Derivation of Oil Film Gap Equations

The following is the full derivation method for the equations presented in the article ”Experimental Measurements of Oil Films in a Dynamically Loaded Journal Bearing”. These equations allow the calculation of film thickness at any point around a bearing circumference given only minimum film thickness, attitude angle and film thickness at any another known angle. Note that values for clearance, bearing radius or shaft radius are not required. Also, this method assumes the geometries are perfectly circular, with no out-of-roundness due to deformation or machining tolerances

First, the distance between two eccentric circles at a given angle around the circumference may be approximated by the following sine function:

$$f(\phi) = A\ sin(\phi + k) + b \tag{1}$$

Where A is amplitude, ϕ is angle, k is x-offset and b is y-offset. This approximation is only applicable when $R_{shaft} >> C$, which is the case in practically all journal bearings. The accuracy of this approximation will be evaluated later in a worked example.

At this point A, k and b are unknown, therefore boundary conditions are required to obtain a solution. In this derivation minimum film thickness, attitude angle and film thickness at bearing top will be defined as known, which matches the directly measurable values from the BETTY test platform. These boundary conditions can be expressed mathematically as:

$$f(\theta) = h_{min} = A\ sin(\theta + k) + b \tag{2}$$

$$f(\phi_{top}) = h_{top} = A\ sin(\phi_{top} + k) + b \tag{3}$$

Where θ is attitude angle, h_{min} is minimum film thickness and h_{top} is film thickness at the top of the bearing.

The gradient at the point of minimum film is zero. Thus, taking the derivative of Equation 1 provides an additional boundary condition:

$$f'(\theta) = 0 = A\ cos(\theta + k) \tag{4}$$

Equation 4 may be rearranged to make k the subject:

$$0 = A\ cos(\theta + k) = cos(\theta + k)$$

$$\frac{(2n+1)\pi}{2} = \theta + k$$

$$k = \frac{(2n+1)\pi}{2} - \theta \tag{5}$$

This shows k has multiple solutions, with two solutions per complete revolution. This corresponds to one solution at minimum film and one at maximum film. The correct n value for a given case depends on how the axes are defined.

For journal bearings rotating in one direction, the attitude angle would not change by any more than $\pi/2$ radians. Thus, defining 0 radians as opposite to the loading vector and increasing in a clockwise direction means the minimum film will always occur at a greater angle than maximum film if the shaft is rotating clockwise, so n should equal 1. Conversely, if the shaft is rotating anti-clockwise minimum film angle will always be smaller than maximum film angle, so n should equal 0. Besides, using the incorrect value for n would result in a nonsensical result, making the mistake obvious.

Equation 5 can be substituted into Equation 2 to obtain:

$$h_{min} = A\ sin\left(\theta + \frac{(2n+1)\pi}{2} - \theta\right) + b$$

$$h_{min} = A\ sin\left(\frac{(2n+1)\pi}{2}\right) + b \tag{6}$$

Which can be rearranged to make b the subject:

$$b = h_{min} - A\ sin\left(\frac{(2n+1)\pi}{2}\right) \tag{7}$$

Similarly, Equation 5 can be substituted into Equation 3 to obtain:

$$h_{top} = A\ sin\left(\phi_{top} + \frac{(2n+1)\pi}{2} - \theta\right) + b \tag{8}$$

Substituting 7 into 8 gives:

$$h_{top} = A\ sin\left(\phi_{top} + \frac{(2n+1)\pi}{2} - \theta\right) + h_{min} - A\ sin\left(\frac{(2n+1)\pi}{2}\right) \tag{9}$$

This can be rearranged to make A the subject:

$$A = \frac{h_{top} - h_{min}}{sin\left(\phi_{top} + \frac{(2n+1)\pi}{2} - \theta\right) - sin\left(\frac{(2n+1)\pi}{2}\right)} \tag{10}$$

Worked Example

The following worked example is supplied to support understanding of the method and to demonstrate its accuracy. This example uses values typical to the BETTY test platform presented in the article.

The system geometry, along with values for minimum film thickness, attitude angle and film thickness at the top of the bearing, is shown in Figure 1. These values can be applied to Equations 5, 7 and 10 to obtain coefficients k, b and A respectively:

$$\begin{gathered} k = \frac{(2\times 1+1)\pi}{2} + \frac{5\pi}{4} = 0.7854 \\ b = 5\times 10^{-6} - 4.511\times 10^{-5} sin\left(\frac{(2\times 1+1)\pi}{2}\right) = 5.011\times 10^{-5} \\ A = \frac{82\times 10^{-6} - 5\times 10^{-6}}{sin\left(0 + \frac{(2\times 1+1)\pi}{2} - \frac{5\pi}{4}\right) - sin\left(\frac{(2\times 1+1)\pi}{2}\right)} = 4.511\times 10^{-5} \end{gathered} \tag{11}$$

Note that in this example minimum film occurs after the point of maximum film, therefore n is equal to 1, rather than 0. Coefficients k, b and A can now be applied to Equation 1:

$$f(\phi) = 4.511\times 10^{-5} sin(\phi + 0.7854) + 5.011\times 10^{-5} \tag{12}$$

This equation can now be used to find the film thickness at any angle around the bearing circumference. This is shown graphically in Figure 2. As previously discussed this method only provides an approximation, albeit a very accurate one. For comparison, the true circumferential film thickness is also plotted, along with the percentage error of the method. The reason for this difference is because Equation 1 is actually calculating the length of the line between the shaft and bearing which is normal to the bearing at a given angle, not the shortest distance between the bearing and the shaft at that angle. However, when $R_{shaft} >> C$ these values are practically identical.

