Supplementary material for penalized empirical likelihood inference for the GINAR(p) model

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In Section 5 of this paper, we examine the performance of order selection and estimation of the PEL method through simulation study. The simulation results about Case II-IV are presented here.

Tables 1-3 report results of order selection for Case II-IV, which includes the percentage of correctly estimated, underestimated and overestimated numbers of models orders. We can see that the PEL performs well in order selection. The percentage of correctly estimated true model structure tends to 100% rapidly as sample size increases.

The means and standard deviations (SD) of the PEL estimators for Case II-IV are summarized in Tables 4-6. The results show that the means tend to the true value and the SD decrease as sample size increases. The estimation results indicate that the PEL estimator is consistent.

Now, we want to further examine on the performance of the penalized empirical likelihood ratio test. A common hypothesis is for the significance of the individual component of $\boldsymbol{\theta}_{10}$. Hence, we choose the null hypothesis as H_0: $\alpha_1 = 0.3 + \delta$, where $\delta = -0.02, -0.01, 0, 0.01, 0.02$, respectively. Using a nominal level $\alpha = 0.05$, we compute the empirical percentage of rejecting the null hypothesis.

The results of empirical size and power are summarized in Table 7. We can see that the size of test appears to be close to the nominal level as sample size increases, and the power increases as either the sample size increases or the value of δ deviates far from 0. Figure 1 shows the QQ plots of the PEL ratio test statistic against the theoretical distribution χ^2_1 distribution under the H_0 for Case I ($n = 800$). We can see that the distribution of the penalized empirical likelihood ratio test statistic is close to the theoretical distribution. The results of the rest of cases are similar to Case I so we omit them.

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Table 1: The order selection results for Case II.

Sample size	Penalty function	Under-estimated		Correctly-estimated		Over-estimated	
		PEL	PCLS	PEL	PCLS	PEL	PCLS
$n = 300$	ALASSO	0.0440	0.1380	0.7450	0.6450	0.2110	0.2170
	SCAD	0.0100	0.0470	0.6590	0.5700	0.3310	0.3830
$n = 500$	ALASSO	0.0070	0.0230	0.8390	0.7900	0.1540	0.1870
	SCAD	0.0000	0.0030	0.7320	0.6760	0.2680	0.3210
$n = 800$	ALASSO	0.0000	0.0040	0.9560	0.8630	0.0440	0.1330
	SCAD	0.0000	0.0000	0.8320	0.7990	0.1680	0.2010

Table 2: The order selection results for Case III.

Sample size	Penalty function	Under-estimated		Correctly-estimated		Over-estimated	
		PEL	PCLS	PEL	PCLS	PEL	PCLS
$n = 300$	ALASSO	0.0500	0.1320	0.7620	0.6280	0.1880	0.2400
	SCAD	0.0120	0.0460	0.6820	0.5240	0.3060	0.4300
$n = 500$	ALASSO	0.0160	0.0200	0.8060	0.7670	0.1780	0.2130
	SCAD	0.0010	0.0020	0.7530	0.7030	0.2460	0.2950
$n = 800$	ALASSO	0.0000	0.0040	0.9230	0.8460	0.0770	0.1500
	SCAD	0.0000	0.0000	0.8340	0.7950	0.1660	0.2050

Table 3: The order selection results for Case IV.

Sample size	Penalty function	Under-estimated		Correctly-estimated		Over-estimated	
		PEL	PCLS	PEL	PCLS	PEL	PCLS
$n = 300$	ALASSO	0.0500	0.1170	0.7230	0.6170	0.2270	0.2660
	SCAD	0.0070	0.0410	0.6220	0.5510	0.3710	0.4080
$n = 500$	ALASSO	0.0150	0.0210	0.8230	0.7570	0.1620	0.2220
	SCAD	0.0030	0.0020	0.7590	0.6750	0.2380	0.3230
$n = 800$	ALASSO	0.0000	0.0020	0.9140	0.8650	0.0860	0.1330
	SCAD	0.0000	0.0000	0.8170	0.7790	0.1830	0.2210

Table 4: The estimation results of the PEL and the PCLS for Case II.

