**Supplemental File**

The relevant mathematical relations

We used multiple linear regression to analyze whether there is a linear relationship between root absorption length and the optimal intrusion force interval, root absorption length and the optimal intrusion force median, root absorption length, and the area percentage of the extreme stress zone. All the analysis processes are similar, and we take the data diagram of the root resorption length and the optimal intrusion force median as an example to illustrate.

Our aim was to analyze whether the fitting line of the optimal intrusion force median for root resorption below 6mm (0-6mm) was the same as that for root resorption above 6mm (7-10mm).

It is assumed that the fitting line of the optimal intrusion force median is the same when the root absorption length is less than 6mm (0mm-6mm) and more than 6mm (7mm-10mm). The optimal intrusion force median() and the root absorption length () can be described by a linear mathematical model described in：

where **, , ,** are partial regression coefficient, is a new independent variable calculated by:

We make a hypothesis test on whether is equal to 0. According to the definition and content of multiple linear regression, when the tests statistic of is significantly different (P<0.05), it can be explained the fitting line of the optimal intrusion force median is not the same when the root absorption length is less than 6mm (0mm-6mm) and more than 6mm (7mm-10mm). The result explains the optimal intrusion force median has increased significantly when the root resorption length is above 6 mm.

As all the stress-strain curves graphs were similar, we illustrated the 0mm root resorption curves as an example (Appendix Figure 5). There were five curves in the graph: extreme stress area, low stress area, effective stress area, low strain area, and effective strain area. Their mathematical expressions were set as , , , , and . The represented intrusion force value.

We selected the lower bound of the optimal intrusion force according to the following two principles:

1) The percentage of the effective strain area to the PDL total area was as large as possible (at least 40% - 60%) (Li 2014; Wu J-l et al. 2018; Wu J et al. 2019).

2) There were obvious points to select lower bound.

The percentage of the effective strain area to the PDL total area（）could be calculated by the following formula:

When x = , represented the effective strain area and represented low strain area.

The first intersection point was obtained by the effective strain area and the low strain area curves. At this point, =, =50%, which was in line with our principles. In the other curves (1-10mm root resorption), =50% could also be calculated.

We selected the upper bound of the optimal intrusion force according to the following two principles:

1) The percentage of the extreme stress area to the PDL total area was as small as possible (less than 30%) (Wu J-l et al. 2018; Wu J et al. 2019).

2) There were obvious points to select upper bound.

The percentage of the extreme stress area to the PDL total area（）could be calculated by the following formula:

When x=, , and represented the extreme stress area, low stress area, and effective stress area, respectively.

The second intersection point was obtained by the low stress area and effective stress area curves. At the second intersection point, =, =3.2%, which was in line with our principles. In the other curves (1-10mm root resorption), could also be calculated as the Appendix Table 2. Almost all the results are in line with our principles.

Comparative verification test of PDL materials

To study the influence of PDL linear elastic materials and nonlinear materials on this study, we selected three nonlinear materials and one classical linear elastic material to test, respectively.

In the upper incisor model with 0 mm horizontal root resorption, four kinds of PDL modeling materials were used without changing other materials. All the tests used the same loading and boundary conditions.

The results of the calculation were shown below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Study | Constitutive model | The calculated  optimal intrusion force | | | Results of animal and human studies |
| The lower bound | The upper bound | The median |
| Hemanth et al (Raghuveer et al. 2015) | Nonlinear | 73.3g | 128.5g | 100.9g | 25g (Steigman and Michaeli 1981; McFadden et al. 1989) |
| Liao et al (Liao et al. 2016)  Zhong et al (Zhong et al. 2019) | Hyperelastic | 5.3g | >150g | >77.7g |
| Wu et al (Wu J-l et al. 2018) | Hyperelastic | 22.8g | 38.3g | 30.55g |
| Our study | Linear-elastic | 22.1g | 28.8g | 25.4g |

The results showed that the results of linear elastic model were closer to the results of animal and human studies, and the results of the nonlinear model were not.

**References**

Li Z. 2014. Oral orthodontic force simulation and tooth movement modulation research. MS Thesis, Harbin Institute of Technology, Harbin, China (in Chinese).

Liao Z, Chen J, Li W, Darendeliler MA, Swain M, Li Q. 2016. Biomechanical investigation into the role of the periodontal ligament in optimising orthodontic force: a finite element case study. Archives of oral biology. 66:98-107.

McFadden WM, Engstrom C, Engstrom H, Anholm JM. 1989. A study of the relationship between incisor intrusion and root shortening. American Journal of Orthodontics and Dentofacial Orthopedics. 96(5):390-396.

Raghuveer H, Hemanth M, Rani M, Hegde C, Vedavathi B, Chaithra D. 2015. An Analysis of the Stress Induced in the Periodontal Ligament during Extrusion and Rotation Movements: A Finite Element Method Linear Study Part I. The Journal of Contemporary Dental Practice. 16(9):740-743.

Steigman S, Michaeli Y. 1981. Experimental intrusion of rat incisors with continuous loads of varying magnitude. American journal of orthodontics. 80(4):429-436.

Wu J-l, Liu Y-f, Peng W, Dong H-y, Zhang J-x. 2018. A biomechanical case study on the optimal orthodontic force on the maxillary canine tooth based on finite element analysis. Journal of Zhejiang University-SCIENCE B. 19(7):535-546.

Wu J, Liu Y, Wang D, Zhang J, Dong X, Jiang X, Xu X. 2019. Investigation of effective intrusion and extrusion force for maxillary canine using finite element analysis. Computer methods in biomechanics and biomedical engineering. 22(16):1294-1302.

Zhong J, Chen J, Weinkamer R, Darendeliler MA, Swain MV, Sue A, Zheng K, Li Q. 2019. In vivo effects of different orthodontic loading on root resorption and correlation with mechanobiological stimulus in periodontal ligament. Journal of the Royal Society Interface. 16(154):20190108.