-r\_{2}^{3Offd\*}=-\frac{2\left(C+∆\_{C}-C\_{1}\right)ht}{7h}<0$, and $b^{Offd\*}-b^{3Offd\*}=-\frac{\left(4C+4∆\_{C}+3C\_{1}-7f\right)}{7h}<0$ respectively, that is, $r\_{1}^{Offd\*}<r\_{1}^{3Offd\*}$, $r\_{2}^{Offd\*}<r\_{2}^{3Offd\*}$, and $b^{Offd\*}<b^{3Offd\*}$ exist. The decisions are increasing in the offline disruption scenario with the hybrid remanufacturing.

Next, we compare the optimal solutions when the online channel or no channel is disrupted respectively. When there is no disruption, we can prove$r\_{1}^{B\*}-r\_{1}^{3B\*}=\frac{t\left(C\_{1}-C\right)}{7}>0$, $r\_{2}^{B\*}-r\_{2}^{3B\*}=\frac{2t\left(C\_{1}-C\right)}{7}>0$, and $b^{B\*}-b^{3B\*}=\frac{t\left(7f-3C\_{1}-4C\right)}{7}>0$. However, we consider $β<\frac{2}{3}$ which is closed to reality when the online channel is disrupted, and prove $r\_{1}^{Ond\*}-r\_{1}^{3Ond\*}=\frac{\left(1-β\right)t\left(C\_{1}-C\right)}{7-8β}>0$, $r\_{2}^{Ond\*}-r\_{2}^{3Ond\*}=\frac{\left(2-3β\right)t\left(C\_{1}-C\right)}{7-8β}>0$, and $b^{Ond\*}-b^{3Ond\*}=\frac{\left[\left(5β-4\right)C+\left(3β-3\right)C\_{1}+\left(7-8β\right)f\right]t}{7-8β}>\frac{\left(7-8β\right)\left(f-C\right)t}{7-8β}>0$. Therefore, there are $r\_{1}^{B\*}>r\_{1}^{3B\*}$, $r\_{2}^{B\*}>r\_{2}^{3B\*}$, $b^{B\*}>b^{3B\*}$, $r\_{1}^{Ond\*}>r\_{1}^{3Ond\*}$, $r\_{2}^{Ond\*}>r\_{2}^{3Ond\*}$, and $b^{Ond\*}>b^{3Ond\*}$, which means the optimal decisions are not improved through the hybrid remanufacturing when there is no disruption and the online disruption.

Finally, we focus on the changes of profits. The optimal profits are obtained through substituting the optimal decisions into the profit functions. We get the difference of profits as follows:

(1) $π\_{t}^{B\*}-π\_{t}^{3Offd\*}=\frac{4\left[\left(1-t\right)∆\_{C}+t\left(C\_{1}-C\right)\right]\left\{h\left[\left(A-∆\_{C}\right)+t\left(∆\_{C}+C-C\_{1}\right)\right]+Q\right\}}{49}>0$,

$π\_{r}^{B\*}-π\_{r}^{3Offd\*}=\frac{\left[\left(1-t\right)∆\_{C}+t\left(C\_{1}-C\right)\right]\left\{h\left[\left(A-∆\_{C}\right)+t\left(∆\_{C}+C-C\_{1}\right)\right]+Q\right\}}{7}>0$;

(2) $π\_{t}^{Offd\*}-π\_{t}^{3Offd\*}=-\frac{4t\left(∆\_{C}+C-C\_{1}\right)\left\{h\left[\left(A-∆\_{C}\right)+t\left(∆\_{C}+C-C\_{1}\right)\right]+Q\right\}}{49}<0$,

$π\_{r}^{Offd\*}-π\_{r}^{3Offd\*}=-\frac{t\left(∆\_{C}+C-C\_{1}\right)\left\{h\left[\left(A-∆\_{C}\right)+t\left(∆\_{C}+C-C\_{1}\right)\right]+Q\right\}}{7}<0$;