Sample size		ALASSO				SCAD			
		PEL		PCLS		PEL		PCLS	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$n = 300$	$\hat{\alpha}_1$	0.2948	0.0649	0.2868	0.0856	0.2937	0.0594	0.2841	0.0819
	$\hat{\alpha}_5$	0.2337	0.0663	0.2197	0.0920	0.2346	0.0655	0.2188	0.0898
	$\hat{\alpha}_8$	0.1654	0.0739	0.1530	0.0954	0.1693	0.0714	0.1624	0.0923
	$\hat{\mu}$	0.9121	0.2341	0.9824	0.3296	0.9464	0.2373	1.0347	0.3348
$n = 500$	$\hat{\alpha}_1$	0.2981	0.0471	0.2950	0.0576	0.2963	0.0427	0.2916	0.0527
	$\hat{\alpha}_5$	0.2375	0.0518	0.2362	0.0603	0.2393	0.0505	0.2360	0.0605
	$\hat{\alpha}_8$	0.1708	0.0576	0.1678	0.0658	0.1727	0.0561	0.1715	0.0651
	$\hat{\mu}$	0.8917	0.1697	0.9074	0.1906	0.9180	0.1723	0.9413	0.2160
$n = 800$	$\hat{\alpha}_1$	0.3007	0.0413	0.3019	0.0442	0.2998	0.0342	0.2990	0.0367
	$\hat{\alpha}_5$	0.2421	0.0426	0.2381	0.0475	0.2444	0.0404	0.2415	0.0450
	$\hat{\alpha}_8$	0.1727	0.0450	0.1740	0.0509	0.1754	0.0447	0.1762	0.0500
	$\hat{\mu}$	0.8753	0.1425	0.8791	0.1430	0.8907	0.1426	0.8992	0.1431

Table 5: The estimation results of the PEL and the PCLS for Case III.

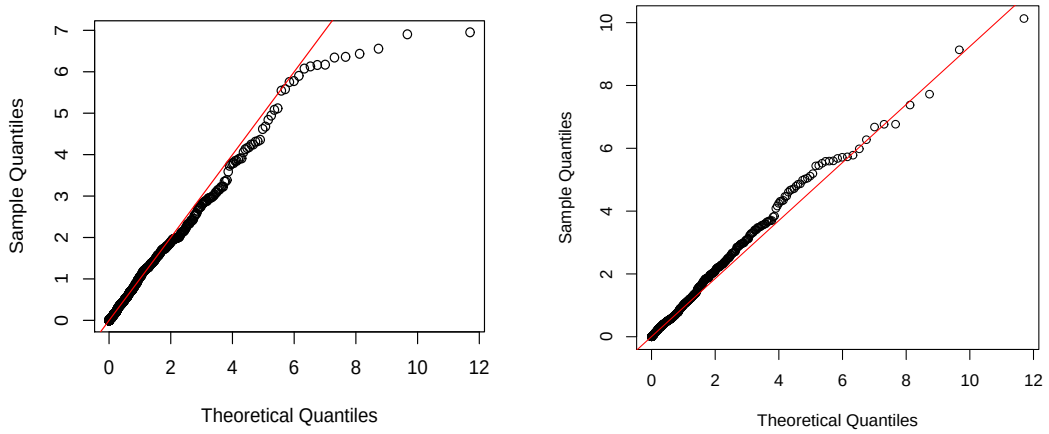
Sample size		ALASSO				SCAD			
		PEL		PCLS		PEL		PCLS	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$n = 300$	$\hat{\alpha}_1$	0.2965	0.0636	0.2834	0.0851	0.2938	0.0603	0.2809	0.0827
	$\hat{\alpha}_5$	0.2311	0.0676	0.2199	0.0916	0.2332	0.0658	0.2225	0.0894
	$\hat{\alpha}_8$	0.1650	0.0733	0.1536	0.0909	0.1694	0.0717	0.1630	0.0879
	$\hat{\mu}$	0.9156	0.2393	0.9879	0.3120	0.9531	0.2517	1.0275	0.3154
$n = 500$	$\hat{\alpha}_1$	0.2998	0.0490	0.2952	0.0600	0.2974	0.0439	0.2930	0.0552
	$\hat{\alpha}_5$	0.2362	0.0543	0.2340	0.0621	0.2375	0.0537	0.2357	0.0619
	$\hat{\alpha}_8$	0.1695	0.0595	0.1666	0.0636	0.1716	0.0584	0.1697	0.0634
	$\hat{\mu}$	0.8881	0.1744	0.9172	0.2032	0.9196	0.1915	0.9455	0.2119
$n = 800$	$\hat{\alpha}_1$	0.2997	0.0417	0.2996	0.0435	0.2985	0.0356	0.2986	0.0358
	$\hat{\alpha}_5$	0.2392	0.0428	0.2414	0.0485	0.2431	0.0398	0.2425	0.0449
	$\hat{\alpha}_8$	0.1746	0.0447	0.1768	0.0492	0.1752	0.0450	0.1761	0.0488
	$\hat{\mu}$	0.8805	0.1208	0.8721	0.1368	0.8982	0.1380	0.8949	0.1497

Table 6: The estimation results of the PEL and the PCLS for Case IV.