These results can also be used to calculate radial clearance by averaging the minimum and maximum film thickness located on opposing sides of the bearing. In this example radial clearance is:

$$C = \frac{h_{min} + h_{max}}{2} = \frac{5 + 95.211...}{2} = 50.1055...\mu m \tag{13}$$

As true clearance is 50.1 μm, the error is 5.50 nm (0.011%) which can generally be considered negligible.

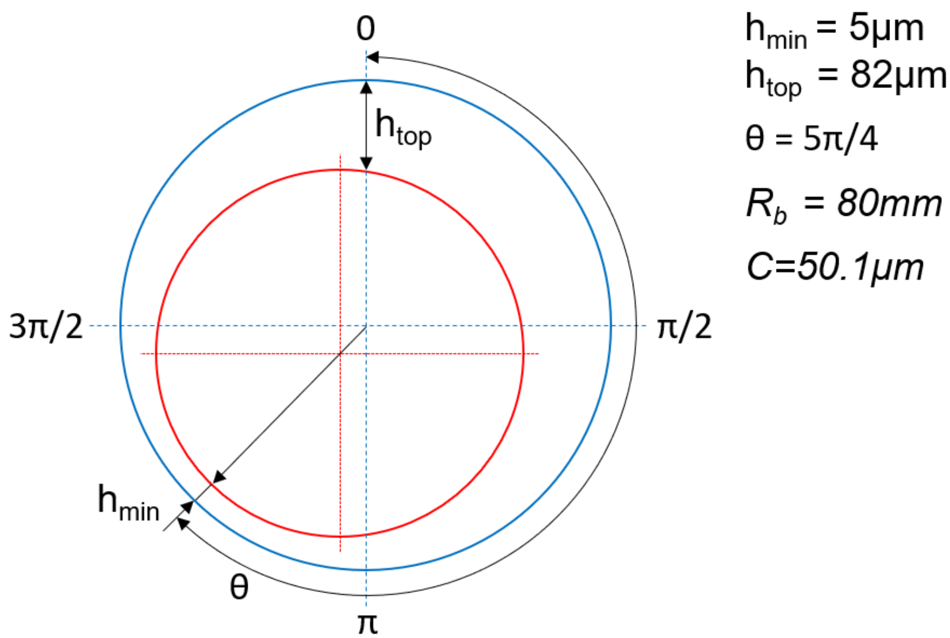

Fig. 1: Shaft-bearing geometry used in worked example. Geometry is intentionally not to scale for clarity.

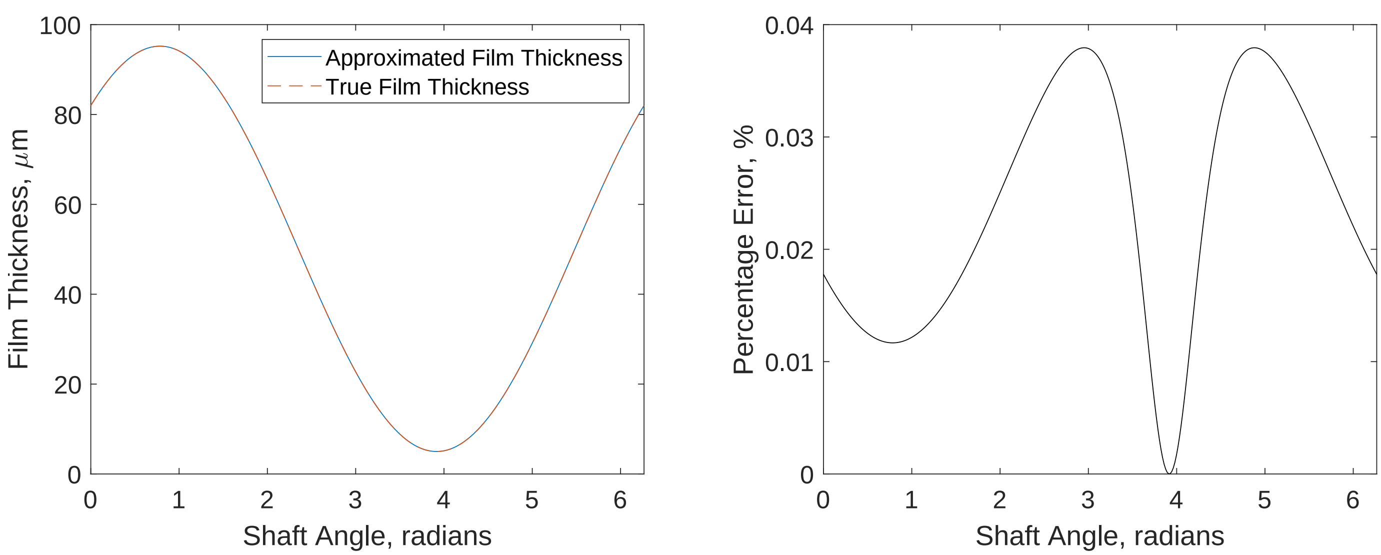

Fig. 2: Circumferential film thickness using approximation method and from true geometry (left). Percentage error in approximation method around bearing circumference (right).