(3) $π\_{t}^{B\*}-π\_{t}^{3B\*}=\frac{4t\left(C\_{1}-C\right)\left\{\left[P-\left(1-t\right)C-tC\_{1}\right]h+Q\right\}}{49}>0$,

$π\_{r}^{B\*}-π\_{r}^{3B\*}=\frac{t\left(C\_{1}-C\right)\left\{\left[P-\left(1-t\right)C-tC\_{1}\right]h+Q\right\}}{7}>0$;

(4) $π\_{t}^{1Ond\*}-π\_{t}^{3Ond\*}=\frac{4t\left(1-β\right)^{2}\left(C\_{1}-C\right)\left\{\left[\left(A-2β∆\_{C}\right)+\left(1-β\right)tC+∆\_{C}-\left(1-β\right)tC\_{1}\right]h+Q\right\}}{\left(7-8β\right)^{2}}>0$,

$π\_{r}^{1Ond\*}-π\_{r}^{3Ond\*}=\frac{t\left(1-β\right)\left(C\_{1}-C\right)\left\{\left[\left(A-2β∆\_{C}\right)+\left(1-β\right)tC+∆\_{C}-\left(1-β\right)tC\_{1}\right]h+Q\right\}}{7-8β}>0$.

***B.7 Threshold values***

To simplify the proof and the expression, we set up some threshold values as follows:

$A=P-C$, $K=h\left[A-\left(2β-1\right)∆\_{C}+Q\right]$;

$t\_{1}=\left(7h-5\right)Q+2Ah$, $t\_{2}=\left(4h-2\right)Ah-\left(3h-5\right)Q$;

$B\_{1}=196h^{2}\left(1-β\right)\left(7-8β\right)$, $B\_{2}=\left[56h^{2}\left(Ah+Q\right)^{2}+t\_{1}t\_{2}\right]\left(7-8β\right)^{2}-196\left(1-β\right)h^{2}K^{2}$;

$β\_{2}=\frac{21h∆\_{C}}{21h∆\_{C}+4\left(Ah+Q\right)}$, $β\_{1}=\frac{14h∆\_{C}}{40\left(Ah+Q\right)-14h∆\_{C}}$, $β\_{3}=0\frac{14h∆\_{C}}{7h∆\_{C}+12\left(Ah+Q\right)}$, $β\_{4}=\frac{7t\_{1}t\_{2}-196h^{4}K^{2}}{8t\_{1}t\_{2}-196h^{4}K^{2}}$;

$∆\_{C1}=\frac{\left(Ah+Q\right)\left[21\left(1-β\right)-\sqrt{\left(1-β\right)\left(β-9\right)\left(β-\frac{7}{8}\right)}\right]}{14\left(1-β\right)\left(2-β\right)h}$,

$∆\_{C2}=\frac{3B\_{1}h\left(Ah+Q\right)-\sqrt{9B\_{1}^{2}h\left(Ah+Q\right)^{2}+4B\_{1}h^{2}\left(2-β\right)\left[B\_{2}-2B\_{1}\left(Ah+Q\right)^{2}\right]}}{2B\_{1}\left(2-β\right)h^{2}}$,

$∆\_{C3}=\frac{4β\left(Ah+Q\right)}{\left(14-13β\right)h}$, $∆\_{C4}=\frac{20β\left(Ah+Q\right)}{\left(21-9β\right)h}$, $∆\_{C5}=\frac{12β\left(Ah+Q\right)}{\left(42-39β\right)h}$,

$∆\_{C6}=\frac{\left(Ah+Q\right)\left[\left(14-13β\right)-\sqrt{-56β^{3}+273β^{2}-420β+196}\right]}{2h\left(7β^{2}-13β+7\right)}$,

$∆\_{C7}=\frac{\left(Ah+Q\right)\left[\left(-276β^{2}+679β-392\right)-7\sqrt{-8448β^{5}+26624β^{4}-35380β^{3}+27497β^{2}-13440β+3136}\right]}{2h\left(588β^{3}-1207β^{2}+826β-196\right)}$.