Sample size		ALASSO				SCAD			
		PEL		PCLS		PEL		PCLS	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
$n = 300$	$\hat{\alpha}_1$	0.2917	0.0683	0.2845	0.0865	0.2889	0.0646	0.2796	0.0850
	$\hat{\alpha}_5$	0.2317	0.0724	0.2213	0.0921	0.2334	0.0702	0.2240	0.0872
	$\hat{\alpha}_8$	0.1613	0.0721	0.1599	0.0897	0.1661	0.0706	0.1678	0.0878
	$\hat{\mu}$	0.9281	0.2406	0.9613	0.3144	0.9603	0.2489	1.0251	0.3278
$n = 500$	$\hat{\alpha}_1$	0.2993	0.0477	0.2942	0.0583	0.2962	0.0439	0.2915	0.0536
	$\hat{\alpha}_5$	0.2392	0.0526	0.2342	0.0627	0.2414	0.0502	0.2371	0.0620
	$\hat{\alpha}_8$	0.1722	0.0574	0.1688	0.0642	0.1743	0.0586	0.1726	0.0635
	$\hat{\mu}$	0.8729	0.1646	0.9081	0.2122	0.9060	0.1792	0.9349	0.2165
$n = 800$	$\hat{\alpha}_1$	0.3028	0.0413	0.2999	0.0442	0.3000	0.0345	0.2971	0.0377
	$\hat{\alpha}_5$	0.2401	0.0432	0.2403	0.0460	0.2436	0.0401	0.2423	0.0435
	$\hat{\alpha}_8$	0.1740	0.0446	0.1733	0.0487	0.1749	0.0439	0.1757	0.0491
	$\hat{\mu}$	0.8782	0.1283	0.8858	0.1396	0.8933	0.1396	0.9061	0.1549

Table 7: Empirical percentages of rejecting different H_0 derived by (7).

Case	n	Penalty function	-0.02	-0.01	0(Size)	0.01	0.02
I	300	ALASSO	0.730	0.543	0.070	0.571	0.743
		SCAD	0.718	0.546	0.084	0.562	0.732
	500	ALASSO	0.852	0.718	0.066	0.730	0.842
		SCAD	0.847	0.727	0.059	0.718	0.833
	800	ALASSO	0.932	0.827	0.050	0.851	0.917
		SCAD	0.930	0.810	0.057	0.842	0.921
II	300	ALASSO	0.715	0.523	0.082	0.545	0.722
		SCAD	0.698	0.506	0.090	0.527	0.715
	500	ALASSO	0.822	0.724	0.061	0.732	0.836
		SCAD	0.817	0.707	0.055	0.718	0.829
	800	ALASSO	0.912	0.837	0.048	0.840	0.928
		SCAD	0.907	0.821	0.053	0.832	0.925
III	300	ALASSO	0.742	0.572	0.078	0.562	0.751
		SCAD	0.728	0.561	0.085	0.557	0.736
	500	ALASSO	0.841	0.742	0.059	0.752	0.855
		SCAD	0.835	0.733	0.066	0.748	0.849
	800	ALASSO	0.927	0.851	0.051	0.867	0.913
		SCAD	0.917	0.857	0.055	0.854	0.905
IV	300	ALASSO	0.738	0.565	0.072	0.581	0.752
		SCAD	0.718	0.557	0.077	0.577	0.739
	500	ALASSO	0.832	0.756	0.061	0.762	0.846
		SCAD	0.829	0.742	0.057	0.751	0.831
	800	ALASSO	0.935	0.862	0.051	0.853	0.933
		SCAD	0.930	0.855	0.054	0.847	0.927



(a) ALASSO (b) SCAD

Figure 1: QQ plots of the PEL ratio statistics against χ^2_1 distribution under null hypothesis H_0 for Case I ($n = 800